Anomalous transport in graphene and Weyl semimetals

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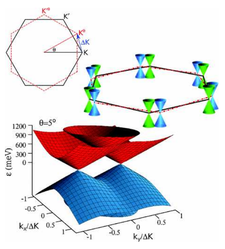
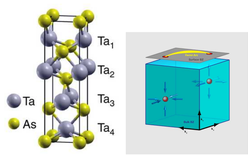
Graphene



C. J. Tabert, E. J. Nicol, PRB 87 (2013) 121402(R)

Weyl semimetals



S.-Y. Xu, et al., Science 349 (2015) 613

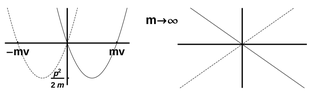
Theoretical Trick

- linear spin-orbit coupling $\vec{b} = v(p_x, p_y)$ leads to splitting

$$\epsilon_\pm = \frac{p^2}{2m} \pm vp$$

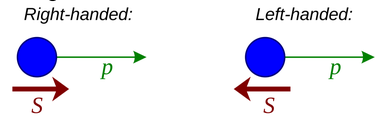
PRB 92 (2015) 245425, 245426

- Dirac dispersion by $m \to \infty$



Lindhard polarization vanishes !

Chiral anomaly



- parallel electric and magnetic field changes chirality
- Fermi momentum of right-h. Fermions increases in electric field $p_f = eEt$
- density is product of longitudinal phase-space density, $\frac{dN_R}{dz} = \frac{p_f}{2\pi\hbar}$

and density of Landau levels in traverse direction, $\frac{d^2N_R}{dxdy} = \frac{eB}{2\pi\hbar}$

- rate of chirality $\frac{dn_5}{dt} = \frac{d^4(N_R - N_L)}{dtd^3x} = 2\frac{\hat{p}_f}{2\pi\hbar}\frac{eB}{2\pi\hbar} = \frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}$
- term $\mathbf{EB}$ is considered as the origin of non-conservation of chiral charge

Chiral symmetry

- left/right-handed projection realized $(1 \mp \gamma_5)/2$ with $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
- chiral or axial transformation $\Psi'(x) = e^{i\alpha(x)\gamma_5}\Psi(x)$ leads to axial current $J_5 = \overline{\Psi}\gamma^\mu\gamma^5\Psi$
- classical action $S' = S + \int \alpha(x)\nabla_\mu J_5^\mu$, from Dirac equation

$$\nabla_\mu J_5^\mu = 2im\overline{\Psi}\gamma^5\Psi \to 0, \quad \text{for} \quad m \to 0$$

- quantum result $\langle\partial_\mu J_5^\mu\rangle = 2im\langle\overline{\Psi}\gamma^5\Psi\rangle \to \frac{e^2}{16\pi^2\hbar^2 c}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$ for $m \to 0$

due to divergences up to fourth adiabatic order (renormalization), anomalous triangle graphs Bardeen, Adler, Parker...

Chiral kinetic theory

two-bands kinetic equation $\dot{f}_\pm + \dot{\mathbf{r}}\,\partial_r f_\pm + \dot{\mathbf{k}}\,\partial_k f_\pm = I_\pm$.
with trajectories of mean-field particles

$$\dot{\mathbf{r}} = \mathbf{v} + \dot{\mathbf{k}} \times \mathbf{\Omega}_\pm, \qquad \dot{\mathbf{k}} = e\mathbf{E} + \dot{\mathbf{r}} \times e\mathbf{B}$$

with Berry curvature term $\mathbf{\Omega}_\pm = \partial_k \times i\langle\pm|\partial_k|\pm\rangle$
Haldane, Son, Stephanov, Yin, Yamamoto, Manuel, Torres-Rincon, Spivak...
disentangled

$$\dot{\mathbf{r}} = \frac{\mathbf{v} + e\mathbf{E}\times\mathbf{\Omega}_\pm + e\mathbf{B}\mathbf{v}\cdot\mathbf{\Omega}_\pm}{1 + e\mathbf{B}\cdot\mathbf{\Omega}_\pm}, \qquad \dot{\mathbf{k}} = \frac{e\mathbf{E} + \mathbf{v}\times e\mathbf{B} + e^2\mathbf{\Omega}_\pm\mathbf{E}\cdot\mathbf{B}}{1 + e\mathbf{B}\cdot\mathbf{\Omega}_\pm}$$

