Currents in a chain of quantum dots

K. Morawetz[1,2,3]

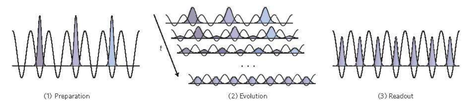
FH MÜNSTER University of Applied Sciences

[1] Münster University of Applied Sciences, Stegerwaldstrasse 39, 48565 Steinfurt, Germany
[2] International Institute of Physics (IIP), Av. Odilon Gomes de Lima 1722, 59078-400 Natal, Brazil
[3] Max-Planck-Institute for the Physics of Complex Systems, 01187 Dresden, Germany

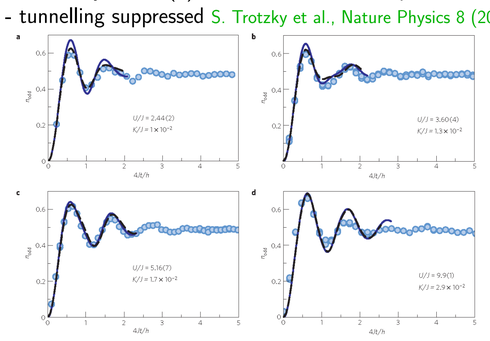
International Institute of Physics, Federal University of Rio Grande do Norte

MAX-PLANCK-GESELLSCHAFT

Optical superlattice each second place filled



(1) prepared density wave, (2) reduction of lattice depth - tunnelling, (3) read-out - tunnelling suppressed S. Trotzky et al., Nature Physics 8 (2012) 325

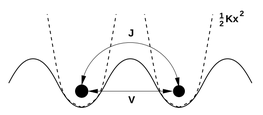


Odd-site population (circles), ensemble-averaged t-DMRG (line), next-nearest neighbour hopping (dashed)

Kinetic theory: model

$$\hat{H} = \sum_p \epsilon_p \hat{c}_p^+ \hat{c}_p + \frac{1}{2}\sum_{p,k,q} V_q \hat{c}_{p+q}^+ \hat{c}_{k-q}^+ \hat{c}_k \hat{c}_p - \frac{K}{2}\sum_p \partial_p^2 \hat{c}_p^+ \hat{c}_p$$

with nearest-neighbor hopping $\epsilon_p = \sum_n e^{-ip(n-n')} H(n-n') = -2J\cos p$, and confining harmonic trap



linearized kinetic equation (Heisenberg equation)

$$\left(s+\frac{1}{\tau}+i\Delta\epsilon\right)\delta f_s - \delta f_0 = i\Delta f \delta V_t^{\rm ext} + i\Delta f V_q \delta n_t + \frac{\Delta f}{\Delta\epsilon}\frac{\delta n_t}{\tau\Pi(q,0,t)} + iK\partial_p\partial_q\delta f_s$$

short-hand $\Delta f = f_0(p+\frac{q}{2}) - f_0(p-\frac{q}{2})$ and $\Delta\epsilon = \epsilon_{p+\frac{q}{2}} - \epsilon_{p-\frac{q}{2}}$

- local equilibrium with variation of chemical potential such that density conserves $n = \sum_p f = \sum_p f^{\rm l.e.}$ N. Mermin, PRB 1 (1970) 2362

more conservations K. Morawetz, Phys. Rev. E 88 (2013) 022148

No time for correlations to be built up

- Linearize kinetic equation to obtain density response

$$\delta n(q,t) = \int_{t_0}^{t} dt' \chi(t,t') V_q^{\rm ext}(t')$$

- one-particle reduced density matrix $\hat{f}$ obeys

$$\dot{\hat{f}} + i[\hat{\mathcal{E}} + \hat{V}^{\rm ind} + \hat{V}^{\rm ext}, \hat{f}] = \frac{\hat{f}^{\rm l.e.} - \hat{f}}{\tau}$$

- linearization leads to analytic solution K. Morawetz, et al., Phys. Rev. B 72 (2005) 233203

$$\frac{1}{\epsilon(\omega,t)} = 1 + \frac{e^2}{\epsilon_0 q^2}\int_0^{t-t_0} dT e^{i\omega T}\chi(t,t-T)$$
$$= 1 - \frac{\omega_p^2}{\sqrt{\omega_p{}^2 - \frac{1}{4\tau^2}}}\int_0^{t-t_0} dt'\, e^{(i\omega-\frac{1}{2\tau})t'} \sin\left(\sqrt{\omega_p{}^2 - \frac{1}{4\tau^2}}\, t'\right)$$

