Enhancement of Pairing near Resonant Cavities

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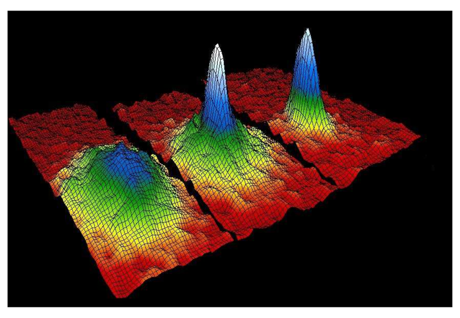
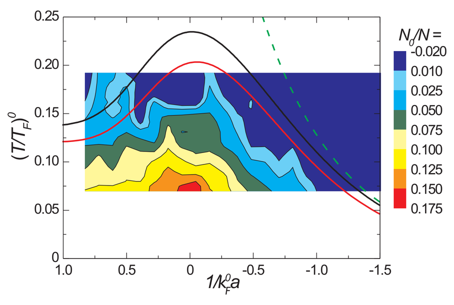
MAX-PLANCK-GESELLSCHAFT

BCS-BEC cross over in trapped Fermi gases

- In trapped ultracold Fermi gases transition from BCS-superfluid to BEC regimes experimentally observed, Greiner et al. Nature 426 (2003) 537, Jochim et al. Science 302 (2003) 2110
- Interaction tuned between fermionic atoms (^6Li, ^{40}K) via Feshbach resonance (by magnetic field), Regal et al. PRL 92 (2004) 040403, Zwierlein et al. PRL 92 (2004) 120403



http://www.colorado.edu/physics/2000/bec/



Contour plot of the measured condensate fraction N_0/N of ^{40}K as a function of the inverse scattering length and the effective temperature. Q. Chen, C. A. Regal, M. Greiner, D. S. Jin, K. Levin cond-mat/0512596

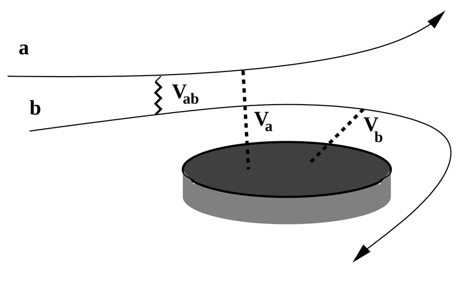
Theoretical challange: additional resonant atom in the system

1. Separable model and results for conventional Cooper pairing
2. Coupled channel scheme of Gell-Mann and Goldberger for many-body systems
3. Results for Cooper pairing with finite cavity

Coupled-channel scattering theory

Coupled T-matrices for 2+1-particle scattering, (Gell-Mann/Goldberger Phys. Rev. 91 (1953) 398)

$$\mathcal{T} = \mathcal{T}_1 + (1+\mathcal{V}_1\mathcal{G}_1)\mathcal{T}_{ab}(1+\mathcal{G}_1\mathcal{V}_1)$$



One-particle T-matrix $\mathcal{T}_1 = \mathcal{V}_1 + \mathcal{V}_1\mathcal{G}_1\mathcal{V}_1$ with $\mathcal{V}_1 = \mathcal{V}_a + \mathcal{V}_b = \mathcal{V}_{01} + \mathcal{V}_{02}$

Correlated two-particle T-matrix: $\mathcal{T}_{ab} = \mathcal{V}_{ab} + \mathcal{V}_{ab}\mathcal{G}_1\mathcal{T}_{ab}$

One-particle wave function to be known: $\mathcal{H}_1|n_1\rangle = E_{n_1}|n_1\rangle$, introduce complete set $|n_1n_2\rangle$

$$\langle p_1P|\mathcal{T}_{ab}|p_2P'\rangle = V_{p_1p_2}\delta_{PP'} + \sum_{n_1n_2p\bar{p}\bar{P}} V_{p_1p}\frac{\langle pP|n_1n_2\rangle(1-f_{n1}-f_{n2})\langle n_1n_2|p'\bar{P}\rangle}{\omega+i\eta-E_{n_{12}}}\langle p'\bar{P}|\mathcal{T}_{ab}|p_2P'\rangle$$

with $p=\frac{p_a-p_b}{2}$, $P=p_a+p_b$, now generalized to many-body systems Phys. Rev. **B 72** (2005) 174504

