Charged liquid bridges

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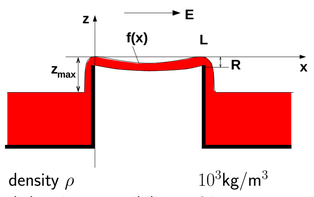
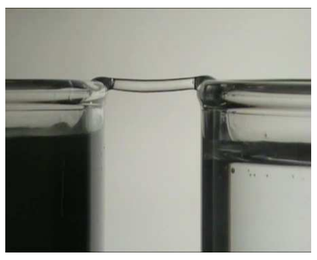
Formation of floating water bridges



density ρ	10^3kg/m^3
dielectric susceptibility ϵ	81
surface tension σ_s	7.27×10^{-2}N/m
viscosity η	1.5×10^{-3}Ns/m^2
conductivity of clean water σ_0	5×10^{-6}A/Vm
molecular conductivity of NaCl λ	12.6×10^{-3}Am^2/Vmol
heat capacity c_p	4.187 J/gK



www.ecfuchs.com
Exp. by Woisetschläger, Fuchs,...

Theoretical questions and concept

1. How is the electric field influencing the height $z_{\max}$ water can creep up?
2. What is the radius $R(x)$ along the bridge?
3. What is the form $z = f(x)$ of the water bridge?
4. What are the static and dynamical constraints for possible bridge formation?

Answers in terms of 4 parameters:

1. capillary height $a = \sqrt{\frac{2\sigma_s}{\rho g}} = 3.8\,\mathrm{mm}$ (surface tension σ_s, particle density ρ, gravity g)
2. creeping height $b(E) = \frac{\epsilon_0(\epsilon-1)E^2}{\rho g} = 7.22\bar{E}^2\,\mathrm{cm}$ (electric field $\bar{E}$ in units of 10^4V/cm)
3. dimensionless ratio of field-force on charges to gravity $c(\rho_c, E) = \frac{\rho_c E}{\rho g} = 15.97\bar{E}\bar{\rho}_c$ (charge density $\bar{\rho}_c$ in units of ng/l)
4. characteristic velocity for dynamical consideration $u_0 = \frac{\sigma_s}{16\eta} \approx 3.02\mathrm{m/s}$

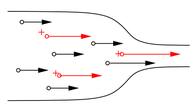
Problem of Ohmic picture

If Ohmic transport would be correct: $j = \rho v = \sigma E \to E \sim v$
-since incompressible $Av = const \to E \sim 1/A$

• constant current density contradicts larger velocity when diameter smaller

Two possible solutions:
1. Accept $E \sim 1/A$, but $D = \epsilon\epsilon_0 E$ constant $\to \epsilon \sim A$, unlikely
2. Non-Omic picture, we will get $j \sim c_1 E + c_2 E^2$

Two fluid picture, charges drag neutral molecules

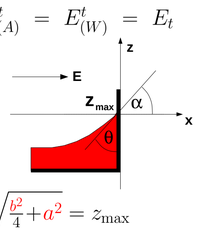


1. Creeping height, 2. Radius

From ressure tensor and $E^n_{(A)} = \epsilon E^n_{(W)} = \epsilon E_n, E^t_{(A)} = E^t_{(W)} = E_t$

$\rho g z + \frac{\sigma_s}{R_1} = \frac{1}{2}\epsilon_0(\epsilon-1)(\epsilon E_n^2 + E_t^2)$ first integral

$\frac{z^2}{a^2} + \frac{1}{\sqrt{1+z'^2}} - \frac{bz}{a^2} = 1;$ $\quad a = \sqrt{\frac{2\sigma_s}{\rho g}}$, $\quad b(E) = \frac{\epsilon_0(\epsilon-1)E^2}{\rho g}$



1. Maximal height: $z = \frac{b}{2} + \sqrt{\frac{b^2}{4} + a^2(1-\sin\theta)} \le \frac{b}{2} + \sqrt{\frac{b^2}{4}+a^2} = z_{\max}$

• without electric field maximal creeping height is capillary length a
• very high fields lead to field-dependent length b

