Short–time universal response function

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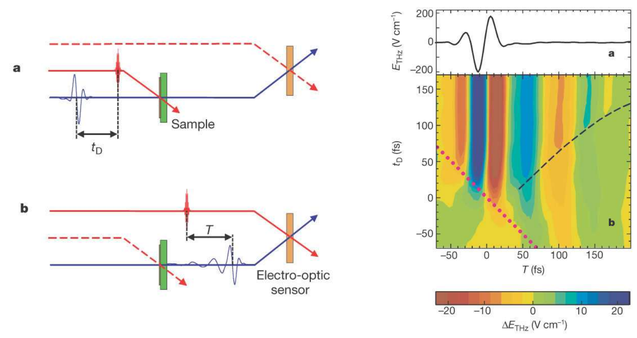
Femtosecond laser irradiation



pump pulse creating charge carriers at time t_0 — probe pulse at time t_D later

A. Leitenstorfer, Nature 426(2003)23, R. Huber et al. Nature 414(2001)286

Time and frequency–dependent dynamical response

$$\frac{1}{\epsilon(\omega,t)} = 1 + \frac{e^2}{\epsilon_0 q^2}\int_0^{t-t_0} dT e^{i\omega T}\chi(t,t-T)$$

time delay after probe pulse $T = t - t_D - t_0$

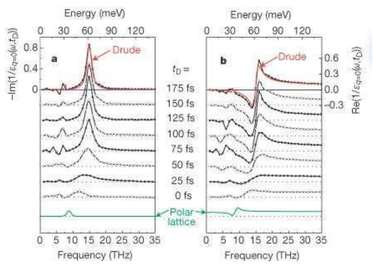
Fourier transformed into frequency

Quantum kinetic equations:

Gartner et al. PRB 66(2002) 075205

Vu and Haug PRB62 (2000) 7179

Kira and Koch PRL 93 (2004) 076402



Short-time response function

density response to external field $\delta n(q,t) = \int_{t_0}^{t} dt' \chi(t,t') V_q^{\rm ext}(t')$

$$\chi(t,t') = \Pi(t,t') + \int_{t'}^{t} d\bar{t}\left\{[\Pi(t,\bar{t})V_q + I(t,\bar{t})]\,\chi(\bar{t},t') + R(t,\bar{t})\right\}$$

polarization is response to induced field

$$\Pi(t,t') = i\sum_p [f_{p+\frac{q}{2}}(t') - f_{p-\frac{q}{2}}(t')] e^{\left(i\varepsilon_{p+\frac{q}{2}} - i\varepsilon_{p-\frac{q}{2}} + \frac{1}{\tau}\right)(t'-t)}$$

Mermin's correction (density conserving local equilibrium)

$$I(t,t') = i\sum_p \frac{f_{p+\frac{q}{2}}(t') - f_{p-\frac{q}{2}}(t')}{\varepsilon_{p+\frac{q}{2}} - \varepsilon_{p-\frac{q}{2}}} \frac{e^{\left(i\varepsilon_{p+\frac{q}{2}} - i\varepsilon_{p-\frac{q}{2}} + \frac{1}{\tau}\right)(t'-t)}}{\tau\bar{\Pi}^{\rm RPA}(t',0)}$$

confining trap potential

$$R(tt') = K\sum_p e^{\left(i\varepsilon_{p+\frac{q}{2}} - i\varepsilon_{p-\frac{q}{2}} + \frac{1}{\tau}\right)(t'-t)} \partial_p\partial_q \delta f(p,q,t')$$

universal two-time response function at short times

$$V\chi(t,t') = -\frac{\omega_p^2}{\gamma} e^{-\frac{t-t'}{2\tau}} \sin\gamma(t-t')$$

but with a different collective mode for Coulomb gas and for cold atoms

$$\gamma = \left\{\begin{array}{c}\sqrt{\omega_p^2 + k - \frac{1}{4\tau^2}} \\ \sqrt{\omega_p^2 + b^2 + k - \frac{1}{4\tau^2}}\end{array}\right\},\ \omega_p^2 = \left\{\begin{array}{c}\frac{ne^2}{m\varepsilon_0} \\ bnaU\end{array}\right\},\ \text{for } V_q = \left\{\begin{array}{c}\frac{e^2\hbar^2}{\varepsilon_0 q^2} \\ Ua\end{array}\right\}$$

where $b = 4J\sin^2\frac{aq}{2\hbar}$, $\epsilon_p = \frac{p^2}{2m}$ / $2J(1-\cos pa/\hbar)$ repsectively

