Electric fields above superconductor surfaces

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History of Bernoulli potential

Equation of motion for condensate, London condition $m\mathbf{v} = -e\mathbf{A}$

$$m\dot{\mathbf{v}} = -e\frac{\partial \mathbf{A}}{\partial t} - e(\mathbf{v}\nabla)\mathbf{A} = e(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + \nabla\left(e\varphi + \frac{1}{2}mv^2\right)$$

Compare with Newton equation of motion, force on condensate $\mathbf{F}_s$

$$\nabla e\varphi = \mathbf{F}_s - \nabla\frac{1}{2}mv^2$$

Thermodynamic corrections:
Kinetic energy $F_{kin} = n_s\frac{1}{2}mv^2$ of condensate determines change in chemical potential

$$e\varphi = -\mu = -\frac{\partial}{\partial n}F_{kin} = -\frac{\partial n_s}{\partial n}\frac{1}{2}mv^2 = -\frac{n_s}{n}\frac{1}{2}mv^2 + 4\frac{n_n}{n}\frac{\partial \ln T_c}{\partial \ln n}\frac{1}{2}mv^2$$

G. Ryckaizen [1]: possibility to measure material parameters
Capacitive coupling (constant electrochemical potential):

J. Bok and J. Klein [2], T. D. Morris, J. B. Brown [3]

$$e\varphi = -\frac{n_s}{n}\frac{1}{2}mv^2 = -\frac{1}{n}\frac{B^2}{2\mu_0}$$

and not $(0.1 + 3.2)\frac{m}{2}v^2$ (following BCS for Pb at $T = 7$K)

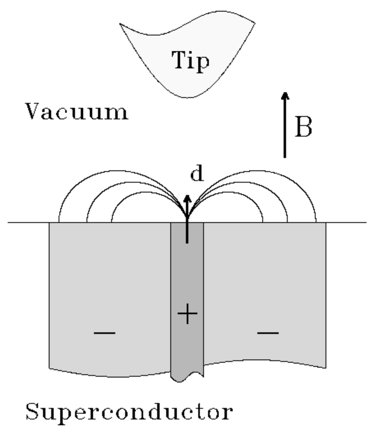
$\longrightarrow$ *No thermodynamic corrections*
Explanation: Compensation due to surface dipoles
Budd-Vannimenus theorem

$$e\varphi_{\rm surf} - e\varphi = n\frac{\partial}{\partial n}\frac{\delta F}{n} \quad\rightarrow\quad e\varphi_{\rm surf} = -\frac{n_s}{n}\frac{1}{2}mv^2$$

P. Lipavský and J. Koláček and J.J. Mareš, K. Morawetz [4]

Suggestion for spatial variations

Blatter et.al [5]
- extreme type II: spatial shape of superconducting fraction sharper image than magnetic field modulation
- space modulation of gap $\triangle$ induces charge transfer so that Bernoulli potential develops inside the superconductor
- since $\phi \sim |\Delta|^2$ observation of gap and GL wave function



Our basic assumptions:
- superconducting films thin on GL scales $L \ll \xi, \lambda$, but thick on BCS coherence length, $L \gg \xi_0$
- magnetic field $\mathbf{B}||z$ penetrates the superconductor in form of vortices
- electrostatic potential scanned close to surface

Electrostatic potential

Electrostatic potential leaking out of superconductor by 3 types of charges

1. bulk charge: transfer of electrons from the inner to outer regions of vortices creating Coulomb force which balances forces
   - electrons rotate around vortex center, inertial (centrifugal)
   - magnetic field pushes electrons via the Lorentz force outward
   - energy of Cooper pairs is lower than free electrons, unpaired electrons in vortex core attracted towards the condensate around core, force points outward
2. surface dipole: all contributions of pairing forces are canceled by surface dipole [6]
   resulting observable surface potential $\quad e\phi_0 = -\frac{f_{\rm el}}{n}$
3. surface charge: distributed on scale of Thomas-Fermi screening length $\xi, \xi_0, \lambda \gg \lambda_{\rm TF} \to 0$ ideal two-dimensional, matching of inner and outer potentials

$$\phi(\mathbf{K}) = \frac{2}{\Omega}\int_\Omega d\mathbf{r}\,\phi_0(\mathbf{r})\cos(\mathbf{K}\mathbf{r})$$

about Abrikosov vortex lattice potential at any distance from surface,

$$\phi(\mathbf{r}, z) = \langle\phi\rangle + \sum_{\mathbf{K}\neq 0}\phi(\mathbf{K})\,\mathrm{e}^{-|\mathbf{K}|z}\cos(\mathbf{K}\mathbf{r})$$

