Nonlocal Kinetic Theory

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MAX-PLANCK-GESELLSCHAFT

Kinetic description

Boltzmann kinetic equation (1872) for distribution f(k,r,t)
(Uehling, Uhlenbeck, Nordheim, Vlassow...), local in time and space

$$\frac{\partial f_1}{\partial t}+\frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r}+\frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k}= \int dpdq\mathcal{P}\left\{f_3(k-q,r,t)f_4(p+q,r,t)N - f_1(k,r,t)f_2(p,r,t)N'\right\}$$

Non-local corrections: Enskog, Bogoliubov, Green, Ernst, Thirring

Generalization of kinetic equation

Generalization of quantum Boltzmann equation - Two extremes

Theory of Gases

$$\frac{\partial f}{\partial t}+\frac{k}{m}\frac{\partial f}{\partial r}-\frac{\partial U}{\partial r}\frac{\partial f}{\partial k} = (1-f)\sigma^<_\varepsilon - f\sigma^>_\varepsilon$$

- Cross section implies finite volume but instant and local approximation → ideal gas
- Clausius and Boltzmann: Virial corrections
- Memory effects Nonlocalities in space and time

Landau Theory

$$\frac{\partial f}{\partial t}+z\frac{k}{m}\frac{\partial f}{\partial r}-\frac{\partial \varepsilon}{\partial r}\frac{\partial f}{\partial k} = z((1-f)\sigma^<_\varepsilon - f\sigma^>_\varepsilon)$$

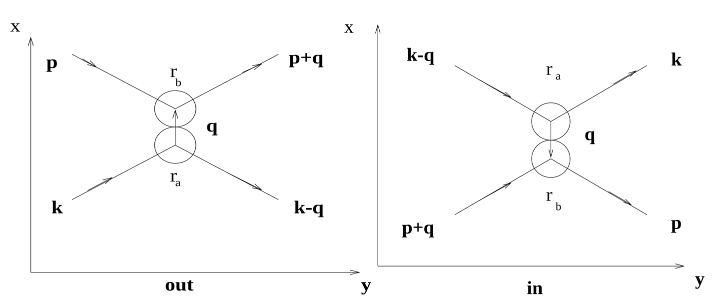
- Wave function renormalisation $z^{-1}=1-\partial_\omega\sigma$
- Pauli-blocking
- Elementary excitations as single particle dispersion relation
- Scattering integral local in space and time

Both treatment covered by extended quasiparticle picture

$$\rho = f+\int\frac{d\omega}{2\pi}\frac{\wp}{\omega-\varepsilon}\frac{\partial}{\partial\omega}((1-f)\Sigma^<_\omega - f\Sigma^>_\omega)$$

Collision of hard spheres

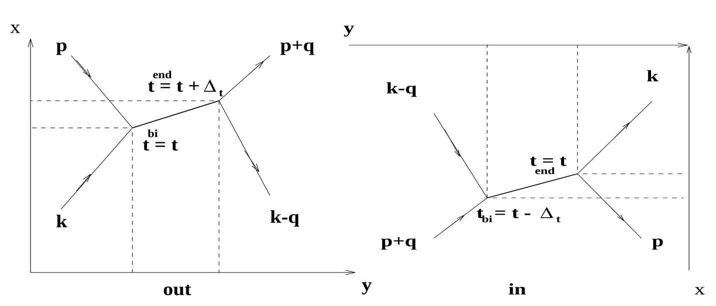
At the instant of the event $\Delta^d = r^{\rm coll}_b - r^{\rm coll}_a; |\Delta^d| = \frac{1}{2}(d_a+d_b)$



$$f_1 = f_a(k,r,t) \qquad f_3 = f_a(k-q,r,t)$$
$$f_2 = f_b(p,r+\Delta^{\rm d},t) \qquad f_4 = f_b(p+q,r-\Delta^{\rm d},t)$$

Collision of sticky point particles

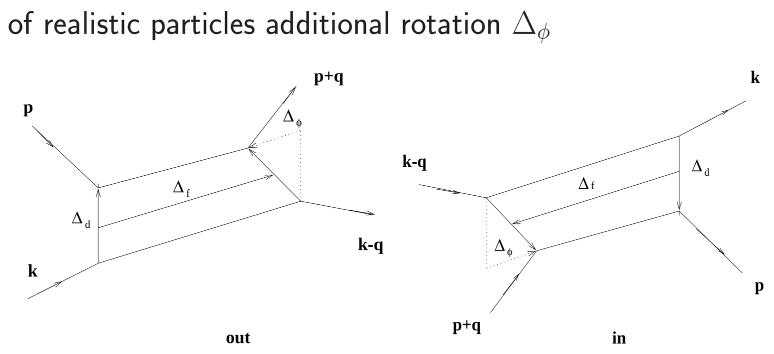
point particles form unstable molecules with velocity $v^{\rm mol}=(k+p)/(m_a+m_b)$ and the collision delay $\Delta^{\rm f}=\Delta_t v^{\rm mol}$.