- Coulomb interactions lead to optical, atoms on lattice to acoustic mode

No time for correlations to be built up

- Linearize kinetic equation to obtain density response

$$\delta n(q,t) = \int_{t_0}^{t} dt' \chi(t,t') V_q^{\rm ext}(t')$$

- one-particle reduced density matrix $\hat{f}$ obeys

$$\dot{\hat{f}} + i[\hat{\mathcal{E}} + \hat{V}^{\rm ind} + \hat{V}^{\rm ext}, \hat{f}] = \frac{\hat{f}^{\rm l.e.} - \hat{f}}{\tau}$$

- linearization leads to analytic solution K. Morawetz, et al., Phys. Rev. B 72 (2005) 233203

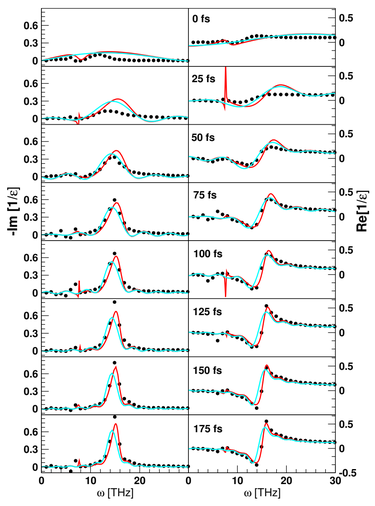
$$\frac{1}{\epsilon(\omega,t)} = 1 + \frac{e^2}{\epsilon_0 q^2}\int_0^{t-t_0} dT e^{i\omega T}\chi(t,t-T)$$
$$= 1 - \frac{\omega_p^2}{\sqrt{\omega_p{}^2 - \frac{1}{4\tau^2}}}\int_0^{t-t_0} dt'\, e^{(i\omega - \frac{1}{2\tau})t'} \sin\left(\sqrt{\omega_p{}^2 - \frac{1}{4\tau^2}}t'\right)$$

Achievement: long-time limit yields the Drude formula

$$\lim_{t\to\infty}\frac{1}{\epsilon} = 1 - \frac{\omega_p{}^2}{\omega_p{}^2 - \omega(\omega + \frac{i}{\tau})}$$

not easy to achieve within short-time expansions, e.q. [ElSayed et al, PRB 49 (1994) 7337] gives the long-time limit of the form $1 - \frac{\omega_p^2}{\omega_p^2 - (\omega + \frac{i}{\tau})^2}$

Comparison with experiment



GaAs

Resonance about 8 THz due to optical phonons, adding intrinsic contribution of crystal lattice

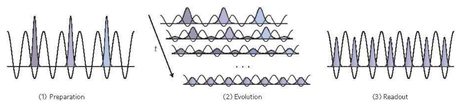
$$\epsilon_{\rm GaAs} = \epsilon_\infty\left(\epsilon + \frac{\omega_{\rm LO}^2 - \omega_{\rm TO}^2}{\omega_{\rm TO}^2 - \omega^2 - i\gamma\omega}\right)$$

$\frac{\omega_{\rm LO}}{2\pi} = 8.8$ THz, $\frac{\omega_{\rm TO}}{2\pi} = 8.1$ THz lattice damping γ =0.2 ps^{-1} and background polarizability of ion lattice $\epsilon_\infty = 11.0$

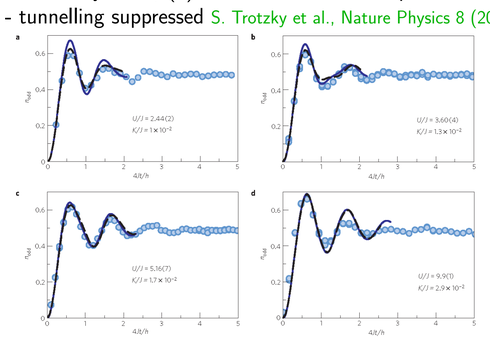
Huber et al, phys.stat.sol.(b) 234 (2002) 207

pumb pulse at $t_0 = -40$fs, probe pulse full-width at half-maximum $27fs$, plasma frequency ω_p =14.4 THz, relaxation time $\tau = 85$ fs

