Asymmetric Bethe–Salpeter equation

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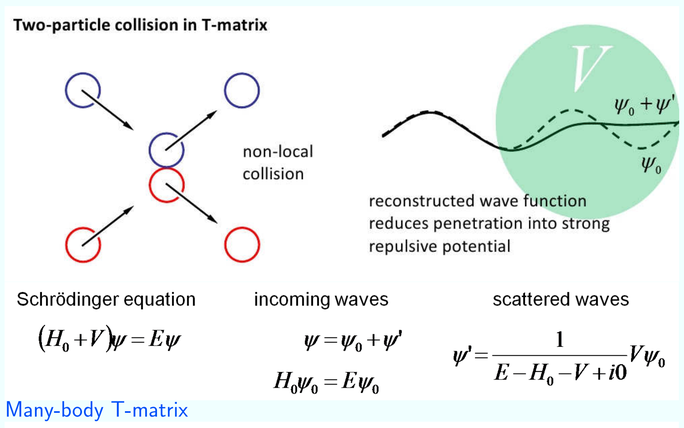
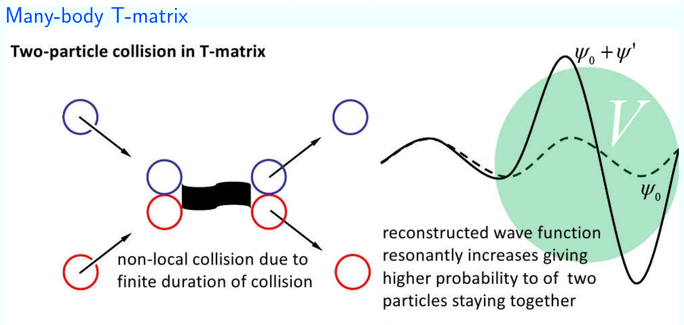
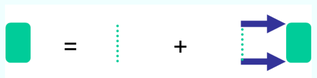
Scattering T-matrix



Many-body T-matrix

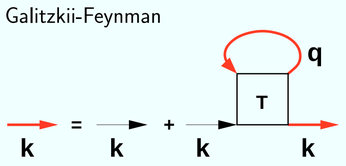


$$\Psi'=\frac{1}{E-H_0-V+i0}V\Psi_0=\frac{V}{E-H_0+i0}V(\Psi_0+\Psi')=\frac{1}{E-H_0+i0}T\Psi_0$$

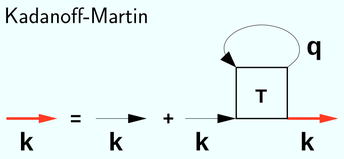


Dichotomy between gap and selfconsistency

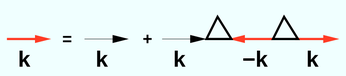
Galitzkii-Feynman

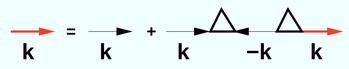


Kadanoff-Martin



Near pole (pairing, bound states) T-matrix is separable $T=\triangle\triangle$





$$G^{-1}(\omega,\mathbf{k})=\omega+\epsilon_{\mathbf{k}}-\Delta^2G(-\omega,-\mathbf{k})$$

no pole no gap equation

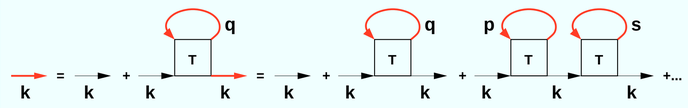
$$G^{-1}(\omega,\mathbf{k})=\omega+\epsilon_{\mathbf{k}}-\frac{\Delta^2}{\omega+\epsilon_{-\mathbf{k}}}$$

two-pole structure, BCS gap equation

- Selfconsistency needed for physical distributions, Goldstone theorem, conservation laws
- Only partial selfconsistency lead to the superconducting gap
- This conflict is known as selfconsistency gap dichotomy:
  - theories satisfying selfconsistency required by Goldstones criterion yield zero gap
  - theories giving the gap do not satisfy selfconsistency
- Worse approximation (Kadanoff - Martin) leads to better results
- Asymmetric selfconsistency is necessary to get gap equation
- Asymmetry violates Kadanoff/Baym criterion B → no particle conservation ?