$$f_1 = f_a(k,r,t) \qquad f_3 = f_a(k-q,r-\Delta^{\rm f},t-\Delta_t)$$
$$f_2 = f_b(p,r,t) \qquad f_4 = f_b(p+q,r-\Delta^{\rm f},t-\Delta_t)$$

Collision of soft particles

collision of realistic particles additional rotation Δ_ϕ



$$f_1 = f_a(k,r,t) \qquad f_3 = f_a(k-q,r-\Delta^{\rm f}-\frac{\Delta_\phi}{2},t-\Delta_t)$$
$$f_2 = f_b(p,r+\Delta^{\rm d},t), \qquad f_4 = f_b(p+q,r-\Delta^{\rm f}+\frac{\Delta_\phi}{2}-\Delta^{\rm d},t-\Delta_t)$$

Kinetic equation

$$\frac{\partial f_1}{\partial t}+\frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r}-\frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k}=\int dpdq\mathcal{P}2\pi\delta\left(\varepsilon^1_a+\varepsilon^2_b-\varepsilon^3_a-\varepsilon^4_b-2\Delta_E\right)$$
$$\times\Big[f_a(k-q-\Delta_k,r+\Delta_3,t-\Delta_t)\,f_b(p+q-\Delta_k,r+\Delta_4,t-\Delta_t) - f_a(k,r,t)\,f_b(p-\Delta_k,r+\Delta_2,t-\Delta_t)\Big]$$

with $\Delta_2=\Delta^d \qquad \Delta_3=\Delta^f-\frac{\Delta^\phi}{2} \qquad \Delta_4=\Delta^f+\frac{\Delta^\phi}{2}+\Delta^d$

Non-local kinetic equation

Non-Markovian (memory) kinetic equation for reduced density ρ

$$\frac{\partial}{\partial t}\rho+\nabla_k\epsilon_{\rm HF}\nabla_r\rho-\nabla_r\epsilon_{\rm HF}\nabla_k\rho = \int_0^{t-t_0}d\tau\Big(\{G^<(t-\frac{\tau}{2},\tau),\Sigma^>(t-\frac{\tau}{2},-\tau)\} - \{G^>(t-\frac{\tau}{2},\tau),\Sigma^<(t-\frac{\tau}{2},-\tau)\}\Big)$$

is transformed by extended quasiparticle picture into precursor of kinetic equation for quasiparticle distribution f

$$\frac{\partial}{\partial t}f+\nabla_k\epsilon\nabla_R f-\nabla_R\epsilon\nabla_k f = z\left((1-f)\sigma^<-f\sigma^>\right)$$

ladder summation: Boltzmann-Uehling-Uhlenbeck, random phase approximation: Lenard- Balescu equation, etc

But: non-local scattering events by

$$\Sigma[G]\to\Sigma[f]$$

Memory or off-shell parts

- compensate the off-shell parts of Kadanoff Baym equation without other neglects
- in quasiparticle energy $\varepsilon_1=\frac{k^2}{2m_a}+{\rm Re}\Sigma^R_{1,\varepsilon_1}$, off-shell part leads to the correct binding energy
- compensation of sequential three particle processes in energy
- direct link between Wigner function (reduced density matrix) and quasiparticle distribution

virial corrections from intrinsic gradients in the scattering integrals.