Consequence nontrivial currents and non-conserving chiral anomaly

$$\dot{n}_\pm + \frac{\partial}{\partial r}\int\frac{d\mathbf{k}}{(2\pi\hbar)^3}\left[v_k \pm (e\times\Omega_\pm) \pm e\mathbf{B}(\mathbf{v}_k\cdot\mathbf{\Omega}_\pm)\right] f_\pm = \frac{e^2}{4}\mathbf{E}\cdot\mathbf{B}\int\frac{d\mathbf{k}}{(2\pi\hbar)^3}\mathbf{\Omega}_\pm\cdot(f_+ - f_-)$$

- ? claim to be chiral anomal of field theory like Bell-Adler-Jackiw ?
- ?? quantum effect violates conservation laws??
- ??? Lorentz invariance broken, gravitation anomaly, CPT violation???

Violation of conservation laws? Critics

Consequence nontrivial currents and non-conserving chiral anomaly

$$\frac{\partial}{\partial t}n_\pm + \nabla(\mathbf{j}_{\rm qp} \pm \mathbf{j}_\Omega) = k\,\mathbf{E}\cdot\mathbf{B} \equiv -\nabla\cdot\mathbf{j}_{\rm anom}$$

since $\nabla(\Phi\nabla\times\mathbf{A}) = \nabla\Phi\cdot(\nabla\times\mathbf{A})$ we have
$\mathbf{B}\cdot\mathbf{E} = -(\nabla\times\mathbf{A})\cdot(\dot{\mathbf{A}} + \nabla\Phi) = -(\nabla\times\mathbf{A})\cdot\dot{\mathbf{A}} - \nabla(\Phi\nabla\times\mathbf{A})$ for any gauge
standard relation $(\nabla\times\mathbf{A})\cdot\dot{\mathbf{A}} = -\nabla\cdot(\dot{\mathbf{A}}\times\mathbf{A}) + \mathbf{A}\cdot(\nabla\times\dot{\mathbf{A}}) = -\nabla\cdot(\dot{\mathbf{A}}\times\mathbf{A}) - \dot{\mathbf{A}}\cdot(\nabla\times\mathbf{A})$ follows $(\nabla\times\mathbf{A})\cdot\dot{\mathbf{A}} = -\frac{1}{2}\nabla\cdot(\dot{\mathbf{A}}\times\mathbf{A})$.
and the anomalous (quantum) current [5]

$$\mathbf{j}_{\rm anom} = k\left(\frac{1}{2}\dot{\mathbf{A}}\times\mathbf{A} - \Phi\nabla\times\mathbf{A}\right)$$

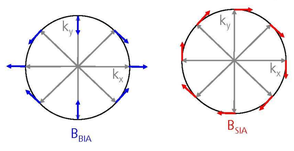
- Expectation: chiral kinetic theory from conserving kinetic theory

Extrinsic vs intrinsic s.o.

intrinsic: c-band (s) coupled to v-bands (v) GaAs/AlGaAs

bulk-inv. asym. (III-V semicond.) Dresselhaus $-k_y\sigma^y + k_x\sigma^x$

structure inversion asym. (macrosc. confining) Rashba $-k_x\sigma^y + k_y\sigma^x$



General form
$H^s = A(\mathbf{k})\sigma_x - B(\mathbf{k})\sigma_y + C(\mathbf{k})\sigma_z = \mathbf{b}\cdot\sigma$

2D − system	$A(k)$	$B(k)$	$C(k)$
Rashba	$\beta_R k_y$	$\beta_R k_x$	
Dresselhaus[001]	$\beta_D k_x$	$\beta_D k_y$	
Dresselhaus[110]	βk_x	$-\beta k_x$	
Rashba − Dresselhaus	$\beta_R k_y - \beta_D k_x$	$\beta_R k_x - \beta_D k_y$	
cubic Rashba(hole)	$i\frac{\beta_R}{2}(k_-^3 - k_+^3)$	$\frac{\beta_R}{2}(k_-^3 + k_+^3)$	
cubic Dresselhaus	$\beta_D k_x k_y^2$	$\beta_D k_y k_x^2$	
Wurtzite type	$(\alpha + \beta k^2)k_y$	$(\alpha + \beta k^2)k_x$	
single − layer graphene	vk_x	$-vk_y$	
bilayer graphene	$\frac{k_+^2 + k_-^2}{4m_e}$	$\frac{k_+^2 - k_-^2}{4m_e i}$	