Achievement: long-time limit yields the Drude formula

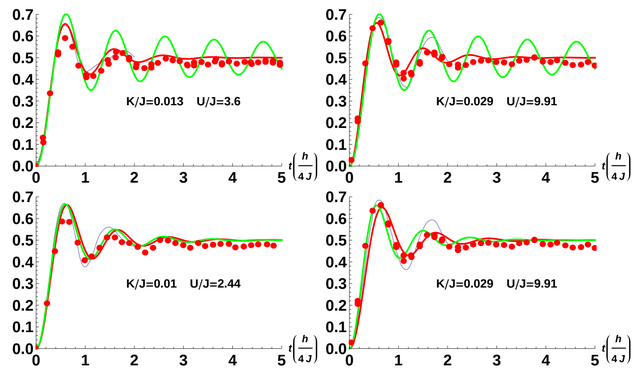
$$\lim_{t\to\infty}\frac{1}{\epsilon} = 1 - \frac{\omega_p{}^2}{\omega_p{}^2 - \omega(\omega + \frac{i}{\tau})}$$

not easy to achieve within short-time expansions, e.g. [ElSayed et al, PRB 49 (1994) 7337] gives the long-time limit of the form $1 - \frac{\omega_p^2}{\omega_p^2 - (\omega+\frac{i}{\tau})^2}$

Density repsonse: sudden quench

with $\gamma^2 = nbV + b^2 - 1/4\tau^2$ and $b = 4J\sin qa/2$

$$\delta n_t = -\frac{n}{2}J_0(\sqrt{4Jbt})e^{\frac{-t}{\tau}} - \frac{n}{4\gamma\tau^2}\int_0^t dx J_0(\sqrt{4Jbx})e^{\frac{-(t+x)}{2\tau}}\left(2\gamma\tau\cos\gamma(t-x) + (1-2bnV_q\tau^2)\sin\gamma(t-x)\right)$$



Comparison of the experimental data of S. Trotzky et al., Nature Physics 8 (2012) 325 (dots) with the RG calculation of A. Flesch et al., Phys. Rev. A78 (2008) 033608 (thin line). Left: Mermin's correction of conserving relaxation time $\tau = 0.6\hbar/J$ approximation without (green) and with interaction (red). Right: With (red) and without (green) the influence of the trapping potential K/J.

Free chains in electric field

1D tight-binding Hamiltonian with lattice distance a

$$\hat{H} = \sum_{nn'} H(n-n')|n\rangle\langle n|' - ea\sum_n nE_n(t)|n\rangle\langle n|$$

time-dependent Schrödinger equation solved by $|\Psi\rangle = \sum_n c_n|n\rangle$ with homogeneous field $E(t)$ and Fourier transformed

$$i\hbar\partial_t c_p = \epsilon_p c_p - ieE(t)a\partial_p c_p$$

exact solution $c_p(t) = c^0\left[p - \frac{a}{\hbar}\int_0^t eE(\bar{t})d\bar{t}\right] e^{-\frac{i}{\hbar}\int_0^t d\bar{t}\,\epsilon\left[p+\frac{a}{\hbar}\int_t^{\bar{t}} eE(t')dt'\right]}$

mean quantum mechanical current

$$\dot{x} = \langle\Psi|\frac{i}{\hbar}[\hat{H},\hat{x}]|\Psi\rangle = 2\frac{a}{\hbar}J\int_0^{2\pi}\frac{dp}{2\pi}c_p^+ c_p \sin p$$

many-body averaging: dots with a spatial distribution $n\sum_{l=-N}^{N}\delta(r-la)$ which Fourier transforms $f_p = n\sum_{l=-N}^{N} e^{-ilp} = n\frac{\sin(N+1/2)p}{\sin(p/2)}$

Bloch oscillations $\langle\dot{x}\rangle = 2nJ\frac{a}{\hbar}\sin\left(e\frac{a}{\hbar}Et\right)$