Optical superlattice each second place filled



(1) prepared density wave, (2) reduction of lattice depth - tunnelling, (3) read-out - tunnelling suppressed S. Trotzky et al., Nature Physics 8 (2012) 325

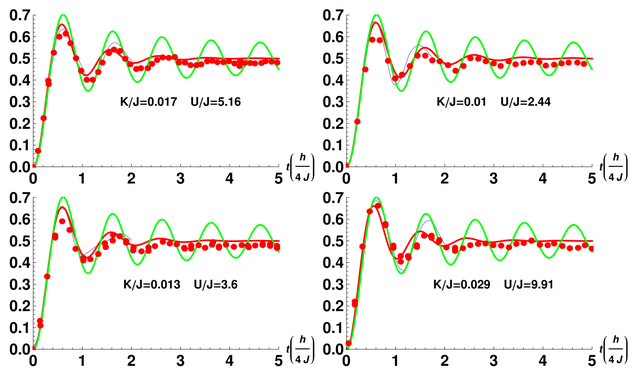


Odd-site population (circles), ensemble-averaged t-DMRG (line), next-nearest neighbour hopping (dashed)

Sudden quench Hubbard

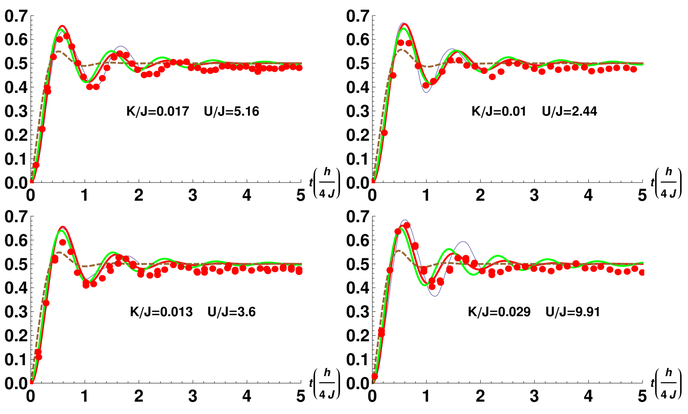
with $\gamma^2 = nbV + b^2 - 1/4\tau^2$ and $b = 4J\sin qa/2$

$$\delta n_t = -\frac{n}{2}J_0(\sqrt{4Jbt})e^{\frac{-t}{\tau}} - \frac{n}{4\gamma\tau^2}\int_0^t dx J_0(\sqrt{4Jbx})e^{\frac{-(t+x)}{2\tau}}\left(2\gamma\tau\cos\gamma(t-x) + (1-2bnV_q\tau^2)\sin\gamma(t-x)\right)$$



experimental data (dots) S. Trotzky et al., Nature Physics 8 (2012) 325 with the RG calculation (thin line) A. Flesch et al., Phys. Rev. A78 (2008) 033608 together with and without interaction, $\tau = 0.6\hbar/J$

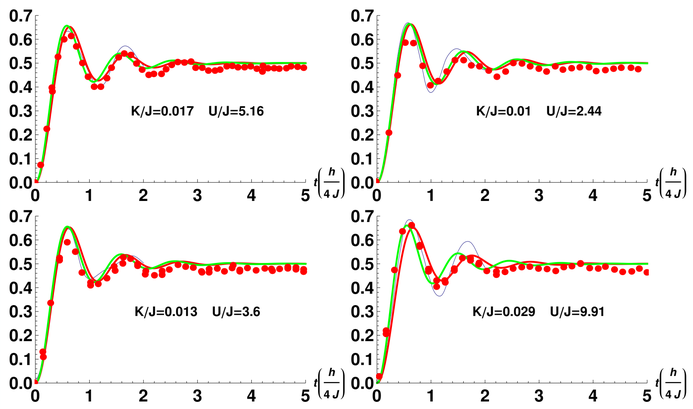
Influence of Mermin's correction



experimental data (dots) with and without Mermin's correction of conserving relaxation time approximation with $\tau = 0.6\hbar/J$. Scaling without Mermin's correction but 4 times relaxation time