References

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[2] J. Bok and J. Klein. *Phys. Rev. Lett.*, 20:660, 1968.
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[7] K. Kumagai, K. Nozaki, and Y. Matsuda. *Phys. Rev. B*, 63:144502, 2001.

Surface potential within the Ginzburg-Landau theory

Bardeen's low temperature extension of GL (free energy by Gorter Casimir two-fluid, subtraction of free energy of normal state)

$$f_{\rm el} = \frac{1}{2}\gamma T^2 + \frac{1}{2m^*}\bar\psi\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi - \varepsilon_{\rm con}\frac{2|\psi|^2}{n} - \frac{1}{2}\gamma T^2\sqrt{1 - \frac{2|\psi|^2}{n}}$$
$$\approx \frac{1}{2m^*}\bar\psi\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 \qquad \text{near } T_c$$

and $\beta = \frac{1}{2n^2}\gamma T^2$, $\alpha = (-4\varepsilon_{\rm con}/n)(1-t)$, where $\varepsilon_{\rm con} = \frac{1}{4}\gamma T_c^2$, $t = T/T_c$

From GL equation $\frac{(-i\hbar\nabla - e^*\mathbf{A})^2}{2m^*}\psi = -\alpha\psi - \beta|\psi|^2\psi$ follows $f_{\rm el} = -\frac{1}{2}\beta|\psi|^4$ and surface potential reads

$$e\phi_0 = \frac{1}{2n}\beta|\psi|^4$$

without surface dipole, surface potential equal to internal potential

$$e\phi = -\frac{1}{2m^* n}\bar\psi\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} - \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\left(\frac{|\psi|^2}{n} + \frac{|\psi|^4}{2n^2}\right)$$

inertial and Lorentz forces neglecting pairing forces
approximation of Khomskii and Kusmartsev adopted by Blatter

$$e\phi_{\rm Bl} = \frac{\gamma T_c}{n}\frac{\partial T_c}{\partial n}|\psi|^2$$

B close to B_{c2} for thin layer

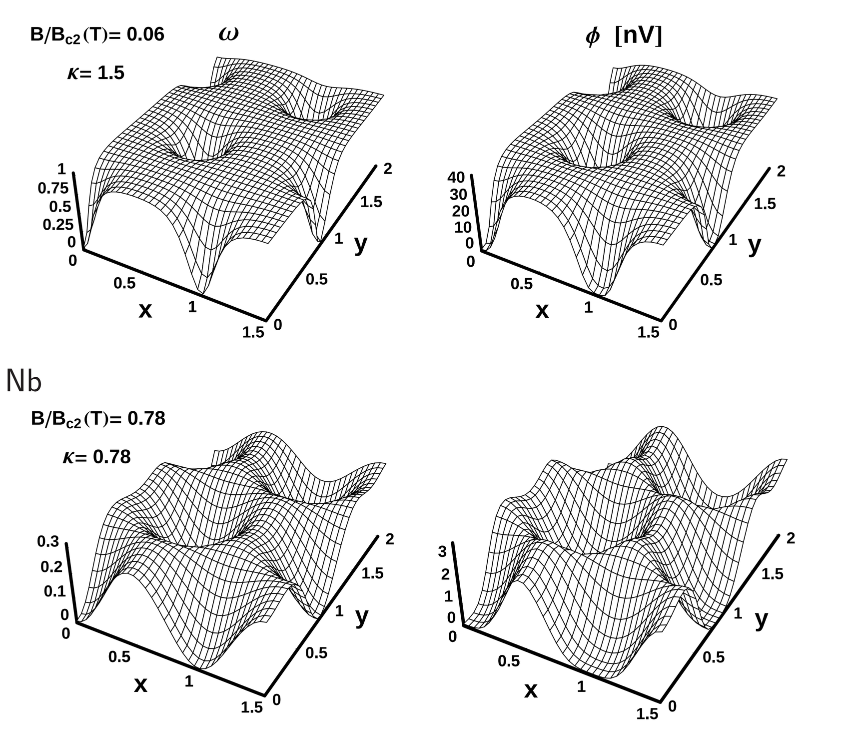
$$\langle\omega\rangle = \frac{(1-b)}{\beta_{\rm A}}, \quad \langle\omega^2\rangle = \frac{(1-b)^2}{\beta_{\rm A}}$$

