Transport with three-particle interaction

Klaus Morawetz[1]

[1]Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Straße 38, 01187 Dresden

Coester line

- Refined two-particle calculations using Bruckner theory [1] and beyond [2] are not crossing the "Coester line" in that they lead to binding energies and/or densities above the saturation ones
- Only density dependent Skyrme parameterisations [3, 4] originally introduced [5, 6], three-particle interactions [7–9] or relativistic treatments [10] can reproduce the correct binding energy and saturation density
- Relativistic approach nonlinear density dependence of scalar density [11]: $(n/n_0)^{8/3}$. Higher order effects [12]: $(n/n_0)^{3.4}$
- No transport theory with three-particle contact interaction formulated, though of wider interest [13]
- Natural density dependence of two particle cross section, Skyrme drift and collision integral should appear from same microscopic footing

Three - particle Kadanoff and Baym equation

Hamiltonian

$$
\begin{aligned}
H = \sum_i a_i^+ \frac{\hbar^2\nabla^2}{2m} a_i &+ \frac{1}{2}\sum_{ij} a_i^+ a_j^+ a_i a_j V_2(i,j) \\
&+ \frac{1}{6}\sum_{ijk} a_i^+ a_j^+ a_k^+ a_i a_j a_k V_3(i,j,k)
\end{aligned}
$$

where a_i^+, a_i are creation and annihilation operators with $i=(x_i,t_i,...)$

$$
\begin{aligned}
V_2(i,j) &= t_0\delta(i-j) \\
V_3(i,j,k) &= t_3\delta(i-j)\delta(i-k).
\end{aligned}
$$

Equation of motion for the causal Green function $G(1,2)=\frac{1}{i} < Ta_1^+ a_2 >$

$$
\begin{aligned}
(i\hbar\partial_{t_1} - \frac{\hbar^2\nabla_1^2}{2m})G(1,1') &= \delta(1-1') + \int d2 V(1,2) G_2(1,2,1',2^+) \\
&+\frac{1}{2}\int d2d3 V(1,2,3) G_3(1,2,3,1',2^+,3^+) \\
&\equiv \delta(1-1') - \int dx_2 \Sigma_{HF}(x_1,x_2,t)G(x_2,t,x_1,t) \\
&+\int d2\,(\Sigma(1,2)G(2,1') - \Sigma^>(1,2)G^<(2,1'))
\end{aligned}
$$

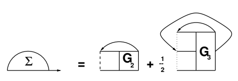
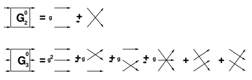


Fig. 1:

Mean field

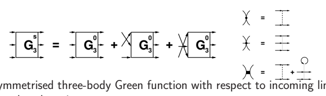


The lowest order expansion of Green functions. The statistical factor g due to degeneracy in the system. Result is usual Skyrme expression

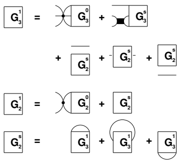
$$
\Sigma^{\rm HF}(1,1') = \left[\frac{g\pm1}{g}t_0 n(1) + \frac{(g\pm1)(g\pm2)}{2g^2}t_3 n(1)^2\right]\delta(1-1')
$$

where the density is $n(1)=G^<(1,1)/g$.

Selfenergy in Born Approximation



Symmetrised three-body Green function with respect to incoming lines and introduced vertices



The first order expansion with respect to interaction. Inserting this into Fig. 1 we obtain the final result $\Sigma^<(1,1')=$

$$
\begin{aligned}
&(g\pm1)\left[t_0^2+(g\pm2)\left(4t_0^2+6t_3^2\frac{n(1)n(1')}{g^2}+5t_0t_3(\frac{n(1)}{g}+\frac{n(1')}{g})\right)\right] \\
&\quad\times G^<(1,1')^2 G^>(1'1) \\
&+\frac{1}{2}(g\pm1)(g\pm2)t_3^2\; G^<(1,1')^3G^>(1'1)^2.
\end{aligned}
$$

Note that it vanishs for Fermions and $g=1$ due to Pauli-principle; for $g=2$ only the two - particle interactions survive.