Finite trap correction

harmonic trap $V^{\rm trap} = \frac{1}{2}Kx^2$ which leads to $\frac{\partial}{\partial t}\delta f_t = ... + iK\partial_p\partial_q\delta f_t$



experimental data (dots) experimental data (dots) with the RG calculation (thin line) with and without trapping potential K/J

Dielectric function

From Response

$$\delta n(q,t) = \int_{t_0}^{t} dt' \chi(t,t') V_q^{\rm ext}(t')$$

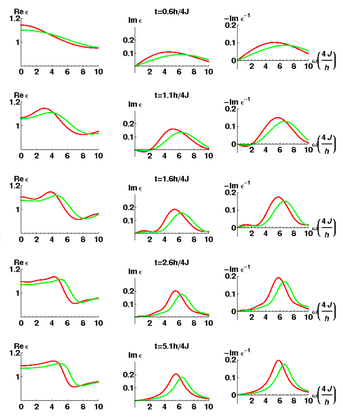
time dependent dielectric function

$$\frac{1}{\varepsilon(\omega,t)} = 1 + \int_0^{t-t_0} dT e^{i\omega T} V\chi(t,t-T)$$

Time evolution of inverse dielectric function

atomic lattice for $U/J = 9.91$

with and without finite trap correction



Time-dependent Singwi-Sjølander local-field corrections

Exact dynamics for $\epsilon_p = 2J(1 - \cos pa/\hbar)$

$$\ddot{\rho}_k = ... - nbV_k\sum_{k_1}\cos(k_1 a)\,(\rho_k f_{k_1,0} \times [1+G_k]$$

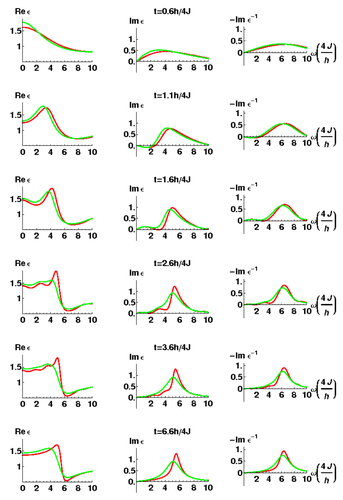
i.e. $V_k \to V_k(1+G_k(t))$ and response becomes

$$\delta n_s(k) = \frac{\chi_s(k)}{1 - V\delta\bar{G}_s}V_s^{\rm ext}(k)$$

Time evolution of inverse dielectric function

atomic lattice for $U/J = 9.91$

with and without local field correction



Summary on short-time behaviour

- collisions have no time to happen yet (correlations are formed by mean-fields)
- separate gross feature of the formation of collective modes at transient times due to mean-field fluctuations
- simple analytic formula for time dependence of the dielectric function and for sudden quench
- Finite size corrections due to trap and density conservation (Mermin) used
- Short time behavior by time-dependent Fermi's golden rule ≡ Finite duration approximation of non-Markovian collision integrals
  - -Formation of quasiparticles universal: $\sim \hbar/\epsilon_F$, $1/\omega_p$
  - -good agreement with solution of Kadanoff and Baym equation
- Initial correlations induce a current and energy contribution (over/under-correlated initial state leads to decrease/increase of kinetic energy)
- quench population dynamics of cold atoms in lattices well described

K. Morawetz, V. Špička, and P. Lipavský: Phys. Lett. A **246**, 311 (1998): *Formation of binary correlations in strongly coupled plasma*

K. Morawetz and H. Köhler: Eur. Phys. J. A **4**, 291 (1999): *Formation of correlations and energy-conservation at short time scales.*

H. S. Köhler and K. Morawetz: Phys. Rev. C **64**, 024613 (2001): *Correlations in Many-Body Systems with Two-time Green's Functions* .

K. Morawetz *et al.*: Phys. Rev. E **63**, R20102 (2001): *Short time dynamics with initial correlations.*

K. Morawetz *et al.*: Phys. Rev. B 72 (2006) 233203: *Femtosecond formation of collective modes due to mean-field fluctuations*

K. Morawetz: Phys. Rev. B 90 (2014) 075303: *Universal short-time response and formation of correlations after quantum quenches*

K. Morawetz, Eur. Phys. J. B 90 (2017) 53: *Quantum currents and pair correlation of electrons in a chain of localized dots*