3D − system	$A(k)$	$B(k)$	$C(k)$
bulk Dresselhaus	$k_x(k_y^2 - k_z^2)$	$k_y(k_x^2 - k_z^2)$	$k_z(k_x^2 - k_y^2)$
Cooperpairs	Δ	0	$\frac{p^2}{2m} - \epsilon_F$
extrinsic $\beta = \frac{i}{\hbar}\lambda^2 V(k)$	$q_y k_z - q_z k_y$	$q_z k_x - q_x k_z$	$q_x k_y - q_y k_x$
neutrons in nuclei $\beta = iW_0(n_n + \frac{n_p}{2})$	$q_z k_y - q_y k_z$	$q_x k_z - q_z k_x$	$q_y k_x - q_x k_y$

Interactions and meanfields

1. Spin-orbit coupling $\sigma\cdot\mathbf{b}(\mathbf{p})$
magn. impurities, intrinsic+ extrinsic

$$\hat{V}_{-p,p'} = \begin{cases} V_0(p' - p) \\ \sigma\cdot\mathbf{V}(p' - p) \\ \frac{i\lambda^2}{\hbar}\sigma\cdot(\mathbf{p}\times\mathbf{p}')V(p' - p) \end{cases}$$

2. Leads to impurity meanfields $\Sigma_0^{\rm imp} = nV_0 + \mathbf{s}\cdot\mathbf{V}$; $\quad \mathbf{\Sigma}^{\rm imp} = \mathbf{s}V_0 + n\mathbf{V}$

Extrinsic spin-orbit coupling meanfield more involved
$\Sigma_0^{\rm ext.} = i\frac{\lambda^2}{\hbar^2}V\left[m(\mathbf{S}_j\times\mathbf{q})_j - \mathbf{s}\cdot(\mathbf{p}\times\mathbf{q})\right]$, $\mathbf{\Sigma}^{\rm ext.} = i\frac{\lambda^2}{\hbar^2}V\left[m(\mathbf{j}\times\mathbf{q}) - n(\mathbf{p}\times\mathbf{q})\right]$
dens. $n = \sum_p f$, curr. $\mathbf{j} = \sum_p \frac{\mathbf{p}}{m}f$, polar. $\mathbf{s} = \sum_p \mathbf{g}$, curr. $S_{ji} = \sum_p \frac{p_j}{m}[\mathbf{g}]_i$

3. Effective (meanfield) Hamiltonian

$$H = \frac{k^2}{2m} + \Sigma_0(\mathbf{k},\mathbf{q},t) + e\Phi(\mathbf{q},t) + \sigma\cdot\mathbf{\Sigma}(\mathbf{k},\mathbf{q},t)$$

with $\mathbf{\Sigma} = \mathbf{\Sigma}_{MF}(\mathbf{k},\mathbf{q},t) + \mathbf{b}(\mathbf{k},t) + \mu\mathbf{B}$

- Search kinetic theory (non-Abelian) $\int\frac{d\omega}{2\pi}G^< = f(\mathbf{k},\mathbf{q},t) + \sigma\cdot\mathbf{g}(\mathbf{k},\mathbf{q},t)$

Kinetic equation with spin-orbit coupling and electric and magnetic fields

$$\boxed{\begin{aligned}(\partial_t + \mathcal{F}\partial_{\mathbf{p}} + \mathbf{v}\partial_{\mathbf{r}})f + \mathbf{A}\cdot\mathbf{g} &= 0 \\ (\partial_t + \mathcal{F}\partial_{\mathbf{p}} + \mathbf{v}\partial_{\mathbf{r}})\mathbf{g} + \mathbf{A}\,f &= 2(\mathbf{\Sigma}\times\mathbf{g})\end{aligned}}$$

coupling of spinor terms $A_i = \partial_{\mathbf{p}}\Sigma_i\partial_{\mathbf{r}} - \partial_{\mathbf{r}}\Sigma_i\partial_{\mathbf{p}} + (\partial_{\mathbf{p}}\Sigma_i\times e\mathbf{B})\partial_{\mathbf{p}}$
with velocity $\mathbf{v} = \frac{\mathbf{k}}{m} + \partial_{\mathbf{k}}\Sigma_0$ and eff. Lorentz force $\mathcal{F} = (e\mathbf{E} - \partial_{\mathbf{r}}\Sigma_0 + e\mathbf{v}\times\mathbf{B})$

