Anomalous Radial Expansion

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MAX-PLANCK-GESELLSCHAFT

Expansion velocity

- In heavy-ion collisions compression is followed by expansion characterised by collective flow [1]
- freeze-out configuration given by geometrical length scale [2] but also by the thermal length scale [3]
- In almost all papers, linear velocity profile $\alpha = 1$:

$$\vec v(\vec r) = v_0 \left(\frac{r}{R_0}\right)^{\alpha} \frac{\vec r}{r}$$

non-relativistic hydrodynamics with the ideal gas equation of state for a slowly expanding fireball [5, 6], exception [9]

- In cosmology Hubble constant [7, 8] : $\dot R(t)/R(t) \approx 65$ km/s/Mpc, expansion of homogeneous and isotropic galaxies [8]
- Experimental data on nuclear multifragmentation within an extended statistical microcanonical model [10], necessary to postulate a non-Hubblean velocity profile with α in between 1.5 and 2 [10], important for coalescence model [4]

Kinetic simulation

Solve numerically the nonlocal kinetic equation [11–13]

$$\frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r} + \frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k} = \int \mathcal{P}^{\pm}\{f_3^- f_4^- (1-f_1)(1-f_2^-) - f_1 f_2^{\pm}(1-f_3^{\pm})(1-f_4^{\pm})\}$$

- Enskog-type shifts of arguments [14, 15]:

$$f_1 \equiv f_a(\mathbf{k},\mathbf{r},t) \qquad f_3 \equiv f_a(\mathbf{k}-\mathbf{q}-\Delta_K, \mathbf{r}-\Delta_3, t-\Delta_t),$$
$$f_2 \equiv f_b(\mathbf{p},\mathbf{r}-\Delta_2,t) \qquad f_4 \equiv f_b(\mathbf{p}+\mathbf{q}-\Delta_K, \mathbf{r}-\Delta_4, t-\Delta_t)$$

- effective scattering measure $\mathcal{P} \propto |\mathcal{T}|^2$, the $\mathcal{T}$-matrix
- quasiparticle energy ε contains mean field and correlated self energy
- Delays are derivatives of scattering phase shift $\phi = \mathrm{Im}\ln\mathcal{T}$ given by realistic potentials [16] :

$$\Delta_t = \left.\frac{\partial\phi}{\partial\Omega}\right|_{\varepsilon_1+\varepsilon_2} \qquad \Delta_2 = \left(\frac{\partial\phi}{\partial\mathbf{p}} - \frac{\partial\phi}{\partial\mathbf{q}} - \frac{\partial\phi}{\partial\mathbf{k}}\right)_{\varepsilon_1+\varepsilon_2}$$

$$\Delta_E = -\left.\frac{1}{2}\frac{\partial\phi}{\partial t}\right|_{\varepsilon_1+\varepsilon_2} \qquad \Delta_3 = -\left.\frac{\partial\phi}{\partial\mathbf{k}}\right|_{\varepsilon_1+\varepsilon_2}$$

$$\Delta_K = \left.\frac{1}{2}\frac{\partial\phi}{\partial\mathbf{r}}\right|_{\varepsilon_1+\varepsilon_2} \qquad \Delta_4 = -\left(\frac{\partial\phi}{\partial\mathbf{k}} + \frac{\partial\phi}{\partial\mathbf{q}}\right)_{\varepsilon_1+\varepsilon_2}.$$

Global thermodynamical variables

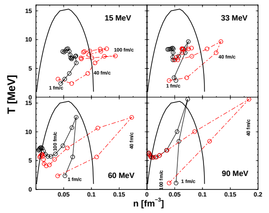
From distribution function local density, current and energy densities

$$\begin{pmatrix} n(\mathbf{r},t) \\ \mathbf{J}(\mathbf{r},t) \\ \mathcal{E}_K(\mathbf{r},t)\end{pmatrix} = \int \frac{d\mathbf{p}}{(2\pi\hbar)^3}\begin{pmatrix} 1 \\ \mathbf{p} \\ p^2/2m\end{pmatrix} f(\mathbf{p},\mathbf{r},t)$$

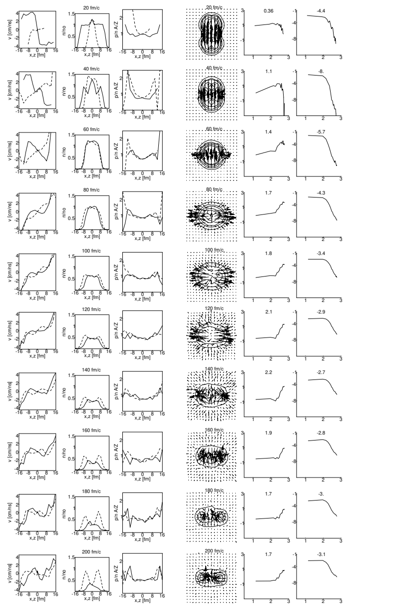
Global variables/particle by spatial integration

$$\text{kin. energy} \quad E_K(t) = \int d\mathbf{r} E_K(\mathbf{r},t)$$
$$\text{Fermi energy} \quad E_F(t) = \int d\mathbf{r} E_f[n(\mathbf{r},t)]n(\mathbf{r},t)/\int d\mathbf{r}\, n(\mathbf{r},t)$$
$$\text{coll. energy} \quad E_{\rm coll}(t) = \int d\mathbf{r}\, J(\mathbf{r},t)^2/mn(\mathbf{r},t)/\int d\mathbf{r}\, n(\mathbf{r},t)$$