Currents

$\delta j_q = \delta j_q^{\rm init} + \delta j_q^c$, initial correlations

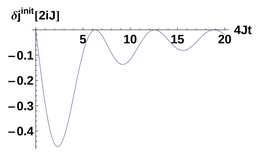
$$\delta j_q^{\rm init} = -2iJ\frac{a}{\hbar}\delta f_0\left[Q_S^s + Q\Pi_S^s\frac{V_q - \frac{1}{\tau S\Pi_0}}{1 - V_q\Pi_S - \frac{1}{\tau S}\left(1-\frac{\Pi_S}{\Pi_0}\right)}\right]$$

$$\delta j_q^c = 2J\frac{a}{\hbar}U_q\left[\Pi_S^s + \Pi\,\Pi_S^s\frac{V_q - \frac{1}{\tau S\Pi_0}}{1 - V_q\Pi_S - \frac{1}{\tau S}\left(1-\frac{\Pi_S}{\Pi_0}\right)}\right]$$

with $b = 4J\sin^2 q/2$

$$\Pi_S = \int_0^{2\pi}\frac{dp}{2\pi}\frac{\Delta f}{\Delta\epsilon - iS}, \quad Q_S = \int_0^{2\pi}\frac{dp}{2\pi}\frac{1}{\Delta\epsilon - iS} = \frac{i}{\sqrt{S^2+4Jb}} \bullet\!\!-\!\!\circ\; iJ_0(\sqrt{4Jbt})e^{-\frac{t}{\tau}}$$

$$\Pi_S^s = \int_0^{2\pi}\frac{dp}{2\pi}\sin p\frac{\Delta f}{\Delta\epsilon - iS}, \quad Q_S^s = \int_0^{2\pi}\frac{dp}{2\pi}\frac{\sin p}{\Delta\epsilon - iS} = \frac{\sqrt{4Jb}}{\sqrt{S^2+4Jb}(\sqrt{S^2+4Jb}+S)} \bullet\!\!-\!\!\circ\; J_1(\sqrt{4Jbt})e^{-\frac{t}{\tau}}$$



imaginary current to decay initial correlations integrated over the Brillouin zone wave vectors

chain of length D has potential $U_q = \lim_{D\to\infty} a\int_{-D/2a}^{D/2a} dx e^{-iqx}[-eE(t)x] = -i2\pi aeE(t)\partial_q\delta(q)$, inhomogeneous electric field with wavelength $q/\hbar$ possesses potential $U_q = eai\frac{E_q(t)}{q}$

Currents: equivalent RCL circuit

single-mode wavelength $U_q = 2\pi i E_{q_0}\frac{a}{q_0}\delta(q-q_0)$, total charge current represented it in terms of resistance in frequency space

$$R^c = \frac{E_{q_0}}{\delta J^c} = \frac{\frac{1}{\tau} + i\left(\frac{b_{q_0}(b_{q_0}+2nV_{q_0})}{\omega} - \omega\right)}{\alpha}$$

with

1. Ohmic resistance per length $R^{-1} = \tau\alpha = \frac{ne^2\tau}{m}\frac{\sin\frac{q_0}{2}}{\frac{q_0}{2}}$
2. inductance $R_L = -i\omega L$ per length $L^{-1} = \alpha$
3. capacitance $R_C = i/\omega C$ per length $C = \frac{\alpha}{\omega_0^2}$

with eigenfrequency $\tau\to\infty$: $\omega_0^2 = \frac{1}{LC} = (b_{q_0} + 2nV_{q_0})b_{q_0}$

in terms of single quantum parameter

$$\alpha = 2e^2 J\frac{a^2 n}{\hbar^2}\frac{\sin\frac{q_0}{2}}{q_0/2} = \frac{ne^2}{m}\frac{\sin\frac{q_0}{2}}{\frac{q_0}{2}}$$

