Response with spin−orbit coupling in magnetic fields

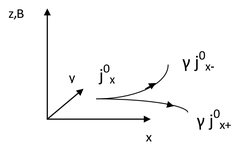
Münster University of Applied Sciences

K. Morawetz[1,2,3]

[1] Münster University of Applied Sciences, Stegerwaldstrasse 39, 48565 Steinfurt, Germany
[2] International Institute of Physics (IIP), Av. Odilon Gomes de Lima 1722, 59078-400 Natal, Brazil
[3] Max-Planck-Institute for the Physics of Complex Systems, 01187 Dresden, Germany

Phenomenological considerations

charge current density $\mathbf{j}_n^0 = -\mu n \mathbf{E} - D\nabla n$
spin polarization current density $S_{ij}^0 = -\mu E_i s_j - D\frac{\partial s_j}{\partial x_i}$
with mobility μ, diffusion D and spin polarization density $s_z = (n_\uparrow - n_\downarrow)/2$



$$j_y^{\rm s.o.} = j_{+y}^{\rm s.o.} + j_{-y}^{\rm s.o.} = \gamma\mu(n_+ - n_-)E_x + \gamma D\frac{\partial(n_+ - n_-)}{\partial x}$$
$$S_{yz}^{\rm s.o.} = j_{+y}^{\rm s.o.} - j_{-y}^{\rm s.o.} = \gamma\mu(n_+ + n_-)E_x + \gamma D\frac{\partial(n_+ + n_-)}{\partial x}$$

Together

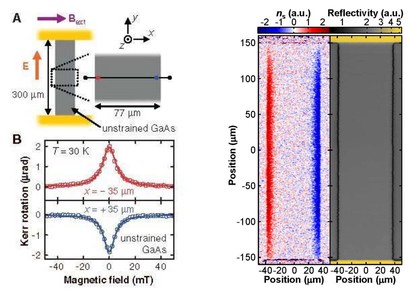
$$j_i = j_i^0 + \gamma\epsilon_{ijk}S_{jk}^0 \qquad \mathbf{j}/e = \mu n\mathbf{E} + D\nabla n + \gamma\mu\mathbf{E}\times\mathbf{s} + \gamma D\nabla\times\mathbf{s}$$
$$S_{ij} = s_{ij}^0 - \gamma\epsilon_{ijk}j_k^0. \qquad S_{ij} = -\mu E_i s_j - D\frac{\partial s_j}{\partial x_i} + \epsilon_{ijk}(\gamma\mu n E_k + \gamma D\nabla_k n)$$

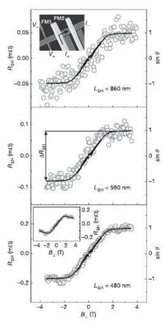
anomalous Hall effect (Karplus, Luttinger 1954)
inverse spin-Hall, inhomog. spin density (Averkiev, Dyakonov, Bakun 1983)
Spin-Hall effect charge current induce spin current

Spin-Hall effect

predicted: Dyakonov, Perel 1971
measured: Kato et al., Science 306 (04) 1910
Sih et. al, Nature Physics 1 (05) 31

Anomalous Hall effect
Valenzuela, Tinkham, Nature 442 (06) 176

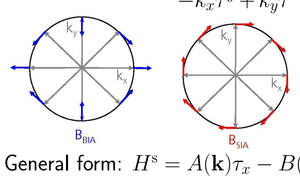




Extrinsic vs intrinsic spin-orbit
intrinsic: c-band (s) coupled to v-bands (v) GaAs/AlGaAs