with $\omega = \frac{|\psi|^2}{|\psi_\infty|^2}$, $b = \frac{B}{B_{c2}}$

mean value becomes

$$\langle e\phi_0\rangle = \frac{\varepsilon_{\rm con}}{n\beta_{\rm A}}\left(1 - t^2\right)^2(1-b)^2$$
$$\langle e\phi_{\rm Bl}\rangle = \frac{\varepsilon_{\rm con}}{n\beta_{\rm A}}\frac{\partial\ln T_c}{\partial\ln n}2\left(1 - t^4\right)(1-b)$$

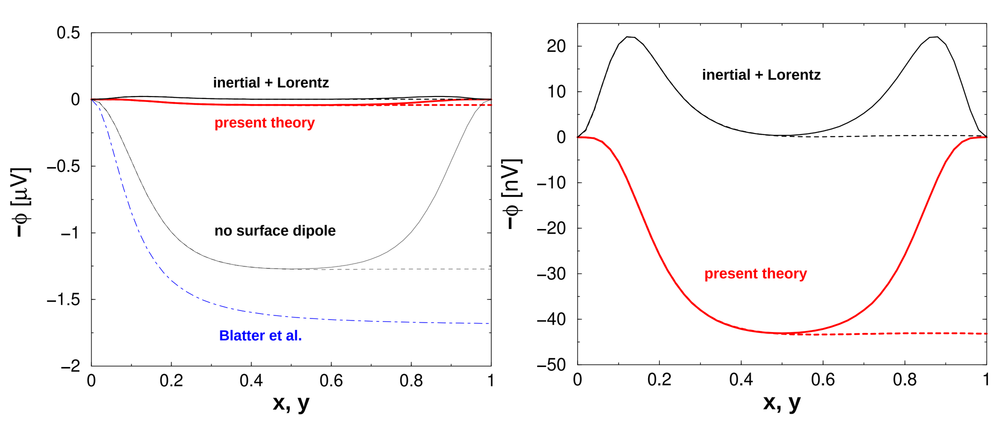
Numerical results for Niobium



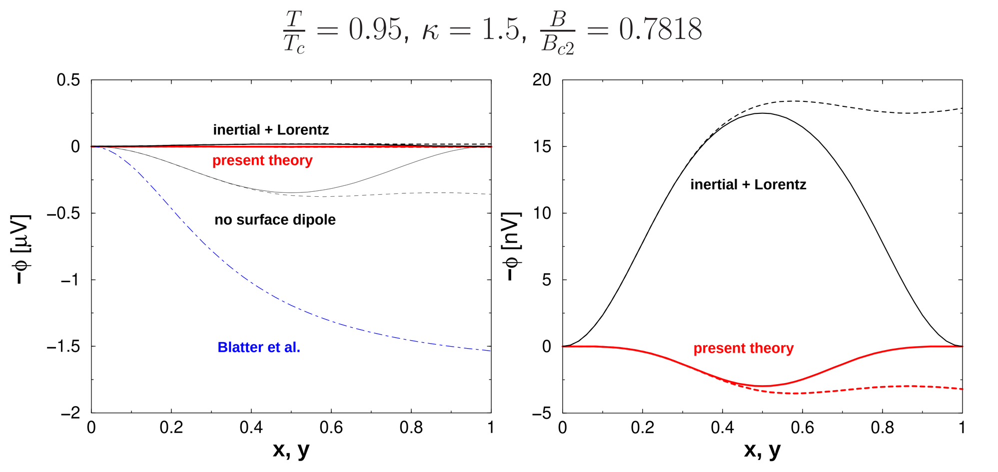
Superconducting fraction $\omega = |\psi|^2/\psi_\infty^2$ (left column) where ψ_∞ in absence of magnetic field, electrostatic potential ϕ (right column) at surface of superconductor with Abrikosov vortex lattice, temperature $T = 0.95\,T_c$

