Deformation of superconductors

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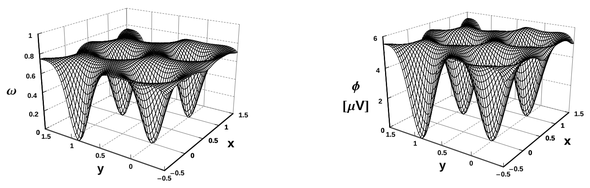
Deformations on crystal lattice

- phase transition to sc state: volume shrinks $\delta V = V_{\rm n} - V_{\rm s} = \alpha_T V$ with $\alpha_T \sim 10^{-7}$
- Density of atomic lattice increases, $\delta n_{\rm lat} = n^{\rm s}_{\rm lat} - n^{\rm n}_{\rm lat} = \alpha_T\, n$
- Strain coefficient α_T depends on T via a fraction of sc electrons, $\alpha_T = \alpha\omega$
- GL theory: in terms of GL function $\omega = 2|\psi|^2/n$, i.e., $\delta n_{\rm lat} = 2\alpha\,|\psi|^2$.
- inhomogeneous lattice causes internal stresses leading to force density [1]

$$\mathbf{F}_{\rm Sim} = K\,\alpha\,\nabla\frac{2|\psi|^2}{n}$$



$\omega = 2|\psi|^2/n$ Electrostatic potential

$\kappa = 1.5$, $T = 0.7\,T_{\rm c}$, $\bar{B} = 0.24\,B_{\rm c2}$

- Diamagnetic currents or around vortex core cause inertial and Lorentz forces balanced by electrostatic field $\mathbf{E} = -\nabla\phi$.
- This electric field transfers Lorentz force from electrons to lattice
- Therefore one can expect that it also causes lattice deformations

$$\mathbf{F} = en\,\nabla\phi$$

- Task is to describe the electrostatic potential at the surface correctly

Surface potential within the Ginzburg-Landau theory

Bardeen's low temperature extension of GL (free energy by Gorter Casimir two-fluid, subtraction of free energy of normal state)

$$f_{\rm el} = \frac{1}{2}\gamma T^2 + \frac{1}{2m^*}\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi - \varepsilon_{\rm con}\frac{2|\psi|^2}{n} - \frac{1}{2}\gamma T^2\sqrt{1-\frac{2|\psi|^2}{n}}$$
$$\approx \frac{1}{2m^*}\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 \qquad \text{near } T_c$$

and $\beta = \frac{1}{2n^2}\gamma T^2$, $\alpha = (-4\varepsilon_{\rm con}/n)(1-t)$, where $\varepsilon_{\rm con} = \frac{1}{4}\gamma T_c^2$, $t = T/T_c$

From GL equation $\frac{(-i\hbar\nabla - e^*\mathbf{A})^2}{2m^*}\psi = -\alpha\psi - \beta|\psi|^2\psi$ follows $f_{\rm el} = -\frac{1}{2}\beta|\psi|^4$ and surface potential reads

$$e\phi_0 = \frac{1}{2n}\beta|\psi|^4$$

without surface dipole, surface potential equal to internal potential

$$e\phi = -\frac{1}{2m^*n}\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} - \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\left(\frac{|\psi|^2}{n} + \frac{|\psi|^4}{2n^2}\right)$$

inertial and Lorentz forces neglecting pairing forces
approximation of Khomskii and Kusmartsev adopted by Blatter

$$e\phi_{\rm Bl} = \frac{\gamma T_c}{n}\frac{\partial T_c}{\partial n}|\psi|^2$$

B close to B_{c2} for thin layer

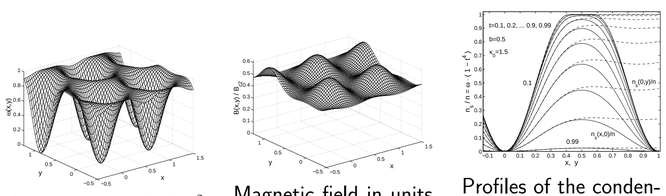
$$\langle\omega\rangle = \frac{(1-b)}{\beta_{\rm A}},\quad \langle\omega^2\rangle = \frac{(1-b)^2}{\beta_{\rm A}}$$

with $\omega = \frac{|\psi|^2}{|\psi_\infty|^2}$, $b = \frac{B}{B_{c2}}$

mean value becomes

$$\langle e\phi_0\rangle = \frac{\varepsilon_{\rm con}}{n\beta_{\rm A}}\left(1-t^2\right)^2(1-b)^2$$
$$\langle e\phi_{\rm Bl}\rangle = \frac{\varepsilon_{\rm con}}{n\beta_{\rm A}}\frac{\partial\ln T_c}{\partial\ln n}\,2\left(1-t^4\right)(1-b)$$

