Stability of condensates and supercurrents

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MAX-PLANCK-GESELLSCHAFT

Problem with paradigma of anomalous functions

Questions:

1. Why appears only one and not two condensates of cooper pairs?
2. Stability limited by pair excitation into bound pairs or pair breaking?

Paradigma:

Cooper pairs need anomalous functions $< a^+a^+ > \neq 0$, cannot conserve density (does not matter)

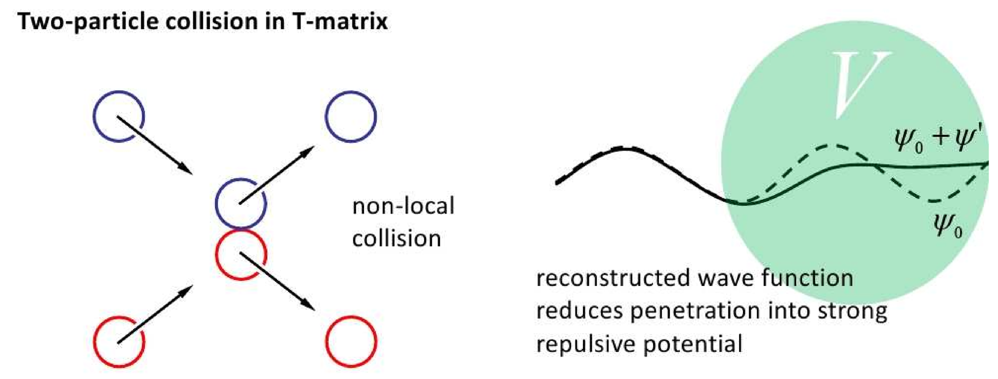
Our view:

- Anomalous functions are short cut to right results (mean field), but same result possible without non-conserving assumptions
- Need unified theory above and below condensation temperature
- Fluctuations and condensation at the same theoretical footing to access stability

Solution:

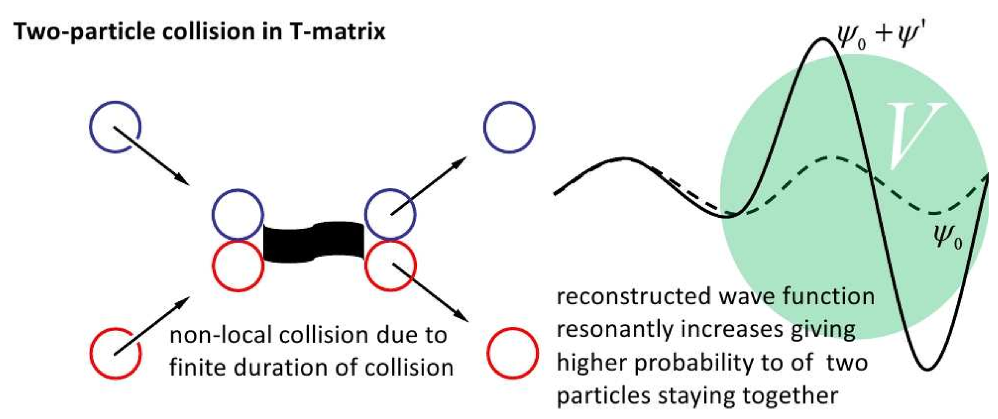
- T-matrix with multiple scattering corrections (MSC)
- critical velocity of pair excitation $\sqrt{3}$-times larger than critical velocity of pair breaking

Scattering T-matrix



Schrödinger equation	incoming waves	scattered waves
$(H_0+V)\psi=E\psi$	$\psi=\psi_0+\psi'$ $H_0\psi_0=E\psi_0$	$\psi'=\frac{1}{E-H_0-V+i0}V\psi_0$

Many-body T-matrix

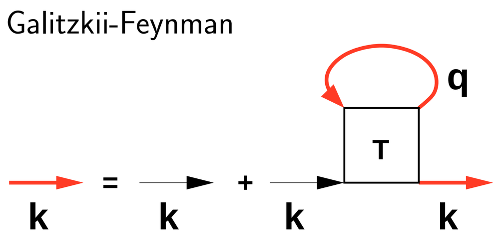


Scattered wave — reconstr. wave function — T-matrix

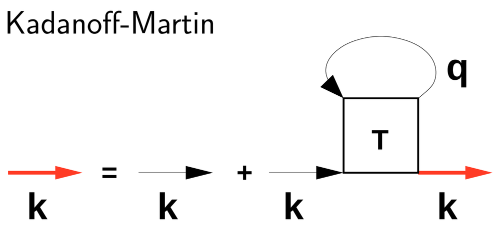
$$\Psi' = \frac{1}{E-H_0-V+i0}V\Psi_0 = \frac{V}{E-H_0+i0}V(\Psi_0+\Psi') = \frac{1}{E-H_0+i0}T\Psi_0$$

