Tunneling and Pairing in high Electric Fields

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Gauge invariance and Problem of ansatz

Fourier-transform of the coordinates $x = x_1 - x_2$ and $X = (x_1 + x_2)/2$

$$\bar G^<(k,X) = \int d^4x \; \mathrm{e}^{\left\{\frac{i}{\hbar}\int\limits_{-1/2}^{1/2} d\lambda\, x_\mu[k^\mu + \frac{e}{c}A^\mu(X+\lambda x)]\right\}} < \Psi^+(X-\frac{x}{2})\Psi(X+\frac{x}{2}) >$$

gauge invariance rule $G(p,t) = G(k - eEt, t) = \bar G(k,t) = \bar G(p + eEt, t)$

Generalized–Kadanoff–Baym ansatz P. Lipavsky, V. Spicka, B. Velicky [2]

$$\begin{aligned} G^<(12) &= G^R(13)\Sigma^<(34)G^A(42) \\ &\approx G^R(12)\circ G^R(23)\Sigma^<(34)G^A(42) \equiv G^R(12)\circ f(2) \\ G^<(\mathbf{p},\tau,r,t) &= A(\mathbf{p},\tau,r,t) f(\mathbf{p},r,t-\frac{|\tau|}{2}) \end{aligned}$$

Gauge invariant

$$G^<(\mathbf{k},\tau,\mathbf{r},t) = \exp\left\{-\frac{i}{\hbar}\left(\epsilon_k\tau + \frac{e^2E^2}{24m}\tau^3\right)\right\} f\left(\mathbf{k} - \frac{e\mathbf{E}|\tau|}{2}, \mathbf{r}, t - \frac{|\tau|}{2}\right)$$

- it has the correct spectral properties
- for high field transport it is gauge invariant
- it preserves causality
- the quantum kinetic equations derived with GKB coincide with those obtained with the density matrix technique [3]
- it reproduces the Debye-Onsager relaxation effect [4]

Gauge invariant spectral properties

Electrons with nonpolar optical phonon interaction
R. Bertoncini, A. P. Jauho [5, 6]

Spectral function

- not positive semidefinite, no probability interpretation
- negative-slope zero crossings are quantized energies towards energy concentrates

Density of states

$$\begin{aligned} \rho_\perp(\Omega) &= \int \frac{d\mathbf{k}_\perp}{(2\pi\hbar)^2} A(\mathbf{k}_\perp,\Omega) \\ \rho_\parallel(\Omega) &= \int \frac{dk_z}{2\pi\hbar} A_E(kz,\Omega) \\ \rho(\omega) &= \int d\Omega\, \rho_\perp(\Omega)\rho_\parallel(\omega-\Omega) \end{aligned}$$

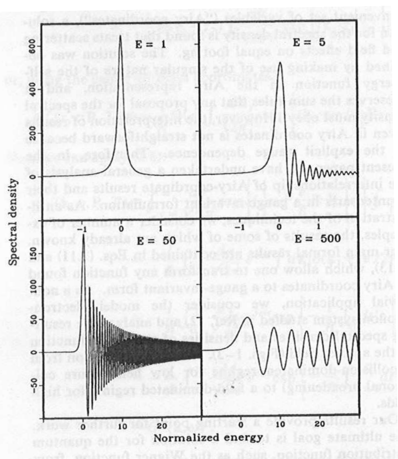


FIG. 1. The spectral density function vs the dimensionless variable $\hbar\omega/\Theta$ for phonon-emission processes with $\mathbf{k}=0$ for four different values of the electric field E given in kV/cm. We use parameters appropriate to Si: an optical-phonon energy $\hbar\omega_0 = 0.039$ eV, $m = 0.295m_0$, where m_0 is the free-electron mass. The electron-phonon interaction is given by a deformation-potential parameter $D_t = 6.85\times10^{10}$ eV m^{-1}.

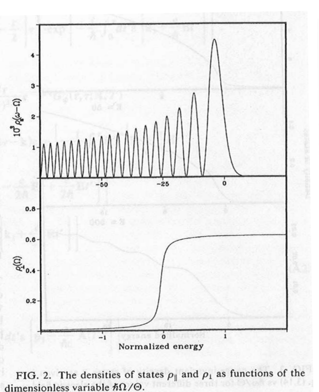


FIG. 2. The densities of states $\rho_\parallel$ and $\rho_\perp$ as functions of the dimensionless variable $\hbar\Omega/\Theta$.

