Electric fields in superconductors

K. Morawetz[1,2], P. Lipavský[3], J. Koláček[3], E. H. Brandt[4], M. Schreiber[1]

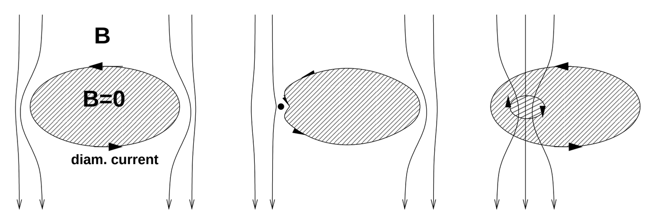
[1]Institute of Physics, Chemnitz University of Technology, Reichenhainer Str.70, 09107 Chemnitz, Germany
[2]Max-Planck-Institute for the Physics of Complex Systems, Nöthnitzer Str. 38, 01187 Dresden, Germany
[3]Academy of Sciences, Cukrovarnická 10, 16200 Praha 6, Czech Republic
[4]Max-Planck-Institute for Metals Research, D-70506 Stuttgart, Germany

CHEMNITZ UNIVERSITY OF TECHNOLOGY

MAX-PLANCK-GESELLSCHAFT

Electric fields ?

Meissner-Ochsenfeld effect: magnetic field expulsion → diamagnetic currents and Lorentz forces



- With increasing field magnetic field penetrates in form of vortices (type II superconductors) → inertial forces
- Internal fields to keep charge distribution close to neutrality, to compensate inhomogeneity due to diamagnetic currents
- Corresponding electrostatic potential has the form of a Bernoulli potential and compensate inertial and Lorentz forces

Hope:
Measurement of electrostatic potentials to access material parameter:

- Surface (Bok&Klein '68): no information (puzzle)
- Charge of vortices (Kumagai& Nozaki& Matsuda'01): bulk properties

Density of free energy

1. Condensation energy Gorter and Casimir (Phys. Z. 35 (1934) 963)

superconducting state characterized by order parameter ϖ

$$f_s = U - \varepsilon_{\rm con}\varpi - \frac{1}{2}\gamma T^2\sqrt{1-\varpi}$$

Equilibrium $\delta\mathcal{F}_s/\delta\varpi = \partial f_s/\partial\varpi = 0$, at critical T_c is $\varpi = 0$

$$\varepsilon_{\rm con} = \frac{\gamma T^2}{4\sqrt{1-\varpi}} \qquad \varepsilon_{\rm con} = \frac{1}{4}\gamma T_c^2 \quad\rightarrow\quad \varpi = 1 - \frac{T^4}{T_c^4} \approx \frac{n_s}{n_s+n_n}$$

2. Kinetic energy

Ginzburg and Landau proposed (Ž. eksper. teor. Fiz. 20,1064 (1950)), normalized $|\psi|^2 = \frac{n_s}{2}$

$$\mathcal{F}_{\rm kin} = \int d\mathbf{r}\frac{1}{2m^*}|(-i\hbar\nabla - e^*\mathbf{A})\,\psi|^2$$

3. Electromagnetic energy

Coulomb interaction, where charge density $\rho = e^*|\psi|^2 + en_n + \rho_{\rm latt}$

$$\mathcal{F}_C = \frac{1}{2}\iint d\mathbf{r}d\mathbf{r}'\frac{1}{4\pi\epsilon}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r})\rho(\mathbf{r}')$$

Magnetic free energy, $\mathbf{B}_a$ is applied magnetic field

$$\mathcal{F}_M = \int d\mathbf{r}\frac{1}{2\mu_0}(\mathbf{B}-\mathbf{B}_a)^2$$

$\mathbf{B}$, φ, n, n_s, ρ or ϖ are subsidiary and functions of ψ, $\mathbf{A}$ and n_n

Lagrange equations

$$\frac{\delta\mathcal{F}_a}{\delta l} = \frac{\partial f_a}{\partial l} - \nabla\frac{\partial f_a}{\partial\nabla l}$$

- $l = \mathbf{A}$ Second Ginzburg-Landau equation

$$\nabla\times\nabla\times\mathbf{A} = \mu_0\mathbf{j}, \qquad \mathbf{j} = \frac{e^*}{m^*}{\rm Re}\,\bar\psi(-i\hbar\nabla - e^*\mathbf{A})\psi$$

- $l = \bar\psi$ First GL-equation $\frac{1}{2m^*}(-i\hbar\nabla - e^*\mathbf{A})^2\psi + \chi\psi = 0$