Dichotomy between gap and selfconsistency

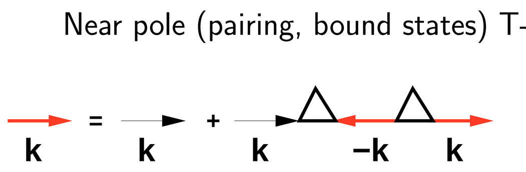
Galitzkii-Feynman

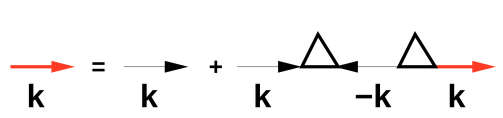


Kadanoff-Martin



Near pole (pairing, bound states) T-matrix is separable $T=\triangle\triangle$





$$G^{-1}(\omega,\mathbf{k}) = \omega+\epsilon_{\mathbf{k}}-\Delta^2 G(-\omega,-\mathbf{k})$$

no pole no gap equation

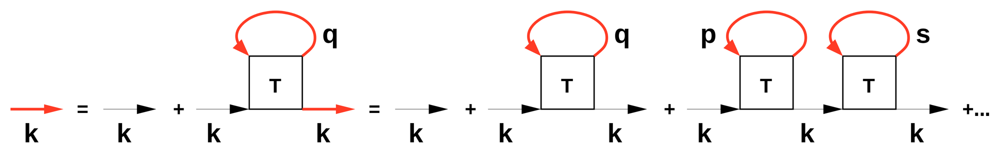
$$G^{-1}(\omega,\mathbf{k}) = \omega+\epsilon_{\mathbf{k}}-\frac{\Delta^2}{\omega+\epsilon_{-\mathbf{k}}}$$

two-pole structure, BCS gap equation

- Selfconsistency needed for physical distributions, Goldstone theorem, conservation laws
- Only partial selfconsistency lead to the superconducting gap
- This conflict is known as selfconsistency gap dichotomy:
  - theories satisfying selfconsistency required by Goldstones criterion yield zero gap
  - theories giving the gap do not satisfy selfconsistency

Removal of double counts

Paradox: The worse approximation yields better result
Wrong conclusion: Superconductor and normal metal not be covered by unified theory
Solution: Galitskii-Feynman approximation includes double-counts fatal in superconducting state



- Third particle ought to be different from the interacting pair: $p \neq q$!
- Each momentum contributes as 1/volume → vanishs for infinite volume
- Normal state ok, but pairing or BE condensates state $q \sim$ volume

1. Asymmetric selfconsistency is necessary to get gap equation
2. Asymmetry violates Kadanoff/Baym criterion B → no particle conservation ?
3. Derivation from cummulant expansion (cluster-cluster diagrams)
   K. Morawetz J. Stat. Phys. 143 (2011) 482

Procedure

subtract own interaction in singular channel
Using the Dyson equation $G_0^{-1}=G^{-1}+\Sigma$
one obtains

$$G_{i\!\!\setminus} = G - G_{i\!\!\setminus}\Sigma_i G$$
$$G_{i\!\!\setminus} = G_0 + G_0(\Sigma-\Sigma_i)G_{i\!\!\setminus}$$

closing with the subtracted propagator $\Sigma_i = T_i\bar G_{i\!\!\setminus}$, short exercise

$$\begin{aligned} G^{-1} = G_0^{-1}-\Sigma &= G_0^{-1}-\Sigma'-\Sigma_i \\ &= G_0^{-1}-\Sigma'-T_i\bar G_{i\!\!\setminus} = G_0^{-1}-\Sigma'-T_i\left(\bar G_0^{-1}-\bar\Sigma'\right)^{-1}\end{aligned}$$

or explicitly

$$G = \frac{\bar G_0^{-1}-\bar\Sigma'}{[G_0^{-1}-\Sigma'][\bar G_0^{-1}-\bar\Sigma']-T_i}$$

free propagator $G_0^{-1}=\omega-\epsilon_p$
"proper" selfenergy $\Sigma_{11}(p)\equiv\Sigma'(p)$
"anomalous" selfenergy $\Sigma_{12}(p)\equiv\Delta(p)$

in matrix form $\mathbf{G}=\mathbf{G^0}+\mathbf{G^0\Sigma G}$ with

$$\mathbf{G} = \begin{pmatrix} G_{11} & G_{12} \\ \bar G_{12} & \bar G_{11}\end{pmatrix},\ \mathbf{G_0} = \begin{pmatrix} G_0 & 0 \\ 0 & \bar G_0\end{pmatrix},\ \mathbf{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \bar\Sigma_{12} & \pm\bar\Sigma_{11}\end{pmatrix}$$