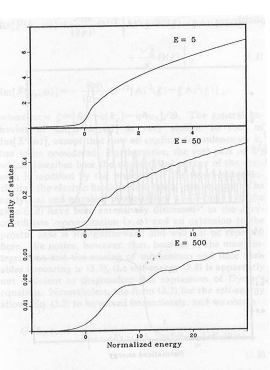


FIG. 3. The gauge-invariant density of states $\bar\rho$ as given in Eq. (3.14) vs $\hbar\omega/\Theta$ for three different values of the electric field.

Formation of subbands

Two-particle effects

On shell two-particle Schroedinger equation

$$(\frac{i}{\hbar}\partial_t + \frac{k_1^2}{2m_1} + \frac{k_2^2}{2m_2} + e_1\mathbf{E}\mathbf{x_1} + e_2\mathbf{E}\mathbf{x_2})\Psi^t_{12} + V_{121'2'}\Psi^t_{1'2'} = 0$$

with $\Psi^t_{12} = \exp(iE_{12}t)\,\Psi_{12}$ and separation $\Psi(k,K) = \phi(k)\Phi(K)$

$$\begin{aligned} \left(\frac{k^2}{2\mu} - \frac{\hbar}{i}(\frac{e_1}{m_1} - \frac{e_2}{m_2})\mu\mathbf{E}\partial_\mathbf{k} - (E_{12} - E_p)\right)\phi(k) + \sum_{k'}\phi(k')V_{k'k} &= 0 \\ \left(\frac{K^2}{2M} - \frac{\hbar}{i}(e_1+e_2)\mathbf{E}\partial_\mathbf{K} - E_p\right)\Phi(K) &= 0 \end{aligned}$$

for $\frac{e_1}{m_1} = \frac{e_2}{m_2}$ no field effects on $\phi(k)$, bound states
Off shell effects from two-particle phase factor $\tau = t_1 - t_2$, $t = (t_1+t_2)/2$

$$\begin{aligned} &\int\limits_{t_2}^{t_1} d\bar t \left(\frac{(\mathbf{p}+e_1\mathbf{E}\bar t)^2}{2m_1} + \frac{(-\mathbf{p}+e_2\mathbf{E}\bar t)^2}{2m_2}\right) \\ &= \left(\frac{p^2}{2\mu_{12}} + \frac{E^2}{2}\left(\frac{e_1^2}{m_1} + \frac{e_2^2}{m_2}\right)t^2\right)\tau + \left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)\mathbf{pE}t\tau + \frac{E^2}{8}\left(\frac{e_1^2}{m_1} + \frac{e_2^2}{m_2}\right)\frac{\tau^3}{3} \end{aligned}$$

- linear term vanishes for equal particles
- quadratic leads to redefinition of energy
- quadratic term gives intrinsic field effect

Ladder approximation in fields

$$\boxed{\mathbf{T}} = \vdots + \vdots\boxed{\mathbf{T}} \qquad T = V + VGG\Phi T$$

Φ is given in terms of Airy-functions [7]

$$\Phi(z) = \frac{\pi}{\lambda_E}\left(\mathrm{Gi}(\frac{z}{\lambda_E}) + i\mathrm{Ai}(\frac{z}{\lambda_E})\right) \to \frac{1}{z+i\eta} \text{ for } E = 0$$

such Bethe-Goldstone equation occurs in field free case

Typical energy scale $\quad \lambda_E = \left(\frac{E^2\hbar^2}{8}(\frac{e_a^2}{m_a} + \frac{e_b^2}{m_b})\right)^{\frac{1}{3}}$

Solution with separable potential $V_{ab}(p,p') = \lambda g(p)g(p')$

$$\langle\mathbf{p}|\mathcal{T}_{ab}(\mathbf{P},\omega)^R|\mathbf{p}'\rangle = \frac{\lambda g(\mathbf{p}' - \frac{e_a-e_b}{2}\mathbf{E}t)g(\mathbf{p} + \frac{e_a-e_b}{2}\mathbf{E}t)}{1 - \lambda J(\mathbf{P},\mathbf{E},\omega,t)}$$

with

$$\begin{aligned} J(\mathbf{P},\mathbf{E},\omega,t) = \int &\frac{d\bar{\mathbf{p}}}{(2\pi\hbar)^3} g(\bar{\mathbf{p}} + \frac{e_a - e_b}{2}\mathbf{E}t)^2 \\ &\times\Phi(\varepsilon_a + \varepsilon_b - \omega)\left(1 - f_a(\frac{\mathbf{P}}{2} - \bar{\mathbf{p}}, t) - f_b(\frac{\mathbf{P}}{2} + \bar{\mathbf{p}}, t)\right) \end{aligned}$$

