Electric fields in superconductors

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Extended Ginzburg-Landau Theory

Following Bardeen (Phys. Rev. 94 (1954) 554) extend Ginzburg-Landau theory towards lower temperatures and include electrostatic potentials, focus on bulk superconductors
The presented theory yields

- non-local Bernoulli potential,
- quasiparticle screening,
- thermodynamic corrections,
- thermoelectric field of normal metal at $T = T_c$,
- Thomas-Fermi screening.

Density of free energy

1. Condensation energy Gorter and Casimir (Phys. Z. 35 (1934) 963)

superconducting state characterized by order parameter ϖ

$$f_s = U - \varepsilon_{\rm con}\varpi - \frac{1}{2}\gamma T^2\sqrt{1-\varpi}$$

Equilibrium $\delta\mathcal{F}_s/\delta\varpi = \partial f_s/\partial\varpi = 0$, at critical T_c is $\varpi = 0$

$$\varepsilon_{\rm con} = \frac{\gamma T^2}{4\sqrt{1-\varpi}} \qquad \varepsilon_{\rm con} = \frac{1}{4}\gamma T_c^2 \quad \rightarrow \quad \varpi = 1 - \frac{T^4}{T_c^4} \approx \frac{n_s}{n_s + n_n}$$

2. Kinetic energy

Ginzburg and Landau proposed (Ž. eksper. teor. Fiz. **20**,1064 (1950)), normalized $|\psi|^2 = \frac{n_s}{2}$

$$\mathcal{F}_{\rm kin} = \int d\mathbf{r}\frac{1}{2m^*}|(-i\hbar\nabla - e^*\mathbf{A})\,\psi|^2$$

3. Electromagnetic energy

Coulomb interaction, where charge density $\rho = e^*|\psi|^2 + e n_n + \rho_{\rm latt}$

$$\mathcal{F}_C = \frac{1}{2}\iint d\mathbf{r}d\mathbf{r}'\frac{1}{4\pi\epsilon}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r})\rho(\mathbf{r}')$$

Magnetic free energy, $\mathbf{B}_a$ is applied magnetic field

$$\mathcal{F}_M = \int d\mathbf{r}\frac{1}{2\mu_0}(\mathbf{B} - \mathbf{B}_a)^2$$

$\mathbf{B}$, φ, n, n_s, ρ or ϖ are subsidiary and functions of ψ, $\mathbf{A}$ and n_n

Lagrange equations

$$\frac{\delta\mathcal{F}_\alpha}{\delta l} = \frac{\partial f_\alpha}{\partial l} - \nabla\frac{\partial f_\alpha}{\partial\nabla l}$$

- $l = \mathbf{A}$ Second Ginzburg-Landau equation

$$\nabla\times\nabla\times\mathbf{A} = \mu_0\mathbf{j}, \qquad \mathbf{j} = \frac{e^*}{m^*}{\rm Re}\,\bar\psi(-i\hbar\nabla - e^*\mathbf{A})\psi$$

- $l = \bar\psi$ First GL-equation $\frac{1}{2m^*}(-i\hbar\nabla - e^*\mathbf{A})^2\psi + \chi\psi = 0$

The effective potential covers forces acting on Cooper pair

$$\chi = \frac{\delta}{\delta|\psi|^2}(\mathcal{F}_C + \mathcal{F}_s) = e^*\varphi + \frac{\partial f_s}{\partial|\psi|^2}$$

- $l = n_n$: $e\varphi = -\frac{\partial f_s}{\partial n_n}$ sum of all potentials on normal electrons vanishes

Effective potential (Bardeen Phys. Rev. 94 (1954) 554)

$$\chi = \frac{\partial f_s}{\partial|\psi|^2} - 2\frac{\partial f_s}{\partial n_n} = -2\frac{\varepsilon_{\rm con}}{n} + \frac{\gamma T^2}{2n}\frac{1}{\sqrt{1-\frac{2|\psi|^2}{n}}}$$

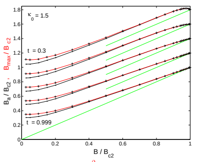
extends GL result towards lower temperatures, close to T_c,

$$\chi \to \alpha + \beta|\psi|^2, \qquad \text{with} \quad \alpha = \frac{\gamma T_c}{n}(T - T_c), \quad \beta = \frac{\gamma T_c^2}{2n^2}$$

Electrostatic forces

- Condensate density is smaller than its non-magnetic value, $n_s^0 = n(1 - T^4/T_c^4)$, also on borders of the elementary cells where the current is zero
- In vortex core magnetic field B is compressed and exceeds applied field B_a
- For all temperatures compression stronger at lower magnetic fields

Applied field B_a and the field $B_{\rm max}$ in the vortex center plotted versus the induction $\bar B$ for $\kappa_0 = 1.5$ at temperatures $t = 0.999$, 0.85, 0.7, 0.5, 0.3. For clarity, each line pair of the next temperature is shifted up by 0.2 and the large-κ-limit is $B_a = \bar B$.