Removal of double counts

Paradox: The worse approximation yields better result Wrong conclusion: Superconductor and normal metal not be covered by unified theory
Solution: Galitskii-Feynman approximation includes double-counts fatal in superconducting state



- Third particle ought to be different from the interacting pair: $p\neq q$!
- Each momentum contributes as 1/volume → vanish for infinite volume
- Normal state ok, but pairing or BE condensates state $q\sim$ volume

Procedure

subtract own interaction in singular channel
Using the Dyson equation $G_0^{-1}=G^{-1}+\Sigma$
one obtains

$$G_{\backslash}=G-G_{\backslash}\Sigma_iG$$
$$G_{\backslash}=G_0+G_0(\Sigma-\Sigma_i)G_{\backslash}$$

closing with the subtracted propagator $\Sigma_i=T_i\bar{G}_{\backslash}$, short exercise

$$G^{-1}=G_0^{-1}-\Sigma=G_0^{-1}-\Sigma'-\Sigma_i$$
$$=G_0^{-1}-\Sigma'-T_i\bar{G}_{\backslash}=G_0^{-1}-\Sigma'-T_i\left(\bar{G}_0^{-1}-\bar{\Sigma}'\right)^{-1}$$

or explicitly

$$G=\frac{\bar{G}_0^{-1}-\bar{\Sigma}'}{[G_0^{-1}-\Sigma'][\bar{G}_0^{-1}-\bar{\Sigma}']-T_i}$$

free propagator	$G_0^{-1}=\omega-\epsilon_p$
"proper" selfenergy	$\Sigma_{11}(p)\equiv\Sigma'(p)$
"anomalous" selfenergy	$\Sigma_{12}(p)\equiv\Delta(p)$

in matrix form $\mathbf{G}=\mathbf{G^0}+\mathbf{G^0\Sigma G}$ with

$$\mathbf{G}=\begin{pmatrix}G_{11} & G_{12}\\ \bar{G}_{12} & \bar{G}_{11}\end{pmatrix},\ \mathbf{G_0}=\begin{pmatrix}G_0 & 0\\ 0 & \bar{G}_0\end{pmatrix},\ \mathbf{\Sigma}=\begin{pmatrix}\Sigma_{11} & \Sigma_{12}\\ \bar{\Sigma}_{12} & \pm\bar{\Sigma}_{11}\end{pmatrix}$$

Bosons S. T. Beliaev, Soviet. Phys. JETP 7 (1958) 289
Fermions L. P. Gorkov, Soviet. Phys. JETP 7 (1958) 505

"normal" $G_{11}\equiv G$ and "anomalous" Green' function

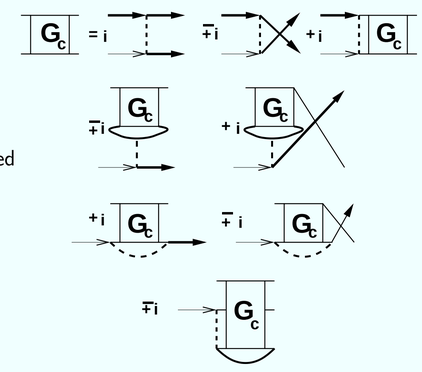
$$G_{12}\equiv\frac{-\Sigma_{12}}{(\omega+\epsilon+\bar{\Sigma}_{11})(\omega-\epsilon-\Sigma_{11})+\Sigma_{12}^2}$$

results, not needed as starting (conservation laws completed)

Diagrammatic derivation

Martin-Schwinger hierarchy coupling one-particle Green's function to two-particle one etc

$$\left[i\partial_{t_1}+\frac{\nabla_{r_1}^2}{2m}-U(1)\right]G(11';U)=\delta(1-1')\mp i\int d\bar{1}V(1,\bar{1})G(1\bar{1}1'\bar{1}^+;U)$$