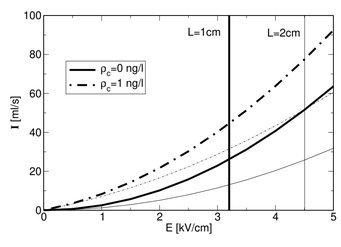
2. Radius of bridge at beaker: equating pressure by surface tension $\frac{\sigma_s}{R}$ with gravitational force density $\rho g h \approx \rho g 2R$:
$R \approx \frac{a}{2} \to \frac{1}{2}\left(\frac{b}{2} + \sqrt{\frac{b^2}{4}+a^2}\right)$

Mass flow of the bridge

laminar (convection $\mathbf{u}\nabla\mathbf{u}$ 0 in Navier Stokes eq. $\eta\nabla^2\mathbf{u} - \nabla p + \rho_c\mathbf{E} = 0$
gradient of electric pressure $-\nabla p = \frac{\epsilon_0(\epsilon-1)E^2}{2L} = \frac{b}{2L}\rho g$. solution $u(r) - u(R) = 2u_0\left(\frac{b}{2L}+c\right)\left(1-\frac{r^2}{R^2}\right)$; $\quad b(E) = \frac{\epsilon_0(\epsilon-1)E^2}{\rho g}$, $\quad c(\rho_c, E) = \frac{\rho_c E}{\rho g}$

mean mass current
$I = 2\pi\rho\int_0^R dr r[u(r)-u(R)] \equiv \rho v \pi R^2$
mean velocity

$$v = u_0\left(\frac{b}{2L}+c\right)$$



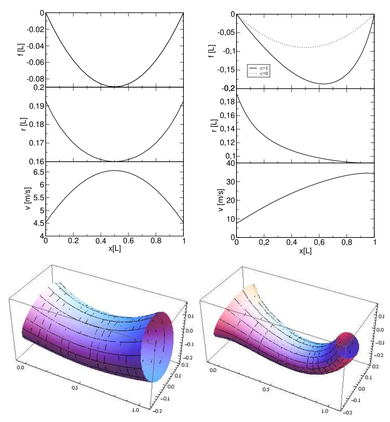
•Ratio of field-dependent creeping height b to bridge length determines mean velocity together with dynamical bulk charges c

AIP Advances 2 (2012) 022146: *The effect of electromagnetic fields on a charged catenary*
Phys. Rev. E 86 (2012) 026302: *Theory of water and charged liquid bridges*
Water 2017, 9(2017), 353: *Reversed currents in charged liquid bridges*

Profile of floating bridge

neglect viscous term, Bernoulli equation
$f(x) - cx = \frac{v^2 - c^2(x)}{2g} + a - \frac{a^2}{2R(x)}$
radius from
$R(x)^2 v(x) = R^2 v$

above: shape
middle: radius
bottom: velocity



no bulk charge: $c = 0$, $b = 1.5$cm
bulk charge: $c = 1$, $b = 1$cm

3. Shape, effect of e.m. fields on charged catenary

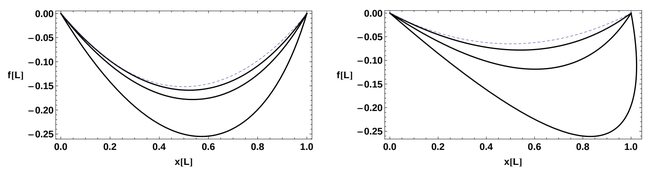
center of mass line $z = f(x)$ with $f(0) = f(L) = 0$
gravity $\rho g f$, volume tension $\rho g b$, force density by dynamical charges $\rho_c E x$ $\int_0^L \mathcal{F}(x)dx = \rho g \int_0^L (f(x) + b - cx)\sqrt{1+f'^2}dx \to$ *extr.*

$$b(E) = \frac{\epsilon_0(\epsilon-1)E^2}{\rho g}$$
$$c(\rho_c, E) = \frac{\rho_c E}{\rho g}$$