Stationary solution: $\hat{\rho}(\hat{\varepsilon}) = \sum_\pm \hat{P}_\pm f_\pm = \frac{f_+ + f_-}{2} + \sigma\cdot\mathbf{e}\,\frac{f_+ - f_-}{2} = f + \sigma\cdot\mathbf{g}$
with effective splitting $f_\pm = f_0(\epsilon_k \pm |\mathbf{\Sigma}|)$
and selfconsistent meanfield $\epsilon_k(r) = \frac{k^2}{2m} + \Sigma_0(k,r)$
and selfconsistent precession $\mathbf{e}(k,r) = \mathbf{\Sigma}/|\mathbf{\Sigma}|$

Transformation to chiral kinetic equation

2 months calculation [4]:

1. in helicity basis $U^+HU = \begin{pmatrix}\epsilon + \Sigma & 0 \\ 0 & \epsilon - \Sigma\end{pmatrix}$ diagonal: $\hat{\rho} = U^+\rho U = \begin{pmatrix} f_{++} & f_{+-} \\ f_{-+} & f_{--}\end{pmatrix}$
2. influence of off-diagonal on diagonal elements, approximately $o(D\mathcal{F})^2$ up to quadratic order in derivatives or forces
3. disentangle diagonal elements $o(D^2, F)$

Results into chiral kinetic equation

$$\left\{\partial_t + \frac{\mathbf{v} \pm e\mathbf{E}\times\mathbf{\Omega} \pm e\mathbf{B}(\mathbf{v}\cdot\mathbf{\Omega})}{1 \pm e\mathbf{B}\cdot\mathbf{\Omega}}\cdot\partial_{\mathbf{r}} + \frac{e\mathbf{E} + v\times e\mathbf{B} \pm \mathbf{\Omega}(e^2\mathbf{E}\times\mathbf{B})}{1 \pm e\mathbf{B}\cdot\mathbf{\Omega}}\cdot\partial_{\mathbf{k}}\right\}f_\pm = 0$$

with Berry curvature by curl of band-diagonal Berry connection

$$\pm\mathbf{\Omega} = i\langle\partial\pm|\times|\partial\pm\rangle = \pm\frac{1}{2\Sigma^3}(\Sigma_x\partial\Sigma_y\times\partial\Sigma_z + \Sigma_z\partial\Sigma_x\times\partial\Sigma_y + \Sigma_y\partial\Sigma_z\times\partial\Sigma_x)$$

and quasiparticle energy $\epsilon_+ = vk - vke\mathbf{B}\cdot\mathbf{\Omega} = vk - \frac{evh\mathbf{B}\cdot\mathbf{k}}{2k^2}$

Chiral anomaly from exact kinetic equation

density balance $\dot{n}_\pm + \nabla(\mathbf{j}_\pm + \mathbf{E}\times\sigma_s) = \xi\frac{e^2}{4\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}$ with ξ Chern number (topological charge), obtain from kinetic equation

$-i\omega\delta(n_+ - n_-) + \frac{i}{\hbar}\mathbf{q}\delta(\mathbf{j}_+ - \mathbf{j}_-) = R(\infty) - R(0)$ with

$$R(k) = \frac{e^2}{2\pi^2\hbar^2}\mathbf{EB}\frac{k^2}{k^2 - \frac{\hbar^2\omega^2}{4v^2}}\int_{-1}^{1}\frac{dx}{2}(x^2 - 1)\,g_0$$

interpret f_- as hole or antiparticle $g_0 = \frac{1}{2}(f_+ - f_-) = \frac{1}{2}(f_+ + \bar{f}_+ - 1)$ with

$$\bar{f}_+(\epsilon_+) = [e^{\frac{\epsilon_+ + \mu}{T}} + 1]^{-1}, \quad f_+(\epsilon_+) = [e^{\frac{\epsilon_+ - \mu}{T}} + 1]^{-1}$$

we obtain $R(0) = \begin{cases} -\frac{2}{3}\frac{e^2}{2\pi^2\hbar^2}\mathbf{EB} & \text{for } \omega = 0 \\ 0 & \text{for } \omega \neq 0 \text{ or } \epsilon = vk \end{cases}$, $\qquad R(\infty) = \frac{1}{3}\frac{e^2}{2\pi^2\hbar^2}\mathbf{EB}$