Quantum kinetic equation

From real-time Greenfunctions

$$\frac{\partial f_1}{\partial t}+\frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r}-\frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k}=\int\frac{dpdq}{(2\pi)^5}\mathcal{P}\,\delta\left(\varepsilon_1+\varepsilon_2^- -\varepsilon_3^- -\varepsilon_4^- -2\Delta_E\right)$$
$$\times\Big[(1-f_1)(1-f_2^-)f_3^-f_4^- - f_1f_2^-(1-f_3^-)(1-f_4^-)\Big]$$

$$f_1 \equiv f(k,r,t) \qquad f_2^- \equiv f(p,r-\Delta_2,t)$$
$$f_3^- \equiv f(k-q-\Delta_K,r-\Delta_3,t-\Delta_t) \qquad f_4^- \equiv f(p+q-\Delta_K,r-\Delta_4,t-\Delta_t)$$

with T-matrix $\quad \mathbf{T} = \cdots + \cdots \mathbf{T} = |\mathbf{T}|\,e^{i\phi}$

$$\mathcal{P}=\mathcal{Z}|\mathbf{T}|^2\Big(\varepsilon_1+\varepsilon_2^- -\Delta_E,k-\frac{\Delta_K}{2},p-\frac{\Delta_K}{2},q,r-\frac{1}{4}(\Delta_2+\Delta_3+\Delta_4),t-\frac{\Delta_t}{2}\Big)$$

$$\Delta_t = \left.\frac{\partial\phi}{\partial\Omega}\right|_{\varepsilon_1+\varepsilon_2} \qquad \Delta_2 = \left(\frac{\partial\phi}{\partial p}-\frac{\partial\phi}{\partial q}-\frac{\partial\phi}{\partial k}\right)_{\varepsilon_1+\varepsilon_2}$$

$$\Delta_E = -\frac{1}{2}\left.\frac{\partial\phi}{\partial t}\right|_{\varepsilon_1+\varepsilon_2} \qquad \Delta_3 = -\left.\frac{\partial\phi}{\partial k}\right|_{\varepsilon_1+\varepsilon_2}$$

$$\Delta_K = \frac{1}{2}\left.\frac{\partial\phi}{\partial r}\right|_{\varepsilon_1+\varepsilon_2} \qquad \Delta_4 = -\left(\frac{\partial\phi}{\partial k}+\frac{\partial\phi}{\partial q}\right)_{\varepsilon_1+\varepsilon_2}$$

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Solution and applications available: http://www.mpipks-dresden.mpg.de/ morawetz

Virial corrections to Observables

Quasiparticle parts (Landautheory – like)

$$n^{\rm qp} = \sum_k f \qquad \mathcal{Q}^{\rm qp} = \sum_k k\,f \equiv m\,j^{\rm qp} = m\sum_k\frac{\partial\varepsilon}{\partial k}f$$
$$\mathcal{E}^{\rm qp} = \sum_k\left(\frac{k^2}{2m}+\frac{1}{2}\sigma_m f\right)f_k \qquad \mathcal{J}^{\rm qp}_{ij} = \sum_k\left(k_j\frac{\partial\varepsilon}{\partial k_i}+\delta_{ij}\varepsilon\right)f-\delta_{ij}\mathcal{E}^{\rm qp}$$

Two-particle correlated parts

$$n^{\rm mol} = \sum_{kpq}\Delta_t\Psi(k,p,q) \qquad j^{\rm mol} = \sum_{kpq}\Delta_3\Psi(k,p,q)$$
$$\mathcal{Q}^{\rm mol} = \sum_{kpq}\frac{k+p}{2}\Delta_t\Psi(k,p,q) \qquad \mathcal{E}^{\rm mol} = \sum_{kpq}\frac{\epsilon_k+\epsilon_p}{2}\Delta_t\Psi(k,p,q)$$
$$\mathcal{J}^{\rm mol}_{ij} = \frac{1}{2}\sum_{kpq}\Psi(k,p,q)\Big\{k_j\Delta_{3i}+p_j(\Delta_{4i}-\Delta_{2i})+q_j(\Delta_{4i}-\Delta_{3i})\Big\}$$

Conservation laws

$$\frac{\partial(n^{\rm free}+n^{\rm mol})}{\partial t}+\frac{\partial(j^{\rm qp}+j^{\rm mol})}{\partial r}=0$$
$$\frac{\partial(\mathcal{Q}^{\rm qp}+\mathcal{Q}^{\rm mol})}{\partial t}+\sum_i\frac{\partial(\mathcal{J}^{\rm qp}_{ij}+\mathcal{J}^{\rm mol}_{ij})}{\partial r_i}=0$$
$$\frac{\partial\mathcal{E}}{\partial t}=\frac{\partial(\mathcal{E}^{\rm qp}+\mathcal{E}^{\rm mol})}{\partial t}=0$$