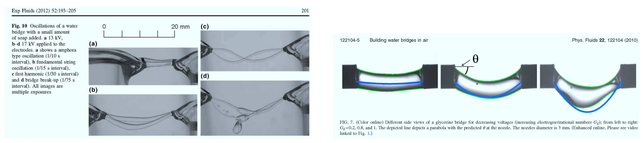
New solution ($t \in (0, L)$)

$$f(t) = \frac{1}{1+c^2}\left\{ct + \xi\left[\cosh\left(\frac{t}{\xi} - \frac{Ld}{2\xi}\right) - \cosh\left(\frac{Ld}{2\xi}\right)\right]\right\}$$
$$x(t) = t - cf(t)$$

with $d = 2\frac{\xi}{L}\mathrm{arcosh}\frac{b}{\xi}$ and ξ to be the solution of $c = c_m(\xi, b) = -\frac{2\xi}{L}\sinh\frac{L}{2\xi}\left(\frac{b}{\xi}\sinh\frac{L}{2\xi} - \sqrt{\frac{b^2}{\xi^2}-1}\cosh\frac{L}{2\xi}\right)$ without bulk charges, $c = 0, d = 1$, solution just well known catenary



$b = 1, 2$ (left, right) corresponding to the maximal values $c_m = 0.41, 1.62$
uncharged catenary (dashed) and $c = c_m(0.5, 0.75, 0.999)$



J. Woisetschläger et al., Exp Fluids 52 (2012) 193; Á. G. Marín, D. Lohse, Phys. Fluids 22 (2010) 122104

Static stability

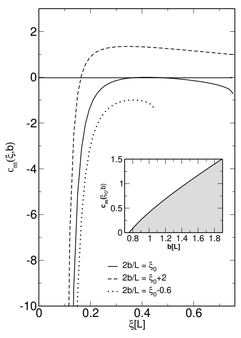
ξ solution of $c = c_m(\xi, b)$
with $b(E) = \frac{\epsilon_0(\epsilon-1)E^2}{\rho g}$, $c(\rho_c, E) = \frac{\rho_c E}{\rho g}$

• Without bulk charges, $c = 0, d = 1$

boundary condition
$\frac{2b}{L} = \frac{2\xi}{L}\cosh\frac{L}{2\xi} \ge \xi_c = 1.5088...$

lower bound for electric field in order to enable length L: $b > \frac{1}{2}L\xi_c$

• With bulk charges new solution $c \le c_m(\xi_0, b)$



Upper critical bound, inset: maximum in dependence on creeping parameter b

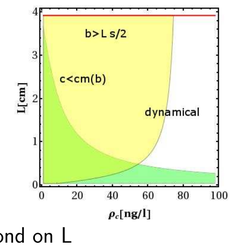
Dynamical stability

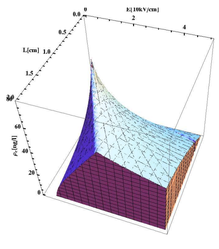
• velocity of charged particles > velocity of dragged water molecules (mean mass velocity)
• total mass current > mass current from charge particles

$$\frac{\sigma E}{\rho_c} > u_0\left(\frac{b}{L}+c\right) > x_i\frac{\sigma E}{\rho_c}$$

with mass ratio of charged (e.g. NaCl) to water molecules $x_i = \frac{\#_i m_{\mathrm{NaCl}}}{\#_w m_{\mathrm{H_2O}}} = \frac{\rho_c m_i}{\rho e_i}$

$E = 0.64$kV/cm

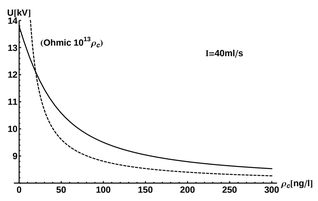




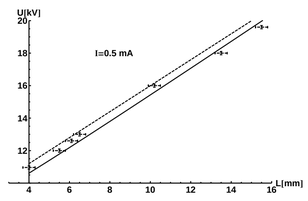
• lower and upper bond on L

Comparison with experiment

J. Woisetschlager, K. Gatterer, E. Fuchs, Exp. in Fluids 48, 121 (2010)
40mg/s, 1cm length, diameter of 2.5mm, necessary 12.5kV



voltage vs bulk charge to maintain 40ml/s,
flow expression (solid)
Ohmic transport (dashed line 10^{13})



vs bridge length to maintain 0.5mA,
bulk charge 2.3ng/l (solid line)
Ohmic transport (dashed line 3×10^3)

• Ohmic $\sigma = \lambda\frac{\rho_c}{eN_A} + \sigma_0$: 13 orders of magnitude higher bulk charges necessary in order to get same range