Field dependent tunneling

Limit of contact potential, $g \to \infty$, fixing interaction strength λ to reproduce scattering length (bound state) $p\cot\delta = p\frac{\mathrm{Re}\mathcal{T}}{\mathrm{Im}\mathcal{T}} = p\frac{1 - \lambda\mathrm{Re}J_0(\varepsilon_p)}{\lambda\mathrm{Im}J_0(\varepsilon_p)} \approx -\frac{\hbar}{a}$

$$\begin{aligned} \mathcal{T}^{-1}_{pp'}(\omega) &\to \mathrm{Re}J_0(\varepsilon_p) - \frac{\hbar}{ap}\mathrm{Im}J_0(\varepsilon_p) - J(\omega), \\ J(\mathbf{P}=0,\omega) &= \int \frac{d\bar{\mathbf{p}}}{(2\pi\hbar)^3}\Phi(2\varepsilon_{\bar{\mathbf{p}}} - \omega)\,(1 - 2f(\varepsilon_{\bar p})) \end{aligned}$$

Without medium $z = -\frac{p^2}{2^{2/3}\lambda_E}, \frac{\omega}{2^{2/3}\lambda_E}$

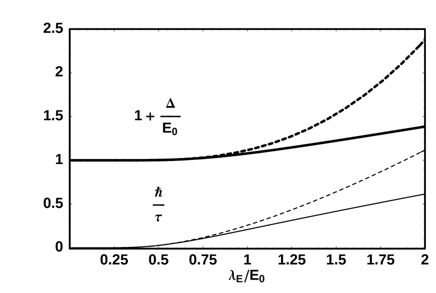
$$\begin{aligned} \mathrm{Im}\,J^0(z) &= \frac{\sqrt{\lambda_E}}{2\hbar^3 2^{5/3}}(Ai'^2(z) - zAi^2(z)) \\ \mathrm{Re}\,J^0(z) &= -\frac{\sqrt{\lambda_E}}{2\hbar^3 2^{5/3}}(Ai'(z)Bi'(z) - zAi(z)Bi(z)) \end{aligned}$$

Bound state solution $\omega = -E_0 = -h^2/ma^2$

$$\mathcal{T}^{-1}(\omega)\frac{4\pi\hbar^3}{m^{3/2}} = -\frac{\hbar}{a\sqrt{m}} + \sqrt{|\omega|} - i\frac{\lambda_E^{3/2}}{4|\omega|}\mathrm{e}^{-\frac{2}{3}\left(\frac{|\omega|}{\lambda_E}\right)^{3/2}} = 0$$

Bound state pole-value of T-matrix turns into resonance $-\omega = E_0 + \Delta + i\frac{\hbar}{\tau}$

$$\begin{aligned} \Delta &= \frac{{\lambda_E}^2}{16E_0^2}\left(3\lambda_E + 4{E_0}^{3/2}\right)\mathrm{e}^{-\frac{4}{3}\left(\frac{E_0}{\lambda_E}\right)^{3/2}} \\ \frac{\hbar}{\tau} &= \frac{{\lambda_E}^{3/2}}{2\sqrt{E_0}}\mathrm{e}^{-\frac{2}{3}\left(\frac{E_0}{\lambda_E}\right)^{3/2}} \end{aligned}$$



Includes WKB–approximation of tunneling through potential wall, $\frac{\hbar}{\tau} \sim \exp\left(\frac{4}{3}\frac{\sqrt{2m}}{e\hbar E}(U_0 - \epsilon)^{3/2}\right)$, if $E_0 = U_0 - \epsilon$, $m_b = \infty$

Field–dependent Pairing

pairing resonance at $\omega = 2\mu + \zeta$

$$\boxed{\begin{aligned} \mathrm{Re}J_0(\varepsilon(p)) - \mathrm{Re}J(2\mu+\zeta) &= \frac{\hbar}{ap}\mathrm{Im}J_0(\varepsilon(p)) \\ \mathrm{Im}J(2\mu+\zeta) &= 0 \end{aligned}}$$

Field free case
$\mathrm{Im}J_0(\varepsilon(p)) = N(p)\pi$, $\mathrm{Im}J_0(2\mu) = 0$
BCS equation for the critical temperature ($V_0 = 4\pi a\hbar^2/m$)

$$N(p_f)\int\limits_0^{\omega_c}\frac{d\xi}{2\xi}\tanh\frac{\xi}{2T_c} = -\frac{1}{V_0}$$