bulk-inversion asymmetry (III-V semicond.) Dresselhaus
$-k_y\tau^y + k_x\tau^x$

structure inversion asym. (macrosc. confining) Rashba
$-k_x\tau^y + k_y\tau^x$



2D − system	$A(k)$	$B(k)$	$C(k)$
Rashba	$\beta_R k_y$	$\beta_R k_x$	
Dresselhaus[001]	$\beta_D k_x$	$\beta_D k_y$	
Dresselhaus[110]	βk_x	$-\beta k_x$	
Rash. − Dressel.	$\beta_R k_y - \beta_D k_x$	$\beta_R k_x - \beta_D k_y$	
k^3 Rashba(hole)	$i\frac{\beta_R}{2}(k_-^3 - k_+^3)$	$\frac{\beta_R}{2}(k_-^3 + k_+^3)$	
k^3 Dresselhaus	$\beta_D^3 k_x k_y^2$	$\beta_D^3 k_y k_x^2$	
Wurtzitetype	$(\alpha+\beta k^2)k_y$	$(\alpha+\beta k^2)k_x$	
graphene	vk_x	$-vk_y$	
bilayergraphene	$(k_-^2 + k_+^2)/4m$	$i(k_-^2 - k_+^2)/4mi$	
3D − system			
bulk Dressel.	$k_x(k_y^2 - k_z^2)$	$k_y(k_x^2 - k_z^2)$	$k_z(k_x^2 - k_y^2)$
Cooperpairs	Δ	0	$\frac{p^2}{2m} - \epsilon_F$
extrinsic $\beta = \frac{1}{\hbar}\lambda^2 V(k)$	$q_y k_z - q_z k_y$	$q_z k_x - q_x k_z$	$q_x k_y - q_y k_x$

General form: $H^{\rm s} = A(\mathbf{k})\tau_x - B(\mathbf{k})\tau_y + C(\mathbf{k})\tau_z = \mathbf{b}\cdot\tau$

Interactions and meanfields

1. Spin-orbit coupling $\tau\cdot\mathbf{b}(\mathbf{p},\mathbf{R})$
magn. impurities, intrinsic+ extrinsic

$$\hat{V}_{-p,p'} = \begin{cases} V_0(p'-p) \\ \tau\cdot\mathbf{V}(p'-p) \\ \frac{i\lambda^2}{\hbar}\tau\cdot(\mathbf{p}\times\mathbf{p}')V(p'-p) \end{cases}$$

2. Leads to impurity meanfields

$$\Sigma_0^{\rm imp} = nV_0 + \mathbf{s}\cdot\mathbf{V}; \quad \mathbf{\Sigma}^{\rm imp} = \mathbf{s}V_0 + n\mathbf{V}$$

Extrinsic spin-orbit coupling meanfield $\Sigma_0^{\rm ext.} = i\frac{\lambda^2}{\hbar^2}V\left[m(\mathbf{S}_j\times\mathbf{q})_j - \mathbf{s}\cdot(\mathbf{p}\times\mathbf{q})\right]$, $\mathbf{\Sigma}^{\rm ext.} = i\frac{\lambda^2}{\hbar^2}V\left[m(\mathbf{j}\times\mathbf{q}) - n(\mathbf{p}\times\mathbf{q})\right]$
density $n = \sum_p f$, curr. $\mathbf{j} = \sum_p \frac{\mathbf{p}}{m}f$, polarization $\mathbf{s} = \sum_p \mathbf{g}$, curr. $S_{ji} = \sum_p \frac{p_j}{m}[\mathbf{g}]_i$

- kinetic theory for 2×2 (non-Abelian) $\int\frac{d\omega}{2\pi}G^< = f(k,R,t) + \tau\cdot\mathbf{g}(k,R,t)$

3. Effective (meanfield) Hamiltonian

$$H = \frac{k^2}{2m} + \Sigma_0(\mathbf{k},\mathbf{q},T) + e\Phi(\mathbf{q},T) + \tau\cdot\mathbf{\Sigma}(\mathbf{k},\mathbf{q},T)$$

with $\mathbf{\Sigma} = \mathbf{\Sigma}_{MF}(\mathbf{k},\mathbf{q},T) + \mathbf{b}(\mathbf{k},\mathbf{q},T) - \mu\mathbf{B}$

Kinetic equation

Wigner function for density and spin-density $\int\frac{d\omega}{2\pi}G^< = \rho = f + \tau\cdot\mathbf{g}$

$$D_T f + (\partial_{\mathbf{k}}\Sigma_i\partial_{\mathbf{R}} - \partial_{\mathbf{R}}\Sigma_i\partial_{\mathbf{k}} + (\partial_{\mathbf{k}}\Sigma_i\times e\mathbf{B})\partial_{\mathbf{k}})g_i = 0$$

$$D_T g_i + (\partial_{\mathbf{k}}\Sigma_i\partial_{\mathbf{R}} - \partial_{\mathbf{R}}\Sigma_i\partial_{\mathbf{k}} + (\partial_{\mathbf{k}}\Sigma_i\times e\mathbf{B})\partial_{\mathbf{k}})f = 2(\mathbf{\Sigma}\times\mathbf{g})_i$$

drift and force of the scalar and vector part $D_T = (\partial_T + \mathcal{F}\partial_{\mathbf{k}} + \mathbf{v}\partial_{\mathbf{R}})$
with the velocity and the effective Lorentz force

$$\mathbf{v} = \frac{\mathbf{k}}{m} + \partial_{\mathbf{k}}\Sigma_0, \qquad \mathcal{F} = (e\mathbf{E} - \partial_{\mathbf{R}}\Sigma_0 + e\mathbf{v}\times\mathbf{B})$$