- Non-local effects given by gradient corrections, $-\frac{\hbar^2}{8m}\frac{1}{\sqrt{n_s}}\nabla^2\sqrt{n_s}$, are important not only at the vortex core but also between the vortices.

Electrostatic potential

$$-e\varphi = \frac{\partial f_s}{\partial n_n} = \frac{\partial U}{\partial n} - \frac{\partial}{\partial n_n}\left(\varepsilon_{\rm con}\varpi + \frac{1}{2}\gamma T^2\sqrt{1-\varpi}\right)$$

If $\varepsilon_{\rm con}$ and γ depend only on the total density

$$e\varphi - \lambda_{TF}^2\nabla^2 e\varphi = = \chi\frac{|\psi|^2}{n} + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} + \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\sqrt{1 - \frac{2|\psi|^2}{n}}$$

with $\frac{\partial U}{\partial n} = E_F = E_F^0 + \frac{\partial E_F}{\partial n}\frac{\rho}{e} = -\lambda_{TF}^2\nabla^2 e\varphi$ and $\lambda_{TF}^2 = \epsilon/(2\mathcal{D}e^2)$

- First term is non-local Bernoulli potential, from GL equation

$$\chi\frac{|\psi|^2}{n} = -\frac{1}{2m^*n}\bar\psi\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi \approx -\frac{1}{2n}m^*v^2|\psi|^2 = -(n_s/n)\frac{1}{2}mv^2$$

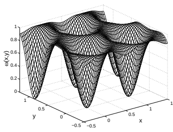
A.G. van Vijfeijken and F. S. Staas, Phys. Lett. **12** 175 (1964)

- Third term finite at T_c: normal state thermoelectric field
- Weak electrical current $\lambda_{TF}^2\nabla^2\varphi \ll \varphi$, $n\,mv^2/2 \ll \varepsilon_{\rm con} - \gamma T^2/4$

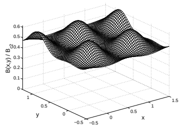
$$e\varphi' = -\frac{n_s}{n}\frac{1}{2}mv^2 + 4\frac{n_n}{n}\frac{\partial\ln T_c}{\partial\ln n}\frac{1}{2}mv^2$$

G. Ryckaizen, J. Phys. C **2**, 1334 (1969)

Numerical results on Nb

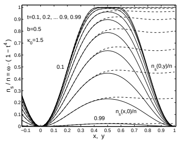


Condensate density $\omega(x,y)$ in the triangular lattice for temperature $t = 0.5$, magnetic induction $\bar B/B_{c2} = 0.5$, and GL parameter $\kappa_0 = 1.5$.

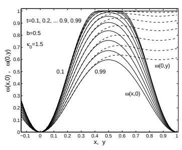


The magnetic field in units of the upper critical field B_{c2} for $t = 0.5$, $\bar B/B_{c2} = 0.5$, and $\kappa_0 = 1.5$. $B(x,y)$ reaches its maximum $B_{\rm max}$ at the vortex centers.

In the vortex centers the condensate density $n_s(x,y)/n = (1-t^4)\omega(x,y)$ goes to zero. Between the vortices $\omega(x,y)$ approaches its equilibrium value 1 (which would be constant in the absence of a magnetic field) yielding $n_s^{\rm eq}/n = 1 - t^4 = 0.94$.



Profiles of the condensate density $n(x,y)$ at various temperatures for $\bar B/B_{c2} = 0.5$ and $\kappa_0 = 1.5$. The solid lines show cuts along the x-axis, and the dashed lines along the y-axis.

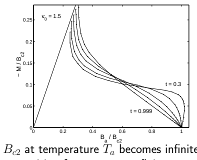


Profiles of the reduced condensate density $\omega(x,y)$ at various temperatures $t = T/T_c$ for $\bar B/B_{c2} = 0.5$ and $\kappa_0 = 1.5$.

Magnetization

$B_a[\omega, \bar B]$ by virial theorem M. M. Doria, J. E. Gurernatis and D. Rainer, Phys. Rev. B **39** (1989) 9573, U. Klein, B. Pöttinger, Phys. Rev. B **44** (1991) 7704

The magnetization $-M = B_a - \bar B$ as a function of the applied magnetic field B_a in units of the upper critical field B_{c2} at temperatures $t = 0.999$, 0.85, 0.7, 0.5, 0.3 for $\kappa_0 = 1.5$.