Surface potential and reversed currents

ζ potential defined by velocity $\vec{v}_i = \frac{e_i}{6\pi\eta r_i}\vec{E} = \frac{\sigma}{\rho_i}\vec{E} = -\epsilon\epsilon_0\frac{\zeta}{\eta}\vec{E}$ describes the electric potential at the surface of the bridge
assume charge density $\rho_c = \rho_b + \rho_r(r)$, and radial-dependent modulation of bulk charge
Poisson equation for the electrostatic potential

$$\nabla^2\Psi = -\frac{\rho_r(r)}{\epsilon\epsilon_0} = -\frac{1}{\epsilon\epsilon_0}\sum_i n_i e_i\left(\mathrm{e}^{-e_i\Psi/T} - 1\right) \approx \kappa^2\Psi$$

solved to obtain radial charge density and additional body force

$$\vec{f}(r) = \rho_r(r)\vec{E} = -\epsilon\epsilon_0\kappa^2\zeta\frac{I_0(\kappa r)}{I_0(\kappa R)}\vec{E}$$

Extension of Navier Stokes integrated to yield velocity

$$v(r) - v(R) = \frac{2J_0}{\pi R^2}\left\{(\kappa R)^2\left(1 - \frac{r^2}{R^2}\right) + 4\frac{\zeta}{\zeta_0}\left[\frac{I_0(\kappa r)}{I_0(\kappa R)} - 1\right]\right\}$$

with

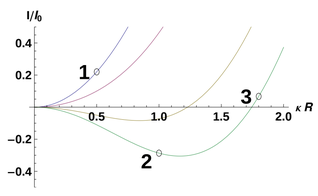
$$J_0 = \frac{\pi v_0}{2\kappa^2}\left(\frac{b}{2L}+c\right) \qquad \zeta_0 = \left(\frac{b}{2L}+c\right)\frac{\rho g}{\epsilon\epsilon_0 E\kappa^2} = \frac{(\epsilon-1)E}{2\epsilon\kappa^2 L} + \frac{\rho_b}{\epsilon\epsilon_0\kappa^2}$$

Total volume flow relative to surface flow

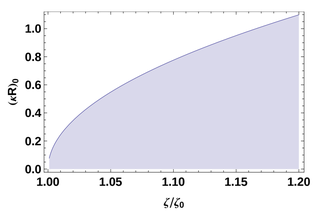
$$J(R) = 2\pi\int_0^R [v(r) - v(R)]\,r dr = J_0\left[(\kappa R)^2 - 8\frac{\zeta}{\zeta_0}\frac{I_2(\kappa R)}{I_0(\kappa R)}\right]$$

Results for reversed currents

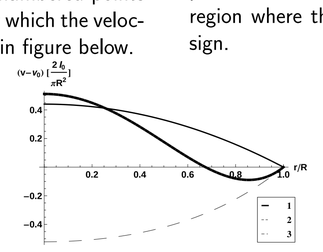
• flow can change direction if ζ potential is exceeding ζ_0



Total mass flow vs. the dimensionless radius of the bridge. The parameter $8\zeta/\zeta_0 = 1, 5, 10, 12$ from left to right. The numbered points sign the regions for which the velocity profile is given in figure below.



Critical parameter $(\kappa R)_0$ where mass flow changes sign vs. the ζ potential. The shaded area is the region where the flow reverses the sign.



The radial velocity profile for the three numbered points of above figure.

Summary

1. electrohydrodynamics sufficient to describe water bridge formation
2. new exact solution of charged catenary: asymmetric profile
3. no bulk charges: maximal length no minimal length
4. bulk charges: also minimal length
5. very small concentrations of bulk charges (¿50 ng/l) destroys bridge
6. dynamical picture: dragged liquid particles due to motion of charges
   - dynamical stability
   - mass flow combines charge transport and neutral mass flow dragged by dielectric pressure in agreement with the experimental data
7. theory applies for charged liquids with small Reynolds numbers (laminar)
8. motivated by recent visualizations of bidirectional flow, additional spatial modulation of radial charge distribution considered
9. surface potential by solving Poisson equation and from Navier-Stokes equation a modified mass flow through the bridge
10. parameter range found where flow is changing its direction