Fermi liquid theory yields temperature $E_K(t) = \frac{3}{5}E_F(t) + E_{\rm coll}(t) + \frac{\pi^2}{4E_F(t)}T(t)^2$



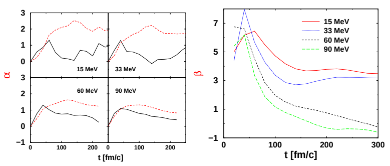
The time evolution of the temperature versus the density for the sphere of a constant radius ($R < 10$ fm) (the solid line) corresponding to the distinction between the bulk and surface regions in Fig. 1, and for the sphere given by the mean square radius of the evolving system (the dotted line). The thick solid lines give the limit of the spinodal region for an infinite nuclear matter.



The time evolution of central $Ta + Au$ collisions at $E_{lab}/A = 33$ MeV . The first column represents the velocity and the second column the density profiles profile. Both transversal (the x-direction) (the dashed line) and the longitudinal (the z-direction) (the solid line) profiles are shown. The third column presents the ratio of proton and neutron densities normalised to the initial Z/A. The $(x - z)$ - density cutis seen in the fourth column where mass momenta are shown by arrows. The fives and six column shows the log - log plot of the angular averaged modulus of the expansion velocity and density versus the radius. The slope of the straight line fit of the surface matter expansion profile for $R > 10$ fm is indicated at each plot.

Radial dependence of velocity and density

$v \propto r^{\alpha}$ $\qquad$ $n \propto r^{-\beta}$



dotted lines: surface matter, solid lines: bulk matter, surface ($R > 10$ fm)

Example: Hubble expansion

total energy h of radially symmetric matter $\qquad \frac{m}{2}\dot R(t)^2 - \frac{G}{R(t)^{\delta}} = h$

homogeneous density n, mass $m = 4\pi n R^3/3$. Escaping matter ($h = 0$) $\qquad \dot R(t) = \sqrt{\frac{6G}{4\pi n}}\, R(t)^{\frac{3-\delta}{2}}$,

$$\alpha = \frac{3-\delta}{2} = \begin{cases} = 1 & \text{corresponds to } \delta = 1 \text{ (Coulomb, Gravitation)} \\ = 2 & \text{corresponds to } \delta = -1 \text{ (String force)}\end{cases}$$

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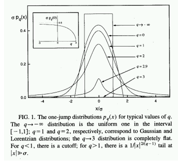
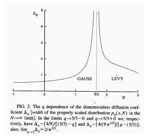
Interpretation in terms of Tsallis statistics [17]

From [18] $\qquad n(r) \propto r^{-\beta} \qquad \beta = \frac{2}{q-1} \approx 3 \to q = \frac{5}{3}$





$v(r) \propto r^{\alpha} \qquad \alpha \approx 2$

Anomalous Diffusion [19] $\qquad \frac{\partial}{\partial t}P(x,t) = D\nabla^{\gamma}P(x,t)^{\nu}$

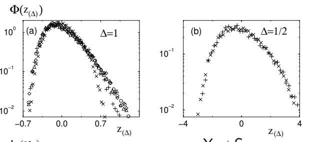
$$\lim_{\gamma\to 2} P(x,t,\gamma) \propto \frac{1}{x^3} \qquad q = \frac{\gamma+3}{\gamma+1} \approx \frac{5}{3}$$

$$\frac{\partial}{\partial t}P(x,t) = \frac{A}{k_1 t^2}\frac{1}{z^2(1+bz)^2} \qquad z = x(|k_1|t)^{-\frac{\gamma+1}{\gamma^2-\gamma+1}}$$

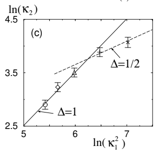
$$v(r) \approx <x\dot P(x,t)> \propto \frac{1}{b^2}\left(\ln(1+bz) + \frac{1}{1+bz}\right) \approx \frac{1}{b^2} + \frac{z^2}{2} + o(bz^3)$$

Universal Fluctuations in exp. Data [20]

Δ −scaling : $\qquad <m>^{\Delta} P_{<m>}[m] = \Phi\left(\frac{m-m^*}{<m>^{\Delta}}\right)$



Xe+Sn



× 25 MeV
+ 32 MeV
△ 39 MeV
◇ 45 MeV
○ 50 MeV

$\kappa_1 =< m >$, $\kappa_2 =< m^2 > - < m >^2$, $K_2 = \kappa_2/\kappa_1^2$

Summary

- Dynamical behaviour of heavy-ion reactions in the Fermi energy domain during the first 200 fm/c is associated with a nonlinear ($\alpha > 1$) radial velocity profile
- Time when compression turns into expansion corresponds to crossing point of bulk α_{bulk} and surface α_{surf} scaling exponents
- Nonlinear radial velocity profile in the surface region is accompanied by long tail of the particle density
- No Gaussian source, power law two-particle Bose-Einstein correlations
- Different behaviour of the surface and bulk matter important for super nova where surface - like ring of matter (crust) is expanding with enormous velocities and separated from the bulk matter collapsing into neutron stars
- Interplay between Coulomb and mean field leads to a string-like forces for surface particles in central HI collisions around the Fermi energy
- Dynamical behaviour creates confinement as non-equilibrium effect ?