for the effective mass for free particles $m = \hbar^2/2Ja^2$

Pair correlation function

in terms of the response function $\beta = 1/T$

$$S_q = -\frac{1}{n}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{{\rm Im}\chi}{1-e^{-\beta\omega}} = {\rm Im}\frac{ib_q}{\pi}\int_{\frac{1}{\tau}-i\infty}^{\frac{1}{\tau}+i\infty} ds\frac{1}{(1-e^{-i\beta(s-\frac{1}{\tau})})(b_q^2+nb_qV+s^2-\frac{s}{\tau})}$$
$$= {\rm Im}\frac{b_q}{2\pi\gamma_q}\left[\Psi\left(-\frac{\beta}{4\pi\tau}+\frac{i\beta\gamma_q}{2\pi}\right) - \Psi\left(-\frac{\beta}{4\pi\tau}-\frac{i\beta\gamma_q}{2\pi}\right) - 2\pi i\frac{\sinh\beta\gamma_q}{\cosh\beta\gamma_q - \cos\frac{\beta}{2\tau}}\right]$$

with the DiGamma function $\Psi(x) = \Gamma'/\Gamma$, zero-temperature result

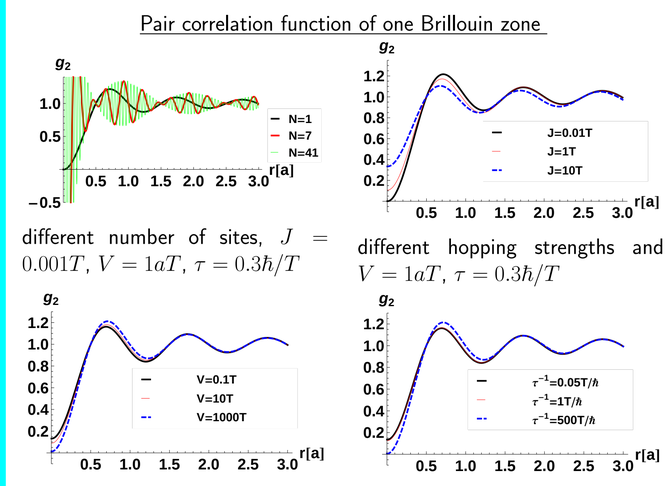
$$S_q(T=0) = {\rm Im}\frac{b_q}{2\pi\gamma_q}\ln\frac{1+2i\gamma_q\tau}{1-2i\gamma_q\tau}$$

The pair correlation function

$$g_r = 1 + \frac{1}{2\pi na}\int_{-\infty}^{\infty} dq e^{iqr}(S_q - 1) = 1 + \frac{1}{2\pi na}F_N(r)\int_0^{2\pi} dq\cos(qr)(S_q - 1)$$

where integrate over $2N+1$ sites which results into a structure factor $F_N(r) = \sin[\pi r(2N+1)]/\sin[\pi r]$

Pair correlation function of one Brillouin zone



different number of sites, $J = 0.001T$, $V = 1aT$, $\tau = 0.3\hbar/T$

different hopping strengths and $V = 1aT$, $\tau = 0.3\hbar/T$

different interactions and $J = 1T$, $\tau = 1\hbar/T$

different relaxation times and $J = 1T$, $V = 1aT$

Summary on transport in wires of localized dots

1. Short-time dynamics
   - collisions have no time to happen yet, correlations are formed at transient times due to mean-field fluctuations
   - simple analytic formula for time dependence of the dielectric function and for sudden quench
   - Finite size corrections due to trap and density conservation (Mermin)
2. Dots in a chain
   - quench population dynamics of cold atoms in lattices well described
   - exact analytical expressions show Bloch oscillations and ballistic transport for non-interacting electrons with only hopping
   - including interactions currents are calculated analytically
   - transport in a chain of dots can be represented by equivalent R-C-L circuit in terms of hopping, interaction strength and relaxation time
   - decay of initial correlations is realized by virtual current
   - analytic expression for pair correlation function:
     - effect of hopping counter-acts effect of interactions and collisions
     - higher collective temperature sharpens feature of 1. Brillouin zone
     - number of dots leads to structure factor inside pair-correlation function which modulates 1. Brillouin-zone, maxims at places of dots

References:

Phys. Rev. B 72 (2006) 233203 *Femtosecond formation of collective modes due to mean-field fluctuations*

Phys. Rev. B 90 (2014) 075303 *Universal short-time response and formation of correlations after quantum quenches*

EPJB in press *Quantum currents and pair correlation of electrons in a chain of localized dots*