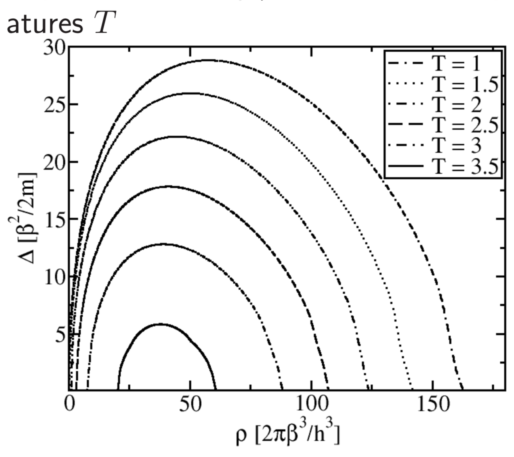
Results for conventional pairing

Greenfunctions and T-matrix provide gap equation:

$$\Delta(\mathbf{k}) = \int\frac{d\mathbf{k}'}{(2\pi)^3}V(\mathbf{k},\mathbf{k}')\frac{1-2f(E_{\mathbf{k}})}{2E_{\mathbf{k}'}}\Delta(\mathbf{k}')$$

- Yamaguchi formfactor for two-particle interaction $V_{\mathbf{kk}'} = \lambda g_k g_{k'}$ with $g_k = \frac{1}{k^2+\beta^2}$
- ρ increases: more particles build pairs, ground state energy is lowered
- medium effects suppress pairing $\Rightarrow$ maximum for gap appears
- temperature T increases: gap Δ decreases because of splitting of cooper pairs

Gap Δ over density ρ for different temperatures T



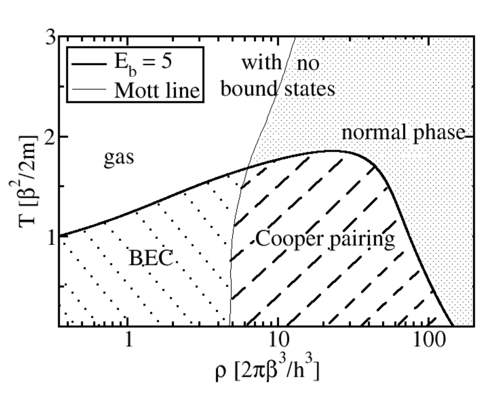
Transition from BEC to Cooper pairing

Anomalous density $\psi(\mathbf{k}) = \langle a^+_{n,\mathbf{k}}a^+_{n,-\mathbf{k}}\rangle = \frac{\Delta(\mathbf{k})}{2E_{\mathbf{k}}}(1-2f(E_{\mathbf{k}}))$

$$\frac{k^2}{m}\psi(\mathbf{k}) + (1-n^B_{\mathbf{k}})\int\frac{d\mathbf{k}'}{(2\pi)^3}V(\mathbf{k},\mathbf{k}')\psi(\mathbf{k}') = 2\mu\psi(\mathbf{k})$$

In low density limit, $n^B_{\mathbf{k}}\to 0$, two-particle Schrödinger equation appears for Bosons $\to$ BEC

- Mott-lines: separate BEC from Cooper pairing
- $T > T_c$: $\rho < \rho_{Mott}$ has only bound states, otherwise normal fluid



Model for cavity: Opaque wall

- Background Hamiltonian

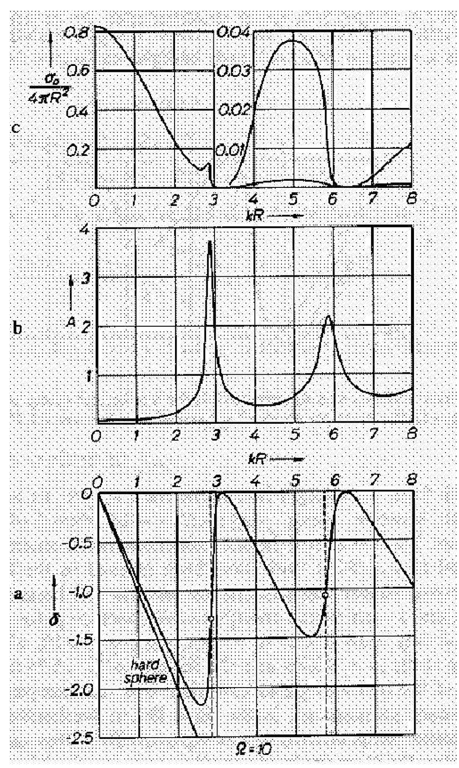
$$\mathcal{H}_1 = -\frac{1}{2m}(\nabla^2 - \frac{\Omega}{R}\delta(r-R))$$