- static limit agrees with chiral anomaly
- finite frequency leads to $1/3$ or topological charge $\xi = 1/3$

($2/3$ by dynamical part from magnetization current [Kharzeev et al. '17])

- anomalous term with $2/3$ from Dirac monopole but $1/3$ from the Dirac sea $k \to \infty$
- compare: in chiral kinetic theory anomaly term comes exclusively from zero momentum or Dirac monopole

Graphene: anomalous currents

Pauli structure $\hat{H}_{\rm eff} = H + \sigma\cdot\mathbf{\Sigma}$ with $\mathbf{\Sigma} = \mathbf{\Sigma}^H(\mathbf{k},\mathbf{r},t) + \mathbf{b}(\mathbf{k},\mathbf{r},t) + \mu_B\mathbf{B}$
particle and pseudospin current densities read

$$\hat{j}_\alpha = \sum_p[\hat{\rho}, v_\alpha]_+ = 2\sum_p\left[f\partial_{p_\alpha}\epsilon + \mathbf{g}\cdot\partial_{p_\alpha}\mathbf{\Sigma} + \sigma\cdot(\partial_{p_\alpha}\epsilon\mathbf{g} + f\partial_{p_\alpha}\mathbf{\Sigma})\right] = j_\alpha^n + j_\alpha^a + \sigma\cdot\mathbf{S}_\alpha$$

scalar: normal and anomaly current, vector part: pseudospin current S_{ij}
Dirac particles by the limit $\epsilon_\pm = \frac{p^2}{2m} \pm |\mathbf{\Sigma}| \to \pm vp$

- graphene only possess an anomalous particle current
- normal pseudospin current, however, possesses a finite $m \to \infty$ limit (unexpected)

Anomalous particle conductivities: universal limits

anomaly particle current $J_\alpha = (\sigma_{\alpha\beta}^{\rm Hall} + \sigma_{\alpha\beta}^{\rm inter} + \sigma_{\alpha\beta}^{\rm intra})E_\beta$
with $\mathbf{e} = \mathbf{\Sigma}/|\Sigma|$, $g = (f_+ - f_-)/2$

$$\sigma_{\alpha\beta}^{\rm Hall} = 2e^2\sum_p\frac{g}{1 - \frac{\omega^2}{4|\Sigma|^2}}\mathbf{e}\cdot(\partial_{p_\alpha}\mathbf{e}\times\partial_{p_\beta}\mathbf{e}) \to \frac{e^2}{8\pi\hbar}\begin{cases}\frac{\Sigma_n}{\mu} + o(\tau_\omega^{-1}), & \mu > \Sigma_n \\ 1 + o(\tau_\omega^{-1}), & \mu < \Sigma_n \\ \frac{\Sigma_n\tau}{\hbar}\left(\pi - \frac{4\tau\mu}{\hbar} + o(\mu^2)\right), & \mu > \Sigma_n\end{cases}$$

$$\sigma_{\alpha\beta}^{\rm inter} = 2e^2\sum_p\frac{g}{1 - \frac{\omega^2}{4|\Sigma|^2}}\frac{i\omega}{2|\Sigma|}\partial_{p_\alpha}\mathbf{e}\cdot\partial_{p_\beta}\mathbf{e} \to \zeta\frac{e^2}{16\hbar}, \qquad (n \to 0)$$