Field dependent case $\lambda_E = \left(\frac{E^2\hbar^2}{8}(\frac{e_a^2}{m_a} + \frac{e_b^2}{m_b})\right)^{\frac{1}{3}}$

From $\mathrm{Im}J(2\mu+\zeta) = 0$ follows $\zeta = -\frac{\lambda_E^3}{24T_c^2} + o(\lambda_E)^6 = -\frac{1}{2}\alpha E^2$ with polarization $\alpha = \frac{\hbar}{96T^2}\left(\frac{e_a^2}{m_a} + \frac{e_b^2}{m_b}\right)$ and Gap equation reads

$$-\frac{1}{N(p_f)V_0} = \int\limits_{-\omega_c}^{\omega_c} d\xi\frac{\pi}{\lambda_E}\mathrm{Gi}(\frac{2\xi}{\lambda_E})\tanh\frac{\xi}{2T_c} \approx \frac{1}{2}\ln\frac{a\omega_c}{T_c} - \frac{\lambda_E}{T}b + o\left(\frac{\lambda_E}{T_c}\right)^5$$

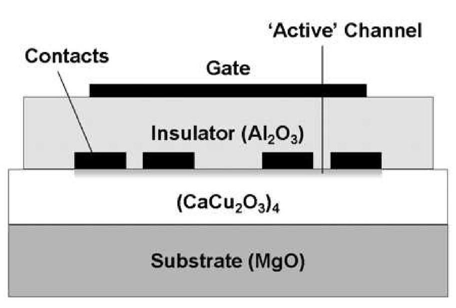
First part is standard $\omega_c \ll T_c$ and $a = 2\mathrm{e}^\gamma/\pi \approx 1.134$. The second part $\lambda_E \ll T_c$ with $b = \sqrt{3\pi}\left(\frac{3}{2}\right)^{1/6}\left(7\Gamma\left(\frac{7}{6}\right) + 6\Gamma\left(\frac{13}{6}\right)\right)/224 \approx 0.190.$

Finally the field–dependent critical temperature [8] with $T_0[n]$ from standard Gap equation

$$\boxed{T_c\ln\frac{T_c}{T_0[n]} = b\lambda_E + o(\lambda_E/T_c)^5}$$

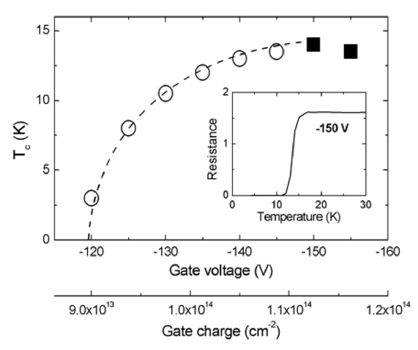
Results

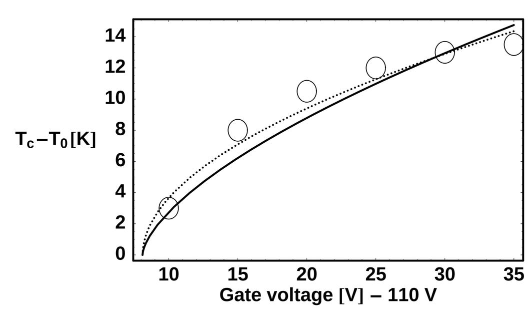
J. H. Schön: *Field induced Superconductivity in a Spin-Ladder Cuprate* [9]



$$\begin{aligned} E &= \frac{Q}{\epsilon_0 A} = K\frac{U}{\epsilon_0} \\ K &= 120\frac{\mathrm{nF}}{\mathrm{cm}^2} \end{aligned}$$

$$\lambda_E = 817.7K\left(\frac{\epsilon_{cup}}{\epsilon_{ins}}\frac{U}{\mathrm{V}}\right)^{2/3}$$





The critical temperature in dependence on the applied voltage. The solid line gives the theoretical value $\epsilon_{\mathrm{ins}}/\epsilon_{\mathrm{cup}} \approx 10$ and $T_0 = 2\times10^{-8}$ K while the dashed line is for $\epsilon_{\mathrm{ins}}/\epsilon_{\mathrm{cup}} \approx 192$ and $T_0 = 1$ K.

Summary two–particle properties

- Off–shell behavior influenced by fields
- Two identical particles suffer an field effect (contradiction to classical picture)
- Bound states become resonances
- Suppression of " Pauli blocking "
- Life time of bound states in fields, pair creation rate
- Pairing transition at higher temperatures due to field
- Additional field effect on top of T_c in FET structures

paper available at http://www.mpipks-dresden.mpg.de/∼morawetz

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