The effective potential covers forces acting on Cooper pair

$$\chi = \frac{\delta}{\delta|\psi|^2}(\mathcal{F}_C + \mathcal{F}_s) = e^*\varphi + \frac{\partial f_s}{\partial|\psi|^2}$$

- $l = n_n$: $e\varphi = -\frac{\partial f_s}{\partial n_n}$ sum of all potentials on normal electrons vanishes

Effective potential (Bardeen Phys. Rev. 94 (1954) 554)

$$\chi = \frac{\partial f_s}{\partial|\psi|^2} - 2\frac{\partial f_s}{\partial n_n} = -2\frac{\varepsilon_{\rm con}}{n} + \frac{\gamma T^2}{2n}\frac{1}{\sqrt{1-\frac{2|\psi|^2}{n}}}$$

extends GL result towards lower temperatures, close to T_c,

$$\chi \to \alpha + \beta|\psi|^2, \qquad \text{with} \quad \alpha = \frac{\gamma T_c}{n}(T - T_c), \quad \beta = \frac{\gamma T_c^2}{2n^2}$$

Electrostatic potential

$$-e\varphi = \frac{\partial f_s}{\partial n_n} = \frac{\partial U}{\partial n} - \frac{\partial}{\partial n_n}\left(\varepsilon_{\rm con}\varpi + \frac{1}{2}\gamma T^2\sqrt{1-\varpi}\right)$$

If $\varepsilon_{\rm con}$ and γ depend only on the total density

$$e\varphi - \lambda_{TF}^2\nabla^2 e\varphi = = \chi\frac{|\psi|^2}{n} + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} + \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\sqrt{1-\frac{2|\psi|^2}{n}}$$

with $\frac{\partial U}{\partial n} = E_F = E_F^0 + \frac{\partial E_F}{\partial n}\frac{\rho}{e} = -\lambda_{TF}^2\nabla^2 e\varphi$ and $\lambda_{TF}^2 = \epsilon/(2\mathcal{D}e^2)$

- First term is non-local Bernoulli potential, from GL equation

$$\chi\frac{|\psi|^2}{n} = -\frac{1}{2m^*n}\bar\psi(-i\hbar\nabla - e^*\mathbf{A})^2\psi \approx -\frac{1}{2n}m^*v^2|\psi|^2 = -(n_s/n)\frac{1}{2}mv^2$$

A.G. van Vijfeijken and F. S. Staas, Phys. Lett. 12 175 (1964)

- Third term finite at T_c: normal state thermoelectric field
- Weak electrical current $\lambda_{TF}^2\nabla^2\varphi \ll \varphi$, $n\,mv^2/2 \ll \varepsilon_{\rm con} - \gamma T^2/4$

$$e\varphi' = -\frac{n_s}{n}\frac{1}{2}mv^2 + 4\frac{n_n}{n}\frac{\partial\ln T_c}{\partial\ln n}\frac{1}{2}mv^2$$

G. Ryckaizen, J. Phys. C 2, 1334 (1969)

Budd-Vannimenus theorem for superconductors

We derive modification of Budd-Vannimenus theorem

$$\rho_{\rm lat}(\varphi_0 - \varphi(0)) = f_{\rm el} - n\frac{\partial f_{\rm el}}{\partial n}$$

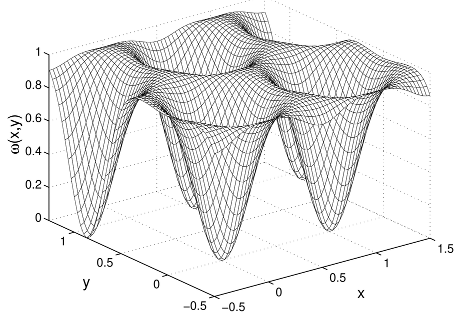
Potential step at surface due to surface dipole in terms of free energy with no regards of potential inside
$e\varphi = -\frac{\partial f_{\rm el}}{\partial n} \to -e\varphi(0)$ for $x = 0$ (includes no surface dipole), to get:

$$\rho_{\rm lat}\varphi_0 = f_{\rm el} + \varphi(0)\,(\rho_{\rm lat} + en)$$

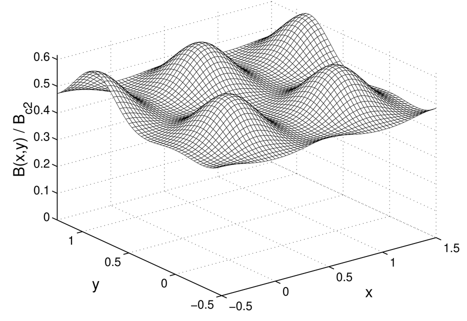
Simplification:
relative charge deviation $(\rho_{\rm lat} + en)/\rho_{\rm lat} \sim 10^{-8} - 10^{-4}$
small corrections in the GL equation, with same accuracy from