Stationary solution: two bands

$$\hat{\rho}(\hat{\varepsilon}) = \sum_\pm \hat{P}_\pm f_\pm = \frac{f_+ + f_-}{2} + \tau\cdot\mathbf{e}\,\frac{f_+ - f_-}{2} = f + \tau\cdot\mathbf{g}$$

with effective splitting $f_\pm = f_0(\epsilon_k \pm |\mathbf{\Sigma}|)$
and selfconsistent meanfield $\epsilon_k(R) = \frac{k^2}{2m} + \Sigma_0(k,R)$
and selfconsistent precession $\mathbf{e}(k,R) = \mathbf{\Sigma}/|\mathbf{\Sigma}|$

Solution with magnetic fields

Linearization of kinetic equations to external electric perturbation $\delta E = -iq\Phi/e$
$\mathbf{v}(\phi) = (v\cos\phi, v\sin\phi, u)$, $\mathbf{B} = B\mathbf{e}_z$, $\phi = \omega_c t$

$$(-i\omega + i\mathbf{q}\cdot\mathbf{v}(\phi) - \partial_t)\,\delta f + \frac{iq\cdot\partial_v\mathbf{\Sigma}}{m}\cdot\delta\mathbf{g} = S_0$$
$$(-i\omega + i\mathbf{q}\cdot\mathbf{v}(\phi) - \partial_t)\,\delta\mathbf{g} + \frac{iq\cdot\partial_v\mathbf{\Sigma}}{m}\delta f - 2(\mathbf{\Sigma}\times\delta\mathbf{g}) = \mathbf{S}$$

with

$$S_0 = \frac{iq\partial_v f}{m}\Phi + \left(i\frac{q}{m} + \omega_c\times\frac{\partial_v^\Sigma}{m}\right)\delta\Sigma_0\partial_v f + \left(i\frac{q}{m} - \omega_c\times\frac{\partial_v^\Sigma}{m}\right)\delta\Sigma_i\partial_v g_i$$
$$S_i = \frac{iq\partial_v g_i}{m}\Phi + 2(\delta\Sigma\times g)_i + \left(i\frac{q}{m} + \omega_c\times\frac{\partial_v^\Sigma}{m}\right)\delta\Sigma_0\partial_v g_i + \left(i\frac{q}{m} - \omega_c\times\frac{\partial_v^\Sigma}{m}\right)\delta\Sigma_i\partial_v f$$

Solution

$$\delta f + \tau\cdot\delta\mathbf{g} = \int_0^\infty dt e^{i(\omega t - \mathbf{q}R_t\mathbf{v})}e^{-it\tau\cdot\mathbf{\Sigma}}(S_0 + \tau\cdot\mathbf{S})e^{it\tau\cdot\mathbf{\Sigma}}$$

magnetic field enters

$$R_t = \frac{1}{\omega_c}\begin{pmatrix} \sin\omega_c t & 1-\cos\omega_c t & 0 \\ \cos\omega_c t - 1 & \sin\omega_c t & 0 \\ 0 & 0 & \omega_c t \end{pmatrix}$$

Linear response

Solution

$$\delta f + \tau\cdot\delta\mathbf{g} = \int_0^\infty dt e^{i(\omega t - \mathbf{q}R_t\mathbf{v})}e^{-it\tau\cdot\mathbf{\Sigma}}(S_0 + \tau\cdot\mathbf{S})e^{it\tau\cdot\mathbf{\Sigma}}$$

work out the formal solution

$$e^{i\tau\cdot(\mathbf{\Sigma}t)}(S_0 + \tau\cdot\mathbf{S})e^{-i\tau\cdot\mathbf{\Sigma}t} =$$

$$S_0 + (\tau\cdot\mathbf{S})\cos(2t|\Sigma|) + \tau(\mathbf{S}\times\mathbf{e})\sin(2t|\Sigma|) + (\tau\cdot\mathbf{e})(\mathbf{S}\cdot\mathbf{e})(1-\cos(2t|\Sigma|))$$

$$\approx S_0 + \tau\cdot\mathbf{S} + 2\tau\cdot(\mathbf{S}\times\mathbf{\Sigma})t$$