$$\Omega = Opacity$$

- Solution of the radial problem $\Psi(r) = \chi(r)/r$

$$\chi(r) = c\begin{cases} A_k\sin kr & ;r<R \\ \sin(kr+\delta) & ;r>R\end{cases}$$

Three cases:
- $\Omega\to\infty$: Bound states in the interior
- $0<\Omega<\infty$: Amplitude A_k maximum $\Longrightarrow$ resonance at virtual levels
- $\Omega = 0$ leads to case without cavity



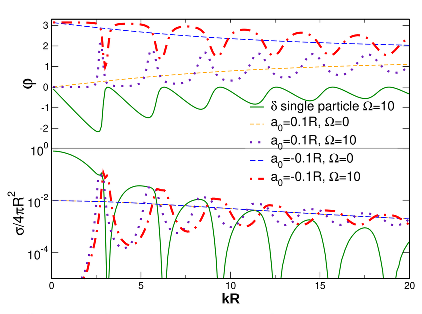
Flügge: Rechenmethoden der Quantenmechanik

Phase shifts for contact interaction

Two-particle phase shift φ from T-matrix: $\tan\varphi = \frac{\mathrm{Im}T}{\mathrm{Re}T}\Big|_{\omega=\frac{p^2}{m}}$,

Cross section (s-wave) $\sigma = \frac{4\pi}{p^2}\sin^2\varphi$

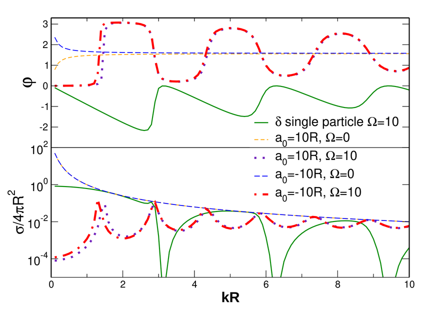
$\Omega = 10$, $a_0 = \pm 0.1R$

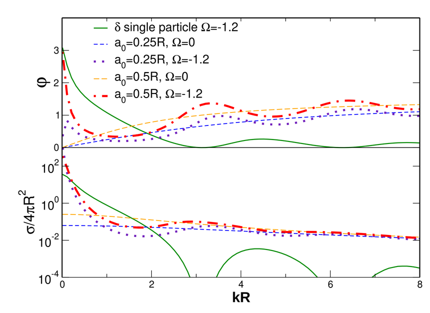


$\delta =$ one-particle phase shift
$\varphi^\infty =$ free two-particle phase shift for contact potentials without cavity
$a_0 =$ two-particle scattering length
$\varphi =$ free two-particle phase shift for contact potentials with cavity

- $\delta < 0$ leads to repulsive potential
- cavity causes oscillations of φ around free particle case

$\Omega = 10$, $a_0 = \pm 10R$ $\qquad$ $\Omega = -1.2$, $a_0 > 0$

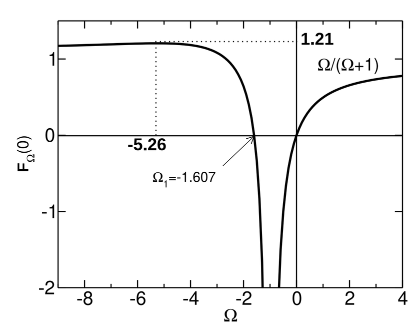




- second maximum in cross section by interference between one- and two-particle channel
- no bound state with cavity
- no bound states for φ^∞
- negative opacity enables bound state for φ

Condition for bound states:

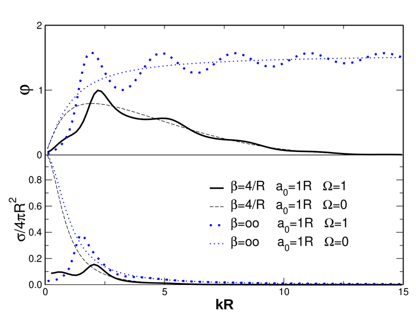
$$-\frac{R}{a_0} \geq F_\Omega(0)$$

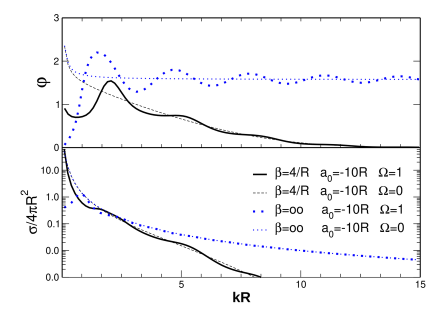


Phase shifts for finite potential range

Consider contact potential (blue) and finite range (black)

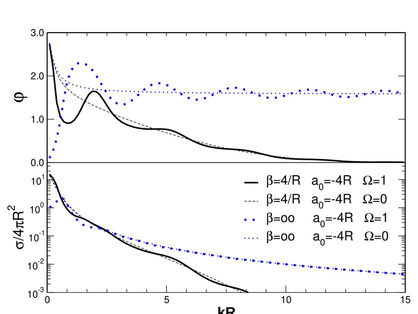
$\beta = 4/R$, $a_0 = 1R$, $\Omega = 1$: $\qquad$ $\beta = 4/R$, $a_0 = -10R$, $\Omega = 1$:

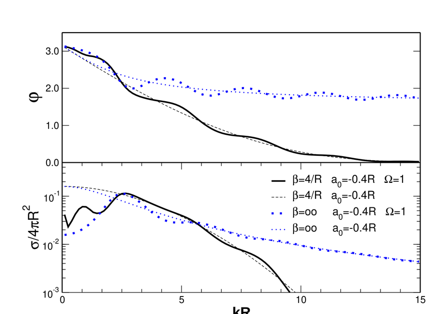




- no bound states for contact interaction and finite potential range
- finite potential range: bound state with and without cavity
- contact interaction: no bound state with cavity

$\beta = 4/R$, $a_0 = -4R$, $\Omega = 1$: $\qquad$ $\beta = 4/R$, $a_0 = -0.4R$, $\Omega = 1$:





- finite potential range: bound state with and without cavity
- contact interaction: no bound state with cavity
- bound states for all cases

Gap equation with cavity

$\Omega \ll kR \Rightarrow$ radial single-particle wave function $\chi_k(r)\approx A_{kR}\sin kr$, 2-particle Schrödinger equation separates

$$|n_1n_2\rangle \approx A_{|K/2-k|R}A_{|K/2+k|R}|pP\rangle, \quad |pP\rangle = \text{plane waves}$$

Solution of T-matrix

$$\langle p_1P|\mathcal{T}_{ab}|p_2P'\rangle = \delta_{PP'}\frac{\lambda g_{p_1}g_{p_2}}{1-\lambda J(P,\omega)}$$

$$J(P,\omega) = \sum_p g_p^2\frac{|A_{|\frac{P}{2}+p|R}A_{|\frac{P}{2}-p|R}|^2(1-f_{\frac{P}{2}+p}-f_{\frac{P}{2}-p})}{\omega - E_{\frac{P}{2}+p} - E_{\frac{P}{2}-p} + \Delta E + i\eta}$$

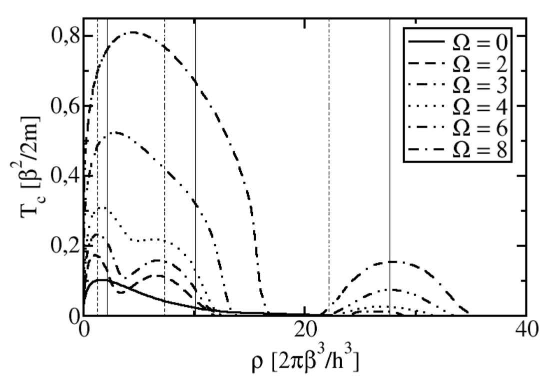
Results for Gap equation with cavity $\xi_{\mathbf{k}} = \frac{k^2}{2m}-\mu$, $E_{\mathbf{k}} = \sqrt{\epsilon^2_{\mathbf{k}}+\Delta^2(\mathbf{k})}$

$$\Delta(\mathbf{k}) = \int\frac{d\mathbf{k}'}{(2\pi)^3}A^2_{kR}A^2_{k'R}V(\mathbf{k},\mathbf{k}')\frac{1}{2E_{\mathbf{k}'}}\Delta(\mathbf{k}')$$

$$\rho = \frac{1}{2}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[1-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(1-2f(E_{\mathbf{k}}))\right]$$