$$\varphi_0 = \frac{f_{\rm el}}{\rho_{\rm lat}}$$

Numerical results on Nb

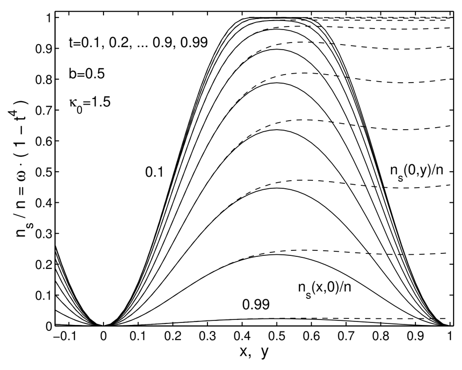


Condensate density $\omega(x,y)$ in the triangular lattice for temperature $t = 0.5$, magnetic induction $\bar B/B_{c2} = 0.5$, and GL parameter $\kappa_0 = 1.5$.

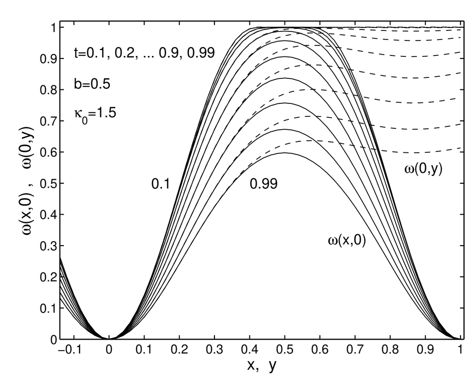


The magnetic field in units of the upper critical field B_{c2} for $t = 0.5$, $\bar B/B_{c2} = 0.5$, and $\kappa_0 = 1.5$. $B(x,y)$ reaches its maximum $B_{\rm max}$ at the vortex centers.

In the vortex centers the condensate density $n_s(x,y)/n = (1-t^4)\omega(x,y)$ goes to zero. Between the vortices $\omega(x,y)$ approaches its equilibrium value 1 (which would be constant in the absence of a magnetic field) yielding $n_s^{\rm eq}/n = 1 - t^4 = 0.94$.



Profiles of the condensate density $n(x,y)$ at various temperatures for $\bar B/B_{c2} = 0.5$ and $\kappa_0 = 1.5$. The solid lines show cuts along the x-axis, and the dashed lines along the y-axis.

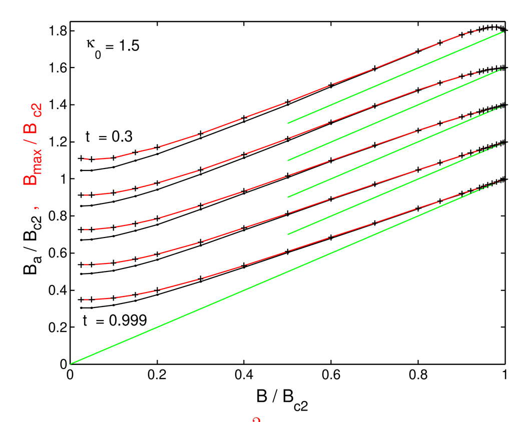


Profiles of the reduced condensate density $\omega(x,y)$ at various temperatures $t = T/T_c$ for $\bar B/B_{c2} = 0.5$ and $\kappa_0 = 1.5$.

Electrostatic forces

- Condensate density is smaller than its non-magnetic value, $n_s^0 = n(1 - T^4/T_c^4)$, also on borders of the elementary cells where the current is zero
- In vortex core magnetic field B is compressed and exceeds applied field B_a
- For all temperatures compression stronger at lower magnetic fields

Applied field B_a and the field $B_{\rm max}$ in the vortex center plotted versus the induction $\bar B$ for $\kappa_0 = 1.5$ at temperatures $t = 0.999$, 0.85, 0.7, 0.5, 0.3. For clarity, each line pair of the next temperature is shifted up by 0.2 and the large-κ-limit is $B_a = \bar B$.