Numerical results on Nb [2]

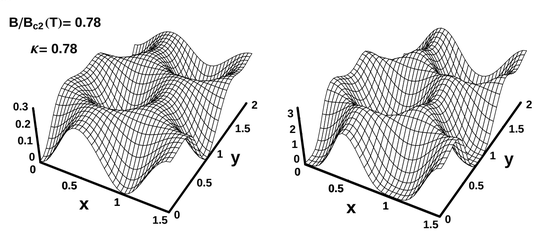


Condensate $|\Psi/\Psi_0|^2$ for $T/T_c = 0.5$, magnetic induction $\bar{B}/B_{c2} = 0.5$, and GL parameter $\kappa_0 = 1.5$.

Magnetic field in units of the upper critical field B_{c2}. $B(x,y)$ reaches its maximum $B_{\rm max}$ at the vortex centers.

Profiles of the condensate density $n(x,y)$ at various temperatures. The solid lines show cuts along the x-axis, and the dashed lines along the y-axis.

- n_s smaller at boarders than nonmagnetic value → nonlocal effects
- B higher than applied field in core → Sc compresses magnetic field in vortices



Superconducting fraction $\omega = |\psi|^2/\psi_\infty^2$ (left column) where ψ_∞ in absence of magnetic field, electrostatic potential ϕ (right column) at surface of superconductor with Abrikosov vortex lattice, temperature $T = 0.95\,T_c$

- At vortex center, $x^2 + y^2 = r^2 \to 0$, superconducting fraction $\omega \propto r^2$, but potential vanishes as $\phi_0 \propto r^4$
- possible to be observed in future measurements

Contributions to force

volume change fully inuced by surface dipole [3] $\mathbf{F} = en\,\nabla\phi$
Electrostatic potentials $e\phi = -\frac{\partial f_s}{\partial n_n}$ (Bernoulli potential)

$$e\phi = -\frac{1}{2m^*n}\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} + \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\left(\sqrt{1-\frac{2|\psi|^2}{n}} - 1\right)$$

Quantum kinetic energy represents gradient corrections, in London limit classical Bernoulli law

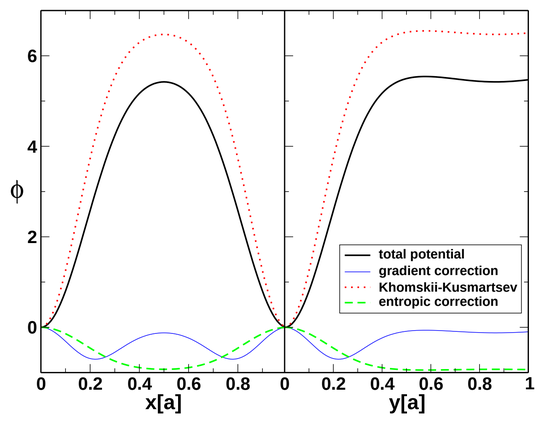
$$\frac{\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi}{2m^*n} \to \frac{e^{*2}A^2\omega}{4m^*} = \frac{1}{2}\omega m v^2$$

Khomskii and Kusmartsev: effect of BCS gap on local density of electronic states, Entropic correction
magnetostriction: $G_{\rm s} = G_{\rm n} - V\varepsilon_{\rm con}$, where $\varepsilon_{\rm con} = \gamma T_{\rm c}^2/4 = B_{\rm c}^2/2\mu_0$
Pressure derivative determines the sample volume, $V_{\rm s,n} = \partial G_{\rm s,n}/\partial p$
one finds $V_{\rm s} = V_{\rm n} - V\frac{\partial\varepsilon_{\rm con}}{\partial p}$
Pressure modifies condensation energy indirectly by increase of density

$$\alpha = \frac{\partial\varepsilon_{\rm con}}{\partial p} = \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{\partial n}{\partial p} = \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{n}{K}$$

and Shimanek result corresponds to second term of the Bernoulli potential

$$\mathbf{F}_{\rm Sim} = n\frac{\partial\varepsilon_{\rm con}}{\partial n}\,\nabla\frac{2|\psi|^2}{n}.$$