Bosons S. T. Beliaev, Soviet. Phys. JETP 7 (1958) 289
Fermions L. P. Gorkov, Soviet. Phys. JETP 7 (1958) 505

"normal" $G_{11}\equiv G$ and "anomalous" Green' function

$$G_{12} \equiv \frac{-\Sigma_{12}}{(\omega+\epsilon+\bar\Sigma_{11})(\omega-\epsilon-\Sigma_{11})+\Sigma_{12}^2}$$

results, not needed as starting (conservation laws completed)
P. Lipavsky, PRB 78 (2008) 214506; K. Morawetz, PRB 82 (2010) 092501

Energy of condensate as poles of the MSC T-matrix

excited bound states ($\mathbf{Q}$-mode), at zero frequency Cooper pairs ($\mathbf{C}$-mode), singular element $\mathcal{T}_{0,\mathbf{C}} = \frac{L^3}{k_BT}\bar\Delta\Delta$, T-matrix of the condensation mode $\mathcal{T}_{0,\mathbf{C}} = V - V\frac{k_BT}{L^3}\sum_k G_\uparrow(\omega,\mathbf{k})G_{\not{\mathcal{C}}\downarrow}(-\omega,\mathbf{C}-\mathbf{k})\mathcal{T}_{0,\mathbf{C}}$

near T_c T-matrix diverges, expand (GL equation Gorkov)

$$\frac{\hbar^2|\mathbf{C}|^2}{2m^*}\bar\Delta+\alpha\bar\Delta+\beta|\Delta|^2\bar\Delta=0$$

T-matrix $\frac{1}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{1}{V}+\frac{k_BT}{L^3}\sum_k G_\uparrow(\omega,\mathbf{k})G_\downarrow(-\omega,\mathbf{Q}-\mathbf{k})$ for non-condensed pairs both propagators depend on gap, leads to energy

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*}+\alpha+2\beta|\Delta|^2$$

in condensation mode

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^*}+\alpha+\beta|\Delta|^2=0$$

identical to GL approximation, eliminate gap $\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*}-\alpha-\frac{|\mathbf{C}|^2}{m^*}$, zero only if $|\mathbf{C}|^2$ compensate $-\alpha$

- T-matrix in the $\mathbf{Q}$-mode remains finite, cannot become singular once the condensation develops in $\mathbf{C}$-mode
- therefore parallel condensation in two competitive modes is excluded
- only a single condensate as tacitly assumed in BCS theory

Critical current
pair momentum $\mathbf{C}$ are limited by the critical current, $|\mathbf{C}|^2 < Q_c^2$, current $\mathbf{j}\propto\mathbf{C}|\Delta|^2\propto\mathbf{C}\left(-\alpha-|\mathbf{C}|^2/2m^*\right)$
critical current as the maximum of $\mathbf{j}$, $Q_c^2 = 2m^*|\alpha|/3$, accordingly, $\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} > \frac{|\mathbf{Q}|^2}{2m^*}-\alpha-\frac{Q_c^2}{m^*} = \frac{|\mathbf{Q}|^2}{2m^*}-\frac{\alpha}{3}>0$

Comparison to Galitsky/Feynman + Kadanoff/Martin

$$\bar\Delta = -V\frac{k_BT}{L^3}\sum_k G_\uparrow(\omega,\mathbf{k})G_{\not{\mathcal{C}}\downarrow}(-\omega,\mathbf{C}-\mathbf{k})\bar\Delta$$

Galitsky-Feynman $G_{\not{\mathcal{C}}\downarrow}\approx G_\downarrow$

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^{*\rm Gal}}+\alpha^{\rm Gal}+2\beta^{\rm Gal}|\Delta|^2 \qquad \frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\rm Gal}}+\alpha^{\rm Gal}+2\beta^{\rm Gal}|\Delta|^2$$

eleminating gap $\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\rm Gal}}-\frac{|\mathbf{C}|^2}{2m^{*\rm Gal}}$
- if persistent current, $\mathbf{C}\neq\mathbf{0}$, dispersion supports nucleation of second condensate at energy minimum $\mathbf{Q}=\mathbf{0}$