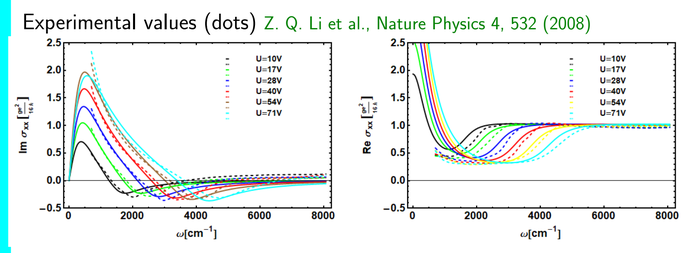
order of limits					$\sigma_{xx}^{\rm inter} = \zeta\frac{e^2}{16\hbar}$
5.	4.	3.	2.	1.	ζ
$\mu \to 0$	$\omega \to 0$	$\Sigma \to 0$	$m \to \infty$	$\tau \to \infty$	-1
$\mu \to 0$	$\tau \to \infty$	$\Sigma \to 0$	$m \to \infty$	$\omega \to 0$	0
$\tau \to \infty$	$\mu \to 0$	$\Sigma \to 0$	$m \to \infty$	$\omega \to 0$	1
$\mu \to 0$	$\Sigma \to 0$	$\tau \to \infty$	$m \to \infty$	$\omega \to 0$	1
$\mu \to 0$	$\tau \to \infty$	$m \to \infty$	$\Sigma \to 0$	$\omega \to 0$	1
$\mu \to 0$	$\tau \to \infty$	$\omega \to 0$	$m \to \infty$	$\Sigma \to 0$	1
$\mu \to 0$	$\Sigma \to 0$	$m \to \infty$	$\tau \to \infty$	$\omega \to 0$	0
$\mu \to 0$	$m \to \infty$	$\tau \to \infty$	$\Sigma \to 0$	$\omega \to 0$	0
$\mu \to 0$	$m \to \infty$	$\Sigma \to 0$	$\omega \to 0$	$\tau \to \infty$	0
$\mu \to 0$	$\Sigma \to 0$	$\omega \to 0$	$m \to \infty$	$\tau \to \infty$	0

- chiral nature of charge carriers leads to minimal finite conductivity even with vanishing density of scatterers
- field has to create first electron-hole pairs before they can be accelerated

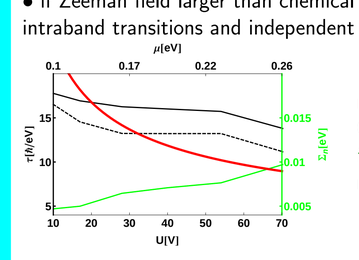
$$\sigma_{\alpha\beta}^{\rm intra} = i2e^2\sum_p\partial_{p_\alpha}\partial_{p_\beta}\Sigma\frac{g}{\omega} = \frac{i\epsilon_0\omega_p^2(n,T,\Sigma_n)}{\omega + \frac{i}{\tau}}$$

Optical conductivity

Experimental values (dots) Z. Q. Li et al., Nature Physics 4, 532 (2008)



- if Zeeman field larger than chemical potential conductivity exclusively by intraband transitions and independent of density (universal)



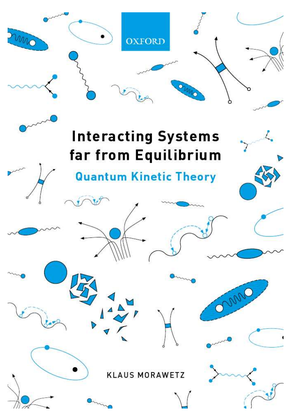
relaxation time for screened Coulomb interaction, fitted effective Zeeman field, only with relaxation time (dashed)

Summary

- Coupled quantum kinetic equation with SU(2) structure including:
  - mean field interaction (scalar+vector), suited for magnetized impurities, spin-flip, ..
  - arbitrary magnetic and electric field strength and spin-orbit interaction (nonlinear)
- Anomalous currents in graphene/Weyl as infinite mass limit of spin-orbit coupled systems
- Chiral terms appear normally without violation of conservation laws
- Graphene
  - influence of magnetic domain puddles and meanfields recast into an effective Zeeman field on intra-, interband longitudinal and Hall conductivities
  - density-independent universal conductivity for large Zeeman fields or small densities
  - experimental optical conductivity well reproduced by intrinsic effective Zeeman field

books and paper: http://www.k-morawetz.de

1. Europhys. Lett., 104 (2013) 27005: *Terrahertz out-of-plane pulses due to spin-orbit coupling*
2. *Quantum kinetic theory of spin-polarized systems in electric and magnetic fields with spin-orbit coupling*:
   Phys. Rev. B 92 (2015) 245425: *I. Kinetic equation and anomalous Hall and spin-Hall effects*,
   Phys. Rev. B 92 (2015) 245426 *II. RPA response functions and collective modes*
3. Phys. Rev. B 94 (2016) 165415: *Dynamical charge and pseudospin currents in graphene and possible Cooper pair formation*
4. Eur. Phys. J. B 92 (2019) 176: *Weyl systems: anomal transport normally explained*
5. Phys. Lett. A383 (2019) 1362: *Chiral anomaly in Weyl systems: no violation of classical conservation laws*
6. arXiv:2004.01507: *Exploring anomalies by many-body correlations*



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