with the direction $\mathbf{e} = \mathbf{\Sigma}/|\Sigma|$ leads to

$$(1 - \Pi_0 V_0 - \mathbf{\Pi}\cdot\mathbf{V})\delta n = \Pi_0\Phi^{\rm ext} + (\Pi_0\mathbf{V} + \mathbf{\Pi}V_0)\cdot\delta\mathbf{s}$$

$$(1 - \Pi_0 V_0)\delta\mathbf{s} = \mathbf{\Pi}_3\Phi^{\rm ext} + (V_0\mathbf{\Pi}_3 + \Pi_0\mathbf{V} + \mathbf{V}\times\mathbf{\Pi}_2 + \mathbf{\Pi}_{\mathbf{e}}\cdot\mathbf{V})\delta n + \mathbf{\Pi}_3(\mathbf{V}\cdot\delta\mathbf{s}) + V_0\mathbf{\Pi}_{\mathbf{e}}\cdot\delta\mathbf{s} + V_0\mathbf{\Pi}_2\times\delta\mathbf{s}$$

with abbreviations $\mathbf{\Pi}_2 = \mathbf{\Pi}_g - \mathbf{\Pi}_{xf}$ and $\mathbf{\Pi}_3 = \mathbf{\Pi} + \mathbf{\Pi}_{xg}$
different magnetic-field dependent polarizations appear

$$\left.\begin{matrix} \Pi_0 \\ \mathbf{\Pi} \\ \mathbf{\Pi}_g \\ \mathbf{\Pi}_{xf} \\ \mathbf{\Pi}_{xg} \\ \mathbf{\Pi}_{\mathbf{e}} \end{matrix}\right\} = \sum_p\int_0^\infty dt e^{i(\omega^+ t - \mathbf{q}\cdot R_t\cdot\mathbf{v})}\left\{\begin{matrix} iq\partial_p f \\ iq\partial_p\mathbf{g} \\ 2\mathbf{g} \\ 2it\mathbf{\Sigma}q\partial_p f \\ 2it\mathbf{\Sigma}\times q\partial_p\mathbf{g} \\ 4t\Sigma g(1 - \mathbf{e}\circ\mathbf{e}) \end{matrix}\right.$$

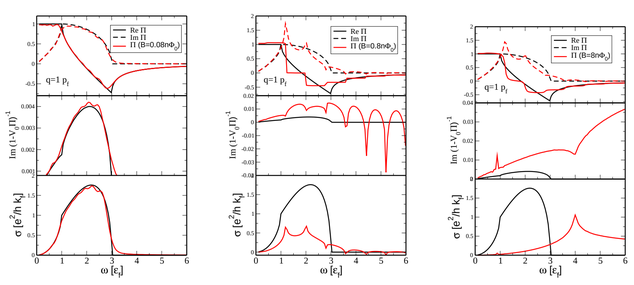
Linear response special cases: 1. no meanfield

Only spin-orbit coupling and magnetic field $V_0 = \mathbf{V} = 0$ obtain decoupled response $\delta n = \Pi_0 U^{\rm ext}$, $\delta\mathbf{s} = (\mathbf{\Pi} + \mathbf{\Pi}_{xg})\Phi^{\rm ext}$
density response unaffected by spin excitation and only modified due to magnetic field, spin response are explicitly dependent on the spin-orbit coupling

Linear response special cases: 2. no spin

Obtain magnetic field-dependent response $\delta n = \kappa\Phi^{\rm ext}$
corresponding to GW (RPA, rainbow, ...), $n = 7\times10^{10}$cm^{-2}

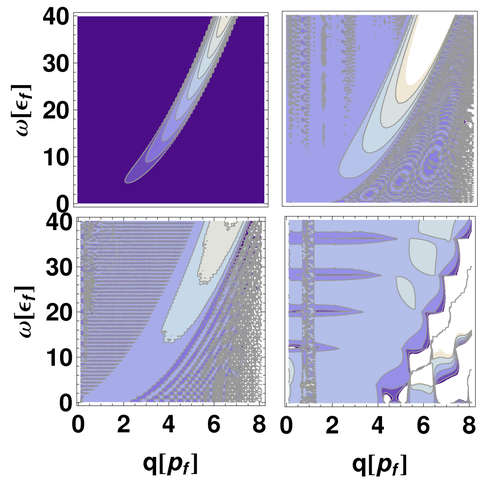
$$\kappa(q\omega) = \frac{\Pi_0(q,\omega)}{1 - V_0(q)\Pi_0(q,\omega)}, \quad \sigma = -i\omega\epsilon_0\epsilon, \quad \epsilon = 1 - V_0\Pi_0$$



real (solid red) and imaginary part (dashed red) of the response function (above) together with the zero magnetic field ones (black lines), the excitation (middle) as well as dynamical conductivity (below) for different magnetic fields for a quasi-2D electron system with a charged-impurity density of 7×10^{10}cm^{-2}, and the magnetic flux $\Phi_0 = h/2e$