Kadanoff-Martin $G_{\not{\mathcal{C}}\downarrow}\approx G^0$

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^{*\rm KM}}+\alpha^{\rm KM}+\beta^{\rm KM}|\Delta|^2 \qquad \frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\rm KM}}+\alpha^{\rm KM}+\beta^{\rm KM}|\Delta|^2$$

eleminating gap $\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\rm KM}}-\frac{|\mathbf{C}|^2}{2m^{*\rm KM}}$
- like Feynman Galitsky nucleation of second condensate at energy minimum $\mathbf{Q}=\mathbf{0}$ possible

Compare MSC T-matrix

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*}+\alpha+2\beta|\Delta|^2, \qquad \frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^*}+\alpha+\beta|\Delta|^2=0$$

eliminate gap $\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*}-\alpha-\frac{|\mathbf{C}|^2}{m^*}$

- therefore parallel condensation in two competitive modes is excluded

Excitation of Cooper pairs from the condensate

N particles (Cooper pairs) in running frame $\mathbf{v}$ excite quasiparticle (bound pair) ϵ_Q, possible if (Cherenkov) $E_i-E_f=\mathbf{v}\cdot\mathbf{Q}-(\epsilon_Q-\epsilon_0)>0$, i.e.

$(\mathbf{v}\cdot\mathbf{Q}) \geq \frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*}-\alpha-\left.\frac{|\mathbf{C}|^2}{m^*}\right|_{C=0} = \frac{|\mathbf{Q}|^2}{2m^*}-\alpha$ solved by real $\mathbf{Q}$ only if

$$|\mathbf{v}| > v_{\rm pe} = \sqrt{\frac{2|\alpha|}{m^*}}$$

since pair breaking velocity $v_{\rm pb}=\frac{\Delta}{k_{\rm F}}=\sqrt{\frac{|\alpha|}{\beta k_{\rm F}}}=\sqrt{\frac{|\alpha|}{3m}}$ we have the relation

$$v_{\rm pe}=\sqrt{3}v_{\rm pb}.$$

- critical velocity of pair breaking lower than critical velocity of pair excitation

→ stability of condensate controlled by pair breaking
- Feynman-Galitsky and Kadanoff-Martin: $(\mathbf{v}\cdot\mathbf{Q})\geq\frac{|\mathbf{Q}|^2}{2m^*}$ fails to justify superconductivity

(zero critical velocity from Landau criterion)

Summary

- The Galitskii-Feynman T-matrix approximation fails for superconducting state because of non-physical repeated collisions
- Separating singular channel from selfenergy avoiding repeated collisions leads to propagators of Beliaev form for Bosons or Nambu-Gorkov form for Fermions (Anomalous Green's function is consequence of theory not assumed ad-hoc)
- MSC -Tmatrix theory of pairing and condensation is valid above and below critical temperature: consistent description of pair-breaking effects
- MSC T-matrix justifies two basic assumptions of BCS theory: condensate single-valued, excitations of bound electron pairs can be neglected

1. Phys. Rev. B 78 (2008) 214506: *Multiple scattering corrections to the T-matrix approximation: Unified theory of normal and superconducting states*, P. Lipavský
2. Phys. Rev. 81 (2010) 092501: *Equivalence of channel-corrected T-matrix and anomalous propagator approach*, K. Morawetz
3. J. Stat. Phys. 143 (2011) 482: *Asymmetric Bethe-Salpeter equation for pairing and condensation*, K. Morawetz

	critical velocities of pair breaking	pair excitation
Galitskii	0	0
KM	$\Delta/k_{\rm F}$	0
TMSC	$\Delta/k_{\rm F}$	$\sqrt{3}\Delta/k_{\rm F}$

- Factor of two in non-linear term: non-condensed pairs feel gap due to condensate twice stronger than by Cooper pairs in condensate (like bosons out of BEC twice stronger)
- Our analyses restricted to conventional superconductors of type I and not to recent multi-gapped materials

4. Phys. Rev. B 84 (2012) 094529-1-13: *Self-consistent T-matrix theory of superconductivity*, B. Šopík, P. Lipavský, M. Männel, K. Morawetz, P. Matlock
5. Eur. Phys. J. B 87 (2014) 8: *Stability of condensate in superconductors*, P. Lipavský, K. Morawetz, B. Šopík, M. Männel