Kinetic equation

$$
\begin{aligned}
&\frac{\partial}{\partial t}f_1(t)+\frac{p_1}{m}\frac{\partial}{\partial r}f_1(t)-\frac{\partial}{\partial r}\sigma^{\rm HF}(t)\frac{\partial}{\partial p_1}f_1(t) \\
&\quad=\frac{2}{\hbar^2}\int\frac{dp_2dp_1'dp_2'}{(2\pi\hbar)^6}T_2{}^2\delta(p_1+p_2-p_1'-p_2') \\
&\qquad\times\int_{t_0}^t d\tau\cos\left[\frac{1}{\hbar}(E_1+E_2-E_1'-E_2')(t-\tau)\right] \\
&\qquad\times\left\{f_1'f_2'\bar f_1\bar f_2-f_1f_2\bar f_1'\bar f_2'\right\}_\tau \\
&\quad+\frac{2}{\hbar^2}\int\frac{dp_2dp_3dp_1'dp_2'dp_3'}{(2\pi\hbar)^{12}}T_3{}^2\delta(p_1+p_2+p_3-p_1'-p_2'-p_3') \\
&\qquad\times\int_{t_0}^t d\tau\cos\left[(E_1+E_2+E_3-E_1'-E_2'-E_3')\frac{t-\tau}{\hbar}\right] \\
&\qquad\times\left\{f_1'f_2'f_3'\bar f_1\bar f_2\bar f_3-f_1f_2f_3\bar f_1'\bar f_2'\bar f_3'\right\}_\tau
\end{aligned}
$$

with $\bar f=1-f$, and the particle dispersion $E=p^2/2m+\sigma^{\rm HF}$. The two - and three- particle cross sections are

$$
\begin{aligned}
T_2{}^2 &= \frac{g\pm1}{g^2}\left[t_0{}^2+(g\pm2)\left(4t_0{}^2+6t_3{}^2\frac{n(1)n(1')}{g^2}+5t_0t_3(\frac{n(1)}{g}+\frac{n(1')}{g})\right)\right] \\
T_3{}^2 &= \frac{1}{2}\frac{g\pm1}{g^3}(g\pm2)t_3{}^2.
\end{aligned}
$$

Balance equation

Density and momentum

$$
\begin{aligned}
&\frac{\partial n}{\partial t}+\frac{\partial}{\partial R}\int\frac{dp}{(2\pi\hbar)^3}\frac{\partial E}{\partial p}f=0 \\
&\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial R_j}\int\frac{dp}{(2\pi\hbar)^3}(p_i\frac{\partial E}{\partial p_j}f+\mathcal{E}\delta_{ij})=0
\end{aligned}
$$

where the mean field energy of the system varies as

$$
\delta\mathcal{E} = \int\frac{dp}{(2\pi\hbar)^3}\frac{\delta\mathcal{E}}{\delta f(p)}\delta f(p)=\int\frac{dp}{(2\pi\hbar)^3}E\delta f(p)
$$

such that follows

$$
\mathcal{E} =< \frac{k^2}{2m} > +\frac{g\pm1}{2g}t_0n^2+\frac{(g\pm1)(g\pm2)}{6g^2}t_3n^3.
$$

With the help of this quantity the balance of energy density reads

$$
\frac{\partial\mathcal{E}}{\partial t}+\frac{\partial}{\partial R}\int\frac{dp}{(2\pi\hbar)^3}E\frac{\partial E}{\partial p}f=-\frac{\partial}{\partial t}E_{\rm corr_2}(t)-\frac{\partial}{\partial t}E_{\rm corr_3}(t)
$$

Correlated flux and density beyond Born approximation [14, 15], for time dependent Born correlation energy see [16, 17]