- At vortex center, $x^2 + y^2 = r^2 \to 0$, superconducting fraction $\omega \propto r^2$, but potential vanishes as $\phi_0 \propto r^4$
- possible to be observed in future measurements

Effect of surface dipole



$\frac{T}{T_c} = 0.95$, $\kappa = 1.5$, $\frac{B}{B_{c2}} = 0.7818$



$\frac{T}{T_c} = 0.95$, $\kappa = 0.78$, $\frac{B}{B_{c2}} = 0.7818$

lower κ: in internal potential GL wave function suppressed by magnetic field compared to Blatter's approach (Clem)

- internal potential similar Blatter's result (Clem model, neglect of $\frac{\partial\gamma}{\partial n}$)
- full theory and inertial/Lorentz forces are much smaller
- surface dipole cancels major part of pairing forces
- full theory and inertial/Lorentz forces result in different profiles and sign

Charged vortices in HTSC probed by NMR [7]

- NMR frequency depends on B, γ and number N of holes per Cu/plane
- Polarization of Cu, coupling of spin with electrical field gradient leads to splitting of quadrupole resonance

$$\nu_Q^{NQR} = E_{\pm 3/2} - E_{\pm 1/2} = AN + C.$$

Therefore the combined Zeeman plus quadrupole splitting

$$\nu_1(B) = E_{\frac{1}{2}} - E_{-\frac{1}{2}} = \gamma B$$
$$\nu_2(B) = E_{-\frac{1}{2}} - E_{-\frac{3}{2}} = \gamma B - \nu_Q^{NQR}$$
$$\nu_3(B) = E_{\frac{3}{2}} - E_{\frac{1}{2}} = \gamma B + \nu_Q^{NQR}$$

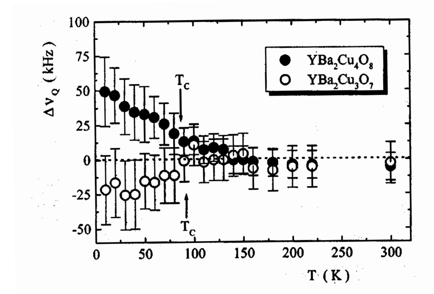
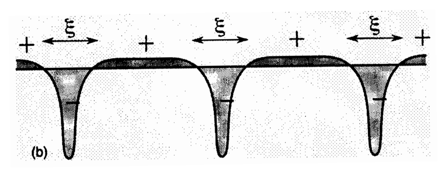


FIG. 7. T dependence of $\Delta\nu_Q = \nu_Q(0) - \nu_Q(H)$ for YBa$_2$Cu$_3$O$_7$ and YBa$_2$Cu$_4$O$_8$. In both materials nonzero $\Delta\nu_Q$ is clearly observed below T_c, showing that the electron density outside the core differs from that in zero field.

from which follows
$$N_B - N_0 = \frac{1}{2A}(\nu_3(B) - \nu_2(B) - 2\nu_3(0))$$

Problems:



Charge accumulated in vortex core per layer (BCS)

$$Q \approx \frac{2ek_F s}{\pi^3}\left(\frac{\lambda_{TF}}{\xi}\right)^2\left(\frac{d\ln T_c}{d\ln\mu}\right)$$
$$\sim 10^{-5}e \quad \exp: 10^{-2}e$$

underdoped regime: $Q > 0$, overdoped: $Q < 0$, contrast to experiment

Space variation of NMR lines

charge transfer between planes and chains
all lines have similar width $\Gamma \approx 200$kHz $>> \delta\nu \approx 20$kHz
averaging of the NMR line over Abrikosov lattice