Internal energy- and momentum–gain

$$\mathcal{I}_{\rm gain} = \sum_{kpq}\Delta_E\Psi(k,p,q) \qquad \mathcal{F}^{\rm gain} = \sum_{kpq}\Delta_K\Psi(k,p,q)$$

with probability per time to form a two-particle molecule
$\Psi(k,p,q)=\delta_{1234}f_1f_2(1-f_3-f_4)|T|^2$
Consistent with extended quasiparticle approximation

Bifurcation in time evolution

Zero momentum transfer channel $q=0$ leads to an additional part absent in usual local Boltzmann kinetic equation

$$\Big[(1-f_1)(1-f_2^-)f_3^-f_4^- - f_1f_2^\pm(1-f_3^\pm)(1-f_4^\pm)\Big]_{q=0} = f_2^-(1-f_2^-)\Big[f_1^- - f_1\Big]$$

For homogeneous systems and relaxation time approximation: additional term in the kinetic equation

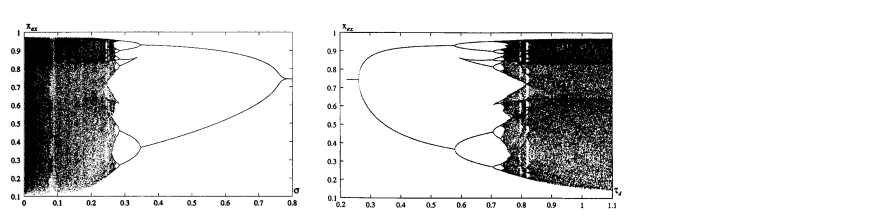
$$\frac{df(k,t)}{dt} = -\frac{f(k,t)-F_0}{\tau}+\frac{f(k,t-\Delta_t)[1-f(k,t-\Delta_t)]}{\tilde\tau}$$

with thermal averaging $<\ldots>$

$$\frac{1}{\tau} = <\frac{2\pi s}{\hbar}\int\frac{dpdq}{(2\pi\hbar)^6}|\mathcal{T}^r(k,p,q)|^2\delta(\varepsilon_k+\varepsilon_p-\varepsilon_{k+q}-\varepsilon_{p-q})f_p(1-f_{k-q})(1-f_{p+q})>$$
$$\frac{1}{\tilde\tau} = <\frac{2\pi s}{\hbar}\int\frac{dpdq}{(2\pi\hbar)^6}|\mathcal{T}^r(k,p,q)|^2\delta((k-p)\cdot\frac{q}{m})f_p(1-f_p)>$$
$$\Delta_t = -\frac{1}{\hbar}<\frac{\partial}{\partial\omega}{\rm Im}\ln\mathcal{T}^r(k,p,q,\omega)|_{\omega=\varepsilon_k+\varepsilon_p}>$$

Rename $\alpha=\frac{\tau}{\tilde\tau} \quad \sigma=\tau \quad \tau_d=\Delta_t$ to obtain
(M. Berezowski, Chaos, Solitons and Fractals **12**, 83 (2001))

$$\sigma\frac{df(t)}{dt}+f(t)=\alpha f(t-\tau_d)[1-f(t-\tau_d)]$$



Feigenbaum diagram $\alpha=3.9,\tau_d=1$ (left) and $\alpha=3.9$, $\sigma=0.2$ (right)
Bifurcation sets in for $|\alpha-2|\gtrless\sqrt{1+(\sigma\Omega)^2}$ for $\tau_d\gtrless 0$ and oscillation appears with a frequency

$$\tau_d\Omega+\arctan(\sigma\Omega)=\pi$$

Summary

1. Dense interacting Fermi system are described by non-local collisions in quantum kinetic equation
   - Neglecting shifts: Landau–Quasiparticle (QP) equation
     Neglecting QP- renormalisation: Enskog–like (virial corrections)
   - Thermodynamically consistent equation of state
     (energy, density, pressure) including quantum 2. virial coefficient
   - Conservation laws proven
   - Applicable recipe for incorporation in simulations
2. Applications
   - Nuclear reactions: more energetic collisions in the early phase, preequilibrium particle emission (high energetic spectra), enhancement of neck and midrapidity distribution, nonequilibrium phase transitions (iso-nothing), anomalous density and velocity distributions
   - Neutral impurities: cancellation of virial and quasiparticle corrections, Fermi - momentum is changed by number of correlated pairs
   - Delay differential equation interpolate between finite time stepping of the logistic map and continuous time derivatives (nucl-th/0012028), Novel observation: distribution function itself shows temporal oscillations around the equilibrium value, bifurcation, signal of onsetting phase transition