- The slope $\partial M/\partial B_a$, close to B_{c2} at temperature T_a becomes infinite
- System undergoes a first order transition from zero to a finite magnetization at a field above B_{c2}, free energy with finite magnetization is lower than free energy of normal state, system jumps to a finite magnetization
- Anomalous magnetic transition observed for Pb doped with Nb

R. Ehrat and L. Rinderer, Journal of Low Temp. Phys. **17**, 255 (1974)

- Here artifact of the Bardeen approximation ? (impurity scattering)

Close to B_{c2}, $\chi \to \alpha + \beta|\psi|^2$ with $\alpha = \frac{\gamma}{2n}(T^2 - T_c^2)$, $\beta = \frac{\gamma T_c^2}{2n^2}$ asymptotic GL parameter $\kappa_{\rm as} = \sqrt{\frac{m^2\beta}{2\mu_0\hbar^2e^2}} = \kappa_0 t$ equals $1/\sqrt{2}$

$$T_a = \frac{T_c}{\sqrt{2}\kappa_0} = 0.47\,T_c \quad \text{for} \quad \kappa_0 = 1.5$$

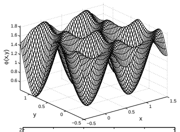
Electrostatic potentials

Use dimensionless values $t = \frac{T}{T_c}$ and $\omega = \frac{2|\psi|^2}{n(1-t^4)}$

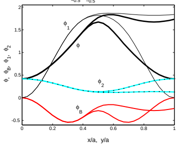
$$\phi = \frac{en}{\frac{1}{4}\gamma T_c^2(1-t^2)(1-t^4)}\varphi = \phi_B + C_1\omega + C_2\sqrt{1-(1-t^4)\omega},$$

with temperature dependent factors

$$C_1 = \frac{1}{1-t^2}\frac{\partial\ln\varepsilon_{\rm con}}{\partial\ln n} \approx \frac{1.9}{1-t^2},$$
$$C_2 = \frac{1}{1-t^2}\frac{\partial\ln\gamma}{\partial\ln n} \approx \frac{0.42}{1-t^2}$$

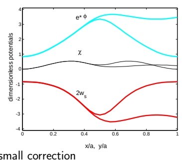


The components of the electrostatic potential: Bernoulli potential ϕ_B, the condensation potential $\phi_1 = C_1\omega$, and the normal thermodynamic potential $\phi_2 = C_2\sqrt{1-(1-t^4)\omega}$ for the temperature $t = 0.5$, the magnetic field $\bar B/B_{c2} = 0.5$ and the GL parameter $\kappa_0 = 1.5$, x-direction (lower curves), y-direction (upper curves).



Effective potential

Effective potential $\chi = 2e\varphi + \frac{\partial f_s}{\partial|\psi|^2}$ electrostatic potential $e^*\varphi$ acting on the Cooper pair and the thermodynamic potential $2w_s$ for $\kappa_0 = 1.5$

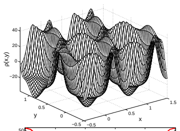


- Electrostatic potential is not a small correction
- Effective potential $\chi = e^*\varphi + 2w_s$ results from strong compensation of thermodynamic potential $2w_s$ and electrostatic potential $e^*\varphi$

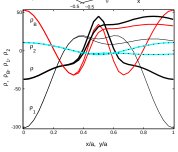
Charge

Dimensionless charge, $\tilde\rho = \frac{\rho}{en}$, measures relative deviation of charge density from crystal value

Poisson: $\tilde\rho = -C_3\frac{\lambda_{TF}^2}{\lambda_{\rm Lon}^2}\tilde\nabla^2\phi$, with $C_3 = \frac{2\mathcal{D}\varepsilon_{\rm con}}{n^2}(1-t^2)(1-t^4)$, $\frac{2\mathcal{D}\varepsilon_{\rm con}}{n^2} \approx 3.8\,10^{-5}$
Dimensionless charge $\tilde\rho = 9.5\,10^{-11}\cdot(1-t^2)(1-t^4)^2\tilde\nabla^2\phi$.



The components of the charge density. The individual charge densities are: the total ρ (solid lines), the Bernoulli ρ_B (dashed lines), the condensation part ρ_1 (dash dotted lines), and the normal thermodynamic part ρ_2 (dotted lines)



Summary

1. Bardeen's extension of the Ginzburg-Landau theory to low temperatures by the two-fluid model of Gorter-Casimir is used to derive three Ginzburg-Landau equations, the Maxwell equation for the vector potential, the Schroedinger equation for the wave function and the Poisson equation for the electrostatic potential
2. The electrostatic and the thermodynamic potential compensate each other to a great extent resulting into an effective potential acting on the superconducting condensate
3. For the Abrikosov vortex lattice numerical solutions are presented and the different contributions to the electrostatic potential and the related charge distribution are discussed. Specifically used parameters are presented for Niobium

P. Lipavský, J. Koláček, K. Morawetz and E. H. Brandt,
Phys. Rev. B **65**, 144511 (2002), Phys. Rev. B **66**, 134525 (2002)