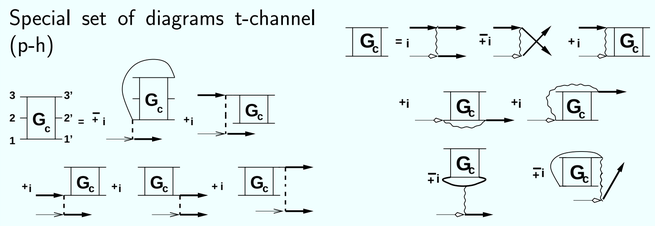


rewritten in terms of correlated Green's functions
$G(121'2';U)=G_c+G_{HF}$

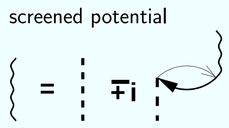
- P. Lipavský: *Multiple scattering corrections to the T-matrix approximation: Unified theory of normal and superconducting states*, Phys. Rev. B 78 (2008) 214506.
- K. Morawetz: *Equivalence of channel-corrected T-matrix and anomalous propagator approach*, Phys. Rev. B 81 (2010) 092501.
- K. Morawetz: *Asymmetric Bethe-Salpeter equation for pairing and condensation*, J. Stat. Phys. 143 (2011) 482.
- B. Šopík, P. Lipavský, M. Männel, K. Morawetz, P. Matlock: *Conserving T-matrix theory of superconductivity*, Phys. Rev. B. 84 (2011) 094529.

Screened ladder channel

Special set of diagrams t-channel (p-h)



screened potential

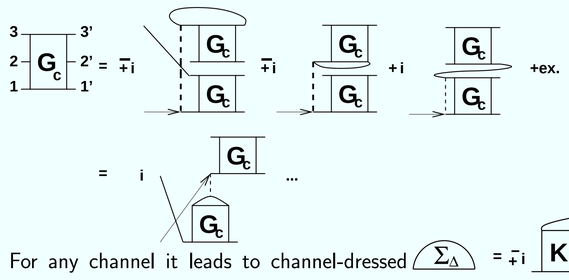


$$G_s=G_{\rm HF}+G_{\rm HF}(\Sigma_s-\Sigma_{\rm F})G_s$$

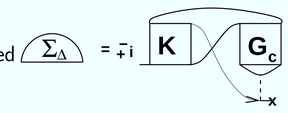
analogously s-channel (p-p) and u-channel (maximally crossed)

$$G_T=G_0+G_0\Sigma_TG_T=G_{\rm HF}+G_{\rm HF}(\Sigma_T-\Sigma_{\rm HF})G_T$$

Cluster-cluster diagrams

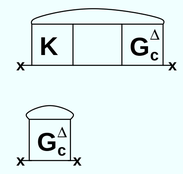


For any channel it leads to channel-dressed propagator



$$G_\Delta=G_p-G_p\Sigma_\Delta G_\Delta=G_0+G_0(\Sigma_T-\Sigma_\Delta)G_\Delta.$$

with $p=s,\tilde{T},T$ denoting the channels and near a pole subtracted T-matrix scheme appears (RSC theory)



Summary

1. The Galitskii-Feynman T-matrix approximation fails in the superconducting state because of non-physical repeated collisions
2. Separating singular channel from selfenergy avoiding repeated collisions leads to propagators of Beliaev form for Bosons or Nambu-Gorkov form for Fermions
3. Auxiliary anomalous Green's function is consequence of theory not assumed ad-hoc, all information derived without this quantity
4. Asymmetric selfconsistency (channel-dressed, totally dressed) required to obtain superconductivity
5. Proof that asymmetric and symmetric selfconsistency can be mapped
6. Developed a microscopic theory of pairing and condensation is valid above and below critical temperature: consistent description of pair-breaking effects
7. Applications for interacting Bose/Fermi systems:

- M. Männel, K. Morawetz, P. Lipavský, *Multiple condensed phases in attractively interacting Bose systems*, New J. Phys. 12 (2010) 033013.
- M. Männel, K. Morawetz, M. Schreiber, P. Lipavský, *Phase diagram and binding energy of interacting Bose gases* , Phys. Rev. B 78 (2008) 054508.
- K. Morawetz, N. H. March and R. H. Squire, *The chemical potential for the inhomogeneous electron liquid in terms of its kinetic and potential parts with special consideration of the surface potential step and BCS-BEC crossover*, Phys. Lett. A 372 (2007) 1707-1711.
- K. Morawetz, M. Männel, M. Schreiber, *Phase diagram for interacting Bose gases* , Phys. Rev. B 76 (2007) 075116.
- K. Morawetz, M. Schreiber, B. Schmidt, P. Lipavský, *Enhancement of fermion pairing due to the presence of resonant cavities*, Phys. Rev. B 72 (2005) 174504-1-5

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