Correlational Energy

Hartree-Fock mean field

$$
\frac{E}{A}=\frac{\mathcal{E}}{n}=\frac{3}{5}\epsilon_f+\frac{3}{8}t_0n+\frac{1}{16}t_3n^2.
$$

Then the nuclear binding at $n_0=0.16$ fm^{-3} with an energy of $E_B=-15.68$MeV is reproduced by the set

$$
\begin{aligned}
t_0 &= -\frac{16}{15n_0}(2\epsilon_f-5E_B)=-1026.67\text{MeV fm}^3 \\
t_3 &= \frac{16}{5n_0^2}(\epsilon_f-5E_B)=14625\text{Mev fm}^6
\end{aligned}
$$

and a compressibility $K=9n^2\frac{\partial^2E/A}{\partial n^2}=\frac{9}{8}t_3n_0^2-\frac{6}{5}\epsilon_f=377\text{MeV}$.

Two- and Three- particle correlational energy

Galitskii result for the two - particle ground state correlation energy [18]

$$
\begin{aligned}
\frac{E_{\rm corr_2}}{n} &= 4\epsilon_f\frac{2\log2-11}{35}(\frac{p_fmT_2}{4\pi^2\hbar^3})^2 \\
&= -\frac{5.7910^{-5}}{\text{MeV fm}^2}n^{4/3}\left[9t_0{}^2+\frac{3}{4}t_3{}^3n^2+5t_3t_0n\right].
\end{aligned}
$$

Three-particle correlational energy not given in the literature

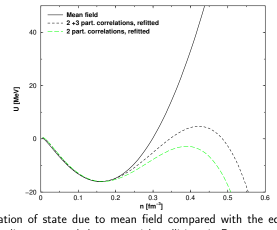
$$
\begin{aligned}
\frac{E_{\rm corr_3}}{n} &= 4\epsilon_f\frac{9013}{2\cdot9\cdot25\cdot77\cdot13}(\frac{p_f^4mT_3}{4\pi^4\hbar^6})^2 \\
&= \frac{2.3710^{-6}}{\text{MeV fm}^2}n^{10/3}t_3{}^2.
\end{aligned}
$$

Saturation properties

Refitting to the nuclear binding of

$$
\begin{aligned}
\tilde{t_0} &= -745.71\text{MeV fm}^3 \\
\tilde{t_3} &= 8272.8.\text{MeV fm}^6
\end{aligned}
$$

which leads to a compressibility of $\tilde K=351\text{MeV}$ somewhat lower than the mean field compressibility.



The equation of state due to mean field compared with the equation of state including two - and three - particle collisions in Born approximation.

Relative importance

$$
\left|\frac{E_2}{E_3}\right| = \frac{2^5\cdot5\cdot11\cdot13}{9013}(11-2\log2)\left[9\frac{t_0{}^2}{n^2t_3{}^2}+\frac{3}{4}+5\frac{t_0}{t_3n}\right]
$$

This ratio is decreasing until it reaches the density $n_{\rm min}=-\frac{18t_0}{5t_3}=0.32\text{fm}^{-3}$ where the ratio has the minimum of 1.4. For higher densities the constant value 18.3 is approached.

Summary

- For two - and three - particle contact interaction the Kadanoff and Baym equation of motion and a Boltzmann kinetic equation is derived
- Two - particle collision integral has density - dependent cross section from the three - particle contact interaction
- Drift side and collision side is derived from the same microscopic footing
- Three- particle orrelational energy first time calculated analytically
- Two- and three- particle correlational energies same leading power of density $(n/n_0)^{10/3}4$
- Maximum in the energy at 3 times nuclear density, above this density Born approximation fails in that the system collapses
- Explicit three-particle collision integral can be neglected below nuclear density, around twice nuclear density the three particle collision integral same importance as two - particle one, for higher densities the three - particle collision integral is negligible

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