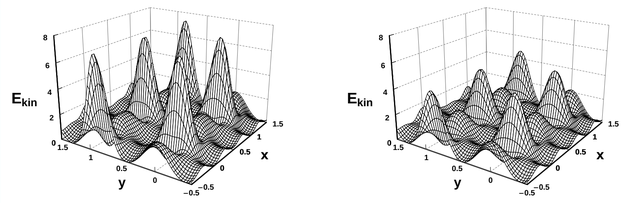


Effective vortex mass

Density of kinetic energy of lattice ions driven by vortices moving with velocity V in the x direction

$$E_{\rm kin} = \tfrac{1}{2}V^2\,n\,M\left[\left(\frac{\partial u_x}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial x}\right)^2\right],$$

where M is the mass of a single ion.
The density of kinetic energy of lattice ions created by vortices moving in the x direction

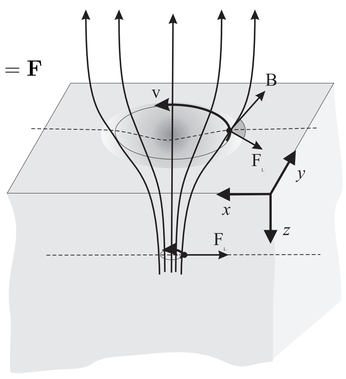


Šimánek force Bernoulli force

Deformation caused by the Abrikosov vortex lattice

displacement field $\mathbf{u}$ obeys

$$\left(K + \frac{4}{3}\mu\right)\nabla(\nabla.\mathbf{u}) - \mu\nabla\times\nabla\times\mathbf{u} = \mathbf{F}$$



where bulk K and shear μ modulus and the force density

$$\mathbf{F} = -\nabla U, \qquad U(\mathbf{r}, z) = \rho\varphi(\mathbf{r}, z)$$

Lorentz force acting on circulating superconducting current. In bulk Lorentz force is parallel to surface. Near surface it is not parallel due to magnetic stray field.

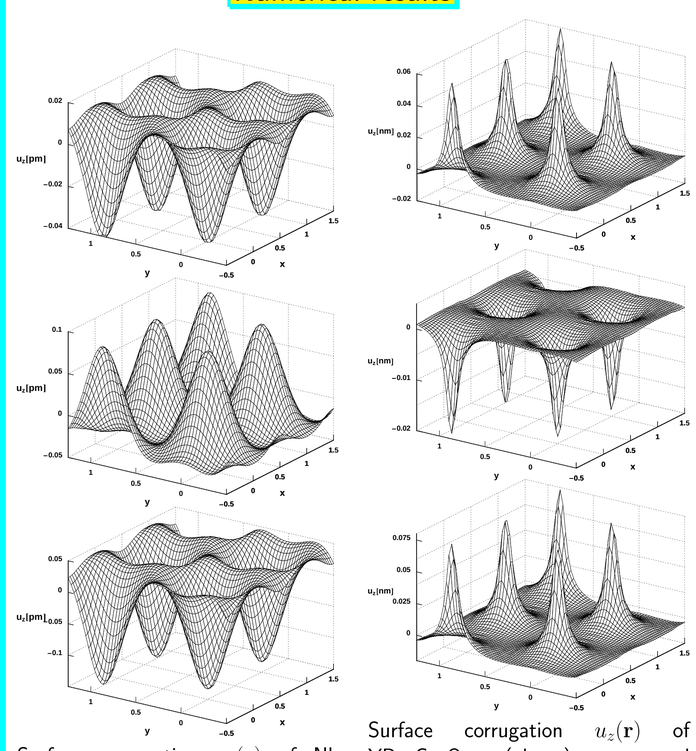
	$\frac{\gamma T_c^2}{4n}$ [μeV]	κ	n [10^{28}m^{-3}]	$\frac{\partial\ln T_c}{\partial\ln n}$	$\frac{\partial\ln\gamma}{\partial\ln n}$	E [GPa]	σ
Nb	4.585	1.5	2.2	0.74 [6]	0.42 [6]	105 [7]	0.4 [7]
YBCO	750	65	0.5	-4.82 [8]	-4.13 [8]	200 [9, 10]	0.2 [10]

Material parameters of Niobium and $YBa_2Cu_3O_7$: condensation energy per particle, GL parameter, particle density and derivatives of critical temperature and linear coefficient of specific heat γ, Young's modulus and Poisson ratio

Solution [11]: $u_z = \frac{(1-2\sigma)(1+\sigma)}{k\,E(1-\sigma)}\left\{(1-\sigma)p(\mathbf{k}) - \rho\sigma\varphi_\infty(\mathbf{k})\right\}$
pressure at the surface is caused by the surface dipole [3]

$$p(\mathbf{r}) = \rho\varphi_+(\mathbf{r}) - \rho\varphi_-(\mathbf{r})$$

Numerical results



Surface corrugation $u_z(\mathbf{r})$ of Nb (above) compared with the value neglecting surface dipoles (middle) and only surface dipoles (below). The temperature and the mean magnetic field are $T = 0.95\,T_{\rm c} = 9$ K and $\bar{B} = 0.21\,B_{\rm c2}(T) = 6.4$ mT. Length unit is the vortex distance $a = 128$ nm.