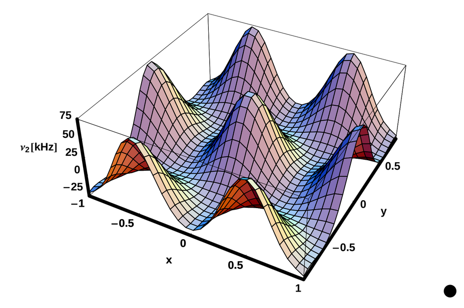
$$F_{2/3}(\tilde\nu) = \frac{1}{\pi\Omega}\int d\mathbf{r}\frac{\Gamma}{(\tilde\nu - \tilde\nu_{2/3}(\mathbf{r}))^2 + \Gamma^2} \qquad \nu_{2/3}(\mathbf{r}) = \gamma B(\mathbf{r}) \mp C \mp AN(\mathbf{r})$$

with $N(\mathbf{r}) = \frac{\Omega_{Cu}}{e}(\rho(\mathbf{r}) - \rho_\infty)$, density is given in terms of electrostatic potential via layered structure screening (Lawrence/Doniach model):

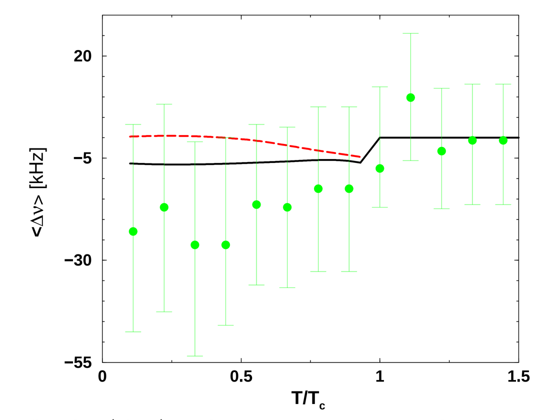
$$\rho(\mathbf{k}) = \frac{2k\epsilon(1 + \mathrm{e}^{-kD})}{(1 - \mathrm{e}^{-2kD_{c-p}})(1 + \mathrm{e}^{-kD_{p-p}})}\phi(\mathbf{k})[\Psi(r)] \qquad \text{compare 3D}: \ -\epsilon k^2\phi(\mathbf{k})$$

$B(\mathbf{r})$ and $\Psi(\mathbf{r})$ from extended GL-theory

Local shifts of lower NMR frequency $\nu_2(\mathbf{r})$ at $T = 0.6\,T_c$



Comparison with experiment



- Space variation of shifts comparable to line width $\to$ no approximation by mean value
- Dominant role of magnetic field

Shift $\langle\Delta\nu\rangle$ of $\Gamma = 140$kHz of a single crystal, and after averaging over grain orientation

Summary

- Electrostatic potential above surface of thin superconducting layer with the Abrikosov vortex lattice calculated
- Surface dipole strongly modifies magnitude of potential, in particular when GL wave function has a small magnitude due to $\phi_0 \propto |\psi|^4$, while without dipole $\phi_{\rm Bl} \propto |\psi|^2$
- Possible cases for which presented theory can be tested:
  - at vortex core $|\psi|^2 \propto r^2$ so that $\phi_0 \propto r^4$ while $\phi_{\rm Bl} \propto r^2$
  - at temperatures close the critical temperature, $t \to 1$, $|\psi|^2 \propto 1 - t$, therefore $\phi_0 \propto (1-t)^2$ while $\phi_{\rm Bl} \propto 1 - t$
  - magnetic fields close to upper critical field, $b \to 1$, $|\psi|^2 \propto 1 - b$ so that $\phi_0 \propto (1-b)^2$ while $\phi_{\rm Bl} \propto 1 - b$
- Hall voltage measurements across the surface yield no thermodynamic contributions, cannot be used to observe material parameter
  - Surface dipole cancels exactly thermodynamic corrections, Bernoulli potential with fountain term remaining
  - Internal probes like NMR are necessary
- Measurements of Kumagai, Nozaki and Matsuda show that the charge transfer in SC is not fully understood
  - We reproduce correct sign and temperature dependence by assuming charge transfer between planes and chains