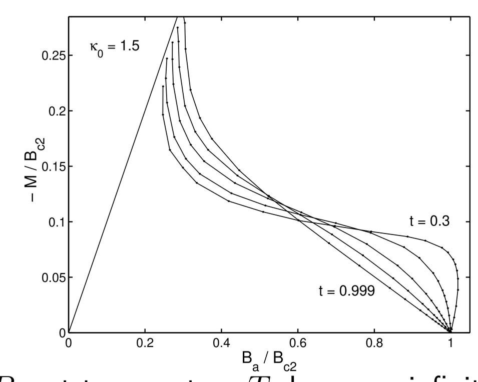
- Non-local effects given by gradient corrections, $-\frac{\hbar^2}{8m}\frac{1}{\sqrt{n_s}}\nabla^2\sqrt{n_s}$,

are important not only at the vortex core but also between the vortices.

Magnetization

$B_a[\omega, \bar B]$ by virial theorem M. M. Doria, J. E. Gurernatis and D. Rainer, Phys. Rev. B 39 (1989) 9573, U. Klein, B. Pöttinger, Phys. Rev. B 44 (1991) 7704

The magnetization $-M = B_a - \bar B$ as a function of the applied magnetic field B_a in units of the upper critical field B_{c2} at temperatures $t = 0.999$, 0.85, 0.7, 0.5, 0.3 for $\kappa_0 = 1.5$.



- The slope $\partial M/\partial B_a$, close to B_{c2} at temperature T_a becomes infinite
- System undergoes a first order transition from zero to a finite magnetization at a field above B_{c2}
- Here artifact of the Bardeen approximation ? (impurity scattering)

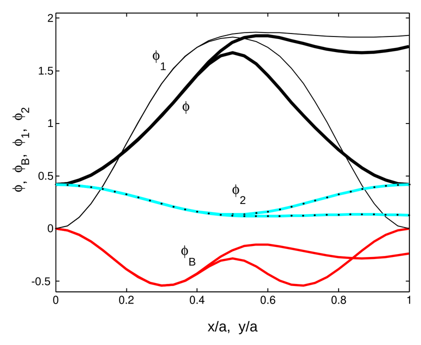
Close to B_{c2}, $\chi \to \alpha + \beta|\psi|^2$ with $\alpha = \frac{\gamma}{2n}(T^2 - T_c^2)$, $\beta = \frac{\gamma T_c^2}{2n^2}$
asymptotic GL parameter $\kappa_{\rm as} = \sqrt{\frac{m^2\beta}{2\mu_0\hbar^2e^2}} = \kappa_0 t$ equals $1/\sqrt{2}$
- lowest temperature where Bardeen approximation works $T_a = \frac{T_c}{\sqrt{2}\kappa_0} = 0.47\,T_c$ for $\kappa_0 = 1.5$

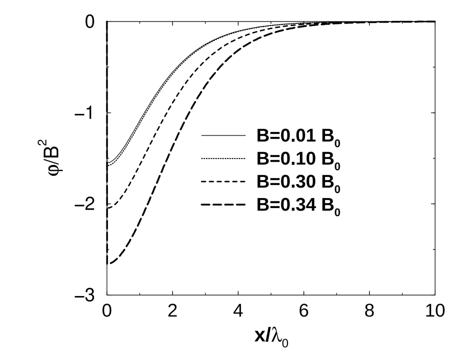
Electrostatic potentials

$$e\varphi - \lambda_{TF}^2\nabla^2 e\varphi = = \chi\frac{|\psi|^2}{n} + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} + \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\sqrt{1-\frac{2|\psi|^2}{n}}$$

Example Nb (hexagonal) $t = \frac{T}{T_c}$, $\omega = \frac{2|\psi|^2}{n(1-t^4)}$

$$\frac{en}{\frac{1}{4}\gamma T_c^2(1-t^2)(1-t^4)}\varphi = \phi_B + C_1\omega + C_2\sqrt{1-(1-t^4)\omega}$$





Linearization with respect to magnetic field

$$e\delta\varphi = -\frac{B^2}{2\mu_0 n}\left(1 + \frac{4t^4}{1-t^4}\frac{\partial\ln T_c}{\partial\ln n}\right)\frac{1}{1-\frac{2}{\kappa^2}}\left(e^{-2x/\lambda} - \frac{\sqrt{2}}{\kappa}e^{-\sqrt{2}x/\xi}\right)$$

- For $\kappa > \sqrt{2}$ Bernoulli potential extends over $\lambda/2$
- For $\kappa < \sqrt{2}$ Bernoulli potential extends on the scale $\xi/\sqrt{2}$