Collective density excitation in magnetic field

$\mathrm{Im}\,(1 - V_0\Pi_0)^{-1}$ measure for position and width of mode $\omega = \omega_0 + I\Gamma$



excitation spectrum for $B = 0, 0.08\Phi_0$n (above) and $B = 0.8, 8\Phi_0$n (below)

Linear response special cases: 3. no magn. impurities

no spin-flip mechanism $\mathbf{V} = 0$ results

$$\delta\mathbf{s} = \left(\frac{\mathbf{\Pi}_3}{1 - 2V_0\Pi_0} - V_0\mathbf{\Pi}_2\times\mathbf{\Pi}_3 + V_0\mathbf{\Pi}_{\mathbf{e}}\cdot\mathbf{\Pi}_3\right)\Phi$$
$$\delta n = \left(\frac{\Pi_0}{1 - 2V_0\Pi_0} + V_0(\mathbf{\Pi}\cdot\mathbf{\Pi}_3 - \Pi_0^2)\right)\Phi$$

spin excitation influences density response only for asymmetric momentum distributions 2D with $\mathbf{s} = (s_x^0, s_y^0, 0)$, $\mathbf{g}(p) = \mathbf{s}f_0(p)$ follows $\mathbf{\Pi}_3\cdot\mathbf{\Pi} = \mathbf{\Pi}^2 = \Pi_0^2$

$$\delta\mathbf{s} = \frac{\mathbf{\Pi}_3}{1 - 2V_o\Pi_o}U^{\rm ext}, \qquad \delta n = \frac{\Pi_0}{1 - 2V_o\Pi_o}U^{\rm ext}$$

and vector polarization for Dresselhaus (Rashba) linear spin-orbit coupling

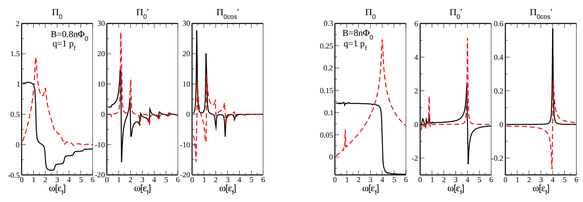
$$\mathbf{\Pi}_3 = \begin{pmatrix} s_x^0 \\ s_y^0 \\ 0 \end{pmatrix}\Pi_0 + \begin{pmatrix} s_x^0 \\ s_y^0 \\ 0 \end{pmatrix}\times\mu\mathbf{B}\Pi_0' - \begin{pmatrix} 0 \\ 0 \\ \beta s_y^0 - \alpha s_x^0 \end{pmatrix}\Pi_{0cos}'$$

out of plane polarization induced by spin precession

$\delta\mathbf{s} = \frac{\mathbf{\Pi}_3}{1-2V_0\Pi_o}U^{\rm ext}$, $\delta n = \frac{\Pi_0}{1-2V_0\Pi_o}U^{\rm ext}$ with

$$\mathbf{\Pi}_3 = \Pi_0\begin{pmatrix} s_x^0 \\ s_y^0 \\ 0 \end{pmatrix} + \Pi_0'\begin{pmatrix} s_x^0 \\ s_y^0 \\ 0 \end{pmatrix}\times\mu\mathbf{B} - \Pi_{0cos}'\begin{pmatrix} 0 \\ 0 \\ \beta s_y^0 - \alpha s_x^0 \end{pmatrix}$$

for Dresselhaus (Rashba) linear spin-orbit coupling



Summary on spin transport

- Coupled quantum kinetic equation derived for spin - $1/2$ - particles including:
  - mean field interaction (scalar+vector), suited for magnetized impurities, spin-flip, ..
  - arbitrary magnetic and electric field strength
  - spin-orbit interaction (nonlinear)
- Anomalous Hall effect, quantum hall effect, spin-Hall effect
- Linear density and spin-density response to external electric field
- Spin response in 2D and linear coupling leads to out-of plane polarization
- Magnetic field induces structure, excitation out of plane follows Landau levels (THz)
- Applications: graphene (pseudospin-orbit coupling), magnetic field induced photocurrent excited by terahertz radiation, magneto-gyrotropic photogalvanic effect MPGE, optical Hall effect, ...

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