Change of critical temperature

- oscillations of critical temperature T_c due to the cavity
- second branch of Cooper-pairing for high densities
- T_c enhanced for increasing opacity Ω



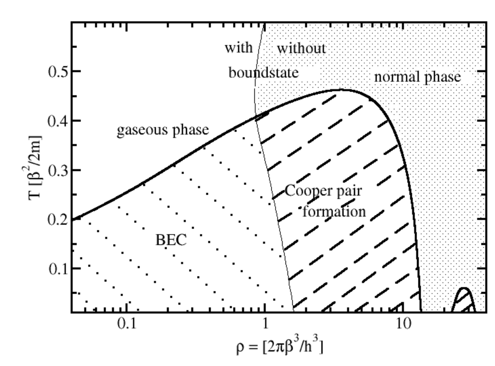
Explanation via BCS theory

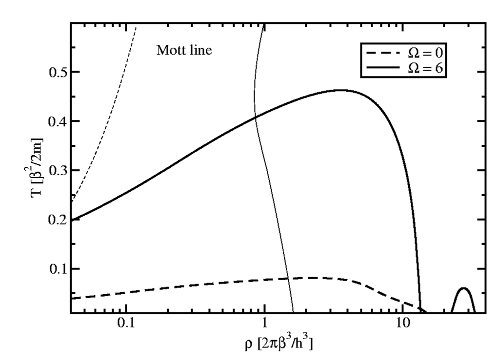
$$T_c = 1.13\omega_c e^{-2/NV_0A^4_{kfR}} = \left(\frac{T^0_c}{1.13\omega_c}\right)^{1/A^4_{kfR}}$$

Amplitude A is resonant: Minima of A provide positions of maxima in T_c

Phase diagram

- binding energy $E_b = 0.005$, $\Omega = 6$
- additional cooper-pairing region for high densities
- repulsive cavity enhances pairing/bound state region
- effective binding energy is higher for $\Omega = 6 \Rightarrow$ Mott transition possible
- $\Omega = 0$ supports no BEC bound states, only bound states for gas phase available





Summary

1. Two particles + one cavity
   - T-matrix for two interacting particles and finite cavity considered
   - Solution with separable potential for two-particle interaction
   - Two-particle phase shift φ shows additional oscillations besides standard ones at virtual levels
   - Interference effects cause splitting of maxima in cross section σ
   - Occurrence of bound states is dependent on scattering length and cavity strength
2. In-medium two particles + one cavity
   - Modification of the pairing temperature of superconductivity due to the presence of resonant cavity
   - Gell-Mann and Goldberger scheme of coupled channel T-matrices is generalized to many-body systems
   - Resonant cavity provides modification of gap equation

$$\Delta(\mathbf{k}) = \int\frac{d\mathbf{k}'}{(2\pi)^3}A^2_{kR}A^2_{k'R}V(\mathbf{k},\mathbf{k}')\frac{1}{2E_{\mathbf{k}'}}\Delta(\mathbf{k}')$$

   - Obtain new range of critical temperatures dependent on density
   - Phase diagram shows Mott transition from Cooper pairing to BEC for $T < T_c$
   - $T > T_c$: transition from normal phase to phase with bound states
   - applicable to (asymmetric) nuclear matter, metals etc

K. Morawetz, M. Schreiber, B. Schmidt, A. Ficker, P. Lipavský Correlated two-particle scattering on finite cavities Phys. Rev. **B 72** (2004) 014301-1-15

K. Morawetz, M. Schreiber, B. Schmidt, P. Lipavský Enhancement of fermion pairing due to the presence of resonant cavities Phys. Rev. **B 72** (2005) 174504-1-5