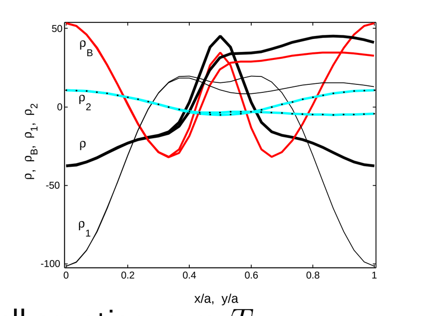
$$e\delta\varphi = -\frac{1}{2}mv^2\left(\frac{n_s}{n} + 4\frac{n_n}{n}\frac{\partial\ln T_c}{\partial\ln n}\right)\frac{1}{1+\frac{\sqrt{2}}{\kappa}}\left(1 + \frac{e^{\left(\frac{2}{\lambda}-\frac{\sqrt{2}}{\xi}\right)x} - 1}{1-\frac{\kappa}{\sqrt{2}}}\right)$$

- Rickayzen's result for $\kappa \to \infty$ (low temperatures)
- For $\kappa \to \sqrt{2}$ potential approaches $e\delta\varphi = ...\frac{1}{2}\left(1 + \frac{2x}{\lambda}\right)$

Charge profile

From Poisson equation

Example Nb:
$\tilde\rho = \frac{\rho}{en}$
$= 9.5\,10^{-11}\cdot(1-t^2)(1-t^4)^2\tilde\nabla^2\phi$
charge density is small compared to total charge density
$\frac{\rho}{en} \approx \frac{\epsilon_0 B^2}{mn} = \frac{B^2}{\mu_0 n}\frac{1}{mc^2} \sim 10^{-10}$



→ neglect of charge profile in Maxwell equation, γ, n, T_c
Linearization with respect to magnetic field

$$\rho = \frac{2e\epsilon_0 B^2}{m\left(1-\frac{2}{\kappa^2}\right)}\left(\frac{n_s}{n} + 4\frac{n_n}{n}\frac{\partial\ln T_c}{\partial\ln n}\right)\left(e^{-2x/\lambda} - \frac{\kappa}{\sqrt{2}}e^{-\sqrt{2}x/\xi}\right)$$

Surface value $x = 0$ always negative, bulk value positive
width of the surface $\rho(w) = 0$

$$w = \frac{\lambda}{2}\frac{\ln\frac{\kappa}{\sqrt{2}}}{\frac{\kappa}{\sqrt{2}} - 1}$$

- $\kappa > \sqrt{2}$ surface charge is formed by the contribution on the scale of the GL coherence length ξ
$\kappa \to \infty$ width $w \to \frac{\xi}{\sqrt{2}}\ln\frac{\kappa}{\sqrt{2}}$ bulk charge extends on scale of λ
- $\kappa < \sqrt{2}$ surface charge is formed by the contribution on the scale of London penetration depth λ
$\kappa \ll \sqrt{2}$ width $w \to \frac{\lambda}{2}\ln\frac{\sqrt{2}}{\kappa}$ bulk charge extends on scale of ξ

Summary

1. Bardeen's extension of the Ginzburg-Landau theory to low temperatures by the two-fluid model of Gorter-Casimir is used
2. Electrostatic and the thermodynamic potential compensate each other to a great extent
3. For Abrikosov vortex lattice numerical solutions are presented and the different contributions to electrostatic potential and charge:
4. Surface charge
   - Bernoulli potential in superconductors by charge build up in surface region $max[\xi/\sqrt{2}, \lambda/2]$
   - Surface charge extends over a range $L = min[\xi/\sqrt{2}, \lambda/2]$
   - Contrast to former theories: surface charge is not localized on Thomas-Fermi screening length $\lambda_{\rm TF}$ since negligible small amplitude $\sim\lambda_{\rm TF}^2/L^2$
5. Surface dipole
   - Budd-Vannimenus theorem generalized to superconductors by:
     - asymptotic extension of internal potential towards surface, $\varphi(0)$, appears instead of bulk potential
     - density of electronic charge differs locally from the lattice charge density due to charge depleted at the surface
   - electrostatic potential of surface from free energy of bulk $\phi_0 = \frac{f_{el}}{\rho_{\rm lat}}$

References

[1] P. Lipavský, J. Kolacek, K. Morawetz, and E. H. Brandt. *Phys. Rev. B*, 65:144511, 2002.

[2] P. Lipavský, K. Morawetz, J. Kolacek, J. J. Mares, E. H. Brandt, and M. Schreiber. *Phys. Rev. B*, 69: 024524, 2004; 70:104518, 2004; 71:024526, 2005,