Surface corrugation $u_z(\mathbf{r})$ of $YBa_2Cu_3O_7$ (above) compared with the value neglecting surface dipoles (middle) and only surface dipoles (below). The temperature and the mean magnetic field are $T = 0.67\,T_{\rm c} = 60$ K and $\bar{B} = 0.01\,B_{\rm c2}(T) = 0.6$ T. Length unit is the vortex distance $a = 58$ nm.

References

[1] J.-M. Duan and E. Šimánek. *Phys. Lett. A*, 190:118, 1992.

[2] P. Lipavský, J. Koláček, K. Morawetz, and E. H. Brandt. *Phys. Rev. B*, 65:144511, 2002.

[3] P. Lipavský, K. Morawetz, J. Koláček, E. H. Brandt, and M. Schreiber. *Phys. Rev. B*, 77:014506, 2008.

[4] R. J. Cava, A. W. Hewat, E. A. Hewat, B. Batlogg, M. Marezio, K. M. Rabe, J. J. Krajewski, W. F. Peck, and L. W. Rupp. *Physica C*, 165:419, 1990.

[5] J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang. *Phys. Rev. Lett.*, 71:1740, 1993.

[6] P. Lipavský, J. Koláček, K. Morawetz, E. H. Brandt, and T. J. Yang. *Bernoulli potential in superconductors*. Springer, Berlin, 2007. Lecture Notes in Physics 733.

[7] See physical properties in www.webelements.com.

[8] The pair-able charge per Cu atoms is $-0.4335\,e$. Cava [4] show that the charge transfer $-0.03\,e$ from chains to planes per Cu site leads to a decrease of the critical temperature by 30 K. This corresponds to $\frac{\partial\ln T_c}{\partial\ln n} = -4.82$. From Fig. 3 of [5] we can see that the specific heat coefficient drops at the same time from 4.4 mJ/gK^2 to 3.0 mJ/gK^2 which gives $\frac{\partial\ln\gamma}{\partial\ln n} = -4.13$.

[9] Y. M. Soifer, A. Verdyan, I. Lapsker, and J. Azoulay. *Physica C*, 408:846, 2004.

[10] N. P. Kobelev, R. K. Nikolaev, N. I. Sidorov, and Ya. M. Soifer. *phys. stat. sol. (a)*, 127:355, 1991.

[11] P. Lipavský, K. Morawetz, J. Koláček, and E. H. Brandt. *Phys. Rev. B*, 77:184509, 2008.

Summary

- Change of volume during superconducting transition can be expressed as a compression caused by surface dipole
- Electrostatic potential above surface of thin superconducting layer with the Abrikosov vortex lattice calculated
- Possible cases for which presented theory can be tested:
  - at vortex core $|\psi|^2 \propto r^2$ so that $\phi_0 \propto r^4$ while $\phi_{\rm Bl} \propto r^2$
  - at temperatures close the critical temperature, $t \to 1$, $|\psi|^2 \propto 1-t$, therefore $\phi_0 \propto (1-t)^2$ while $\phi_{\rm Bl} \propto 1-t$
  - magnetic fields close to upper critical field, $b \to 1$, $|\psi|^2 \propto 1-b$ so that $\phi_0 \propto (1-b)^2$ while $\phi_{\rm Bl} \propto 1-b$
- Theory of vortex motion revised taking gradient corrections into account
- Deformation of a superconductor generates changes of electrostatic potentials at the surface of a sensor - Important applications:
  - Sensor of gravitational waves
  - Gyroscope to measure rotations
- Magnetic field entering superconductor in form of vortices induces corrugation of the surface
- conventional superconductors displacement $\sim 10^{-4}$ Å, high-T_c superconductors $\sim 10^{-1}$ Å detectable with scanning force microscopes
- results for Niobium and $YBa_2Cu_3O_7$ show surface dipole is dominat force
- contribution of bulk potential is opposite to contribution of surface, therefore it reduces magnitude of atomic displacement.