Currents in graphene and pairing

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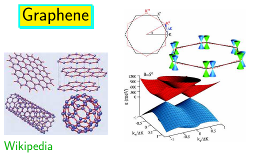
Graphene



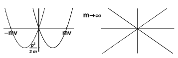
C. J. Tabert, E. J. Nicol, PRB 87 (2013) 121402(R)

Wikipedia

• linear spin-orbit coupling $\vec{b} = v(p_x, p_y)$ leads to splitting

$$\epsilon_\pm = \frac{p^2}{2m} \pm vp$$

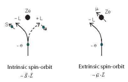
• Dirac dispersion by $m \to \infty$



Lindhard polarization vanishes !

K. Morawetz, PRB 92 (2015) 245425, 245426

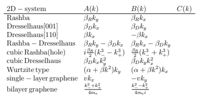
Extrinsic vs intrinsic s.o.

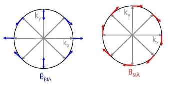


intrinsic: c-band (s) coupled to v-bands (v) GaAs/AlGaAs

bulk-inv. asym. (III-V semicond.) Dresselhaus $-k_y\sigma^y + k_x\sigma^x$

structure inversion asym. (macrosc. confining) Rashba $-k_x\sigma^y + k_y\sigma^x$





3D − system	$A(k)$	$B(k)$	$C(k)$
bulk Dresselhaus	$k_x(k_y^2-k_z^2)$	$k_y(k_x^2-k_z^2)$	$k_z(k_x^2-k_y^2)$
Cooperpairs	Δ	0	$\frac{p^2}{2m}-\epsilon_F$
extrinsic $\beta=\frac{i}{\hbar}\lambda^2 V(k)$	$q_yk_z-q_zk_y$	$q_zk_x-q_xk_z$	$q_xk_y-q_yk_x$
neutrons in nuclei $\beta=iW_0(n_n+\frac{n_p}{2})$	$q_zk_y-q_yk_z$	$q_xk_z-q_zk_x$	$q_yk_x-q_xk_y$

General form
$H^s = A(\mathbf{k})\sigma_x - B(\mathbf{k})\sigma_y + C(\mathbf{k})\sigma_z = \mathbf{b}\cdot\sigma$

Interactions and meanfields

1. Spin-orbit coupling $\sigma\cdot\mathbf{b}(\mathbf{p})$
magn. impurities, intrinsic+ extrinsic

$$\hat{V}_{-p,p'} = \begin{cases} V_0(p'-p) \\ \sigma\cdot\mathbf{V}(p'-p) \\ \frac{i\lambda^2}{\hbar}\sigma\cdot(\mathbf{p}\times\mathbf{p}')V(p'-p) \end{cases}$$

2. Leads to impurity meanfields $\Sigma_0^{\rm imp} = nV_0 + \mathbf{s}\cdot\mathbf{V}; \quad \mathbf{\Sigma}^{\rm imp} = \mathbf{s}\,V_0 + n\mathbf{V}$

Extrinsic spin-orbit coupling meanfield more involved
$\Sigma_0^{\rm ext.} = i\frac{\lambda^2}{\hbar^2}V\,[m(\mathbf{S}_j\times\mathbf{q})_j - \mathbf{s}\cdot(\mathbf{p}\times\mathbf{q})]$, $\mathbf{\Sigma}^{\rm ext.} = i\frac{\lambda^2}{\hbar^2}V\,[m(\mathbf{j}\times\mathbf{q}) - n(\mathbf{p}\times\mathbf{q})]$
dens. $n=\sum_p f$, curr. $\mathbf{j}=\sum_p\frac{\mathbf{p}}{m}f$, polar. $\mathbf{s}=\sum_p\mathbf{g}$, curr. $S_{ji}=\sum_p\frac{p_j}{m}[\mathbf{g}]_i$

3. Effective (meanfield) Hamiltonian

$$H = \frac{k^2}{2m} + \Sigma_0(\mathbf{k},\mathbf{q},t) + e\Phi(\mathbf{q},t) + \sigma\cdot\mathbf{\Sigma}(\mathbf{k},\mathbf{q},t)$$

with $\mathbf{\Sigma} = \mathbf{\Sigma}_{MF}(\mathbf{k},\mathbf{q},t) + \mathbf{b}(\mathbf{k},t) + \mu\mathbf{B}$

• Search kinetic theory (non-Abelian) $\int\frac{d\omega}{2\pi}G^< = f(\mathbf{k},\mathbf{q},t) + \sigma\cdot\mathbf{g}(\mathbf{k},\mathbf{q},t)$

Kinetic equation with spin-orbit coupling and electric and magnetic fields

$$\boxed{\begin{aligned}&(\partial_t + \mathcal{F}\partial_{\mathbf{p}} + \mathbf{v}\partial_{\mathbf{r}})f + \mathbf{A}\cdot\mathbf{g} = 0\\ &(\partial_t + \mathcal{F}\partial_{\mathbf{p}} + \mathbf{v}\partial_{\mathbf{r}})\mathbf{g} + \mathbf{A}\,f = 2(\mathbf{\Sigma}\times\mathbf{g})\end{aligned}}$$

coupling of spinor terms $A_i = \partial_{\mathbf{p}}\Sigma_i\partial_{\mathbf{r}} - \partial_{\mathbf{r}}\Sigma_i\partial_{\mathbf{p}} + (\partial_{\mathbf{p}}\Sigma_i\times e\mathbf{B})\partial_{\mathbf{p}}$
with velocity $\mathbf{v} = \frac{\mathbf{k}}{m}+\partial_{\mathbf{k}}\Sigma_0$ and eff. Lorentz force $\mathcal{F} = (e\mathbf{E}-\partial_{\mathbf{r}}\Sigma_0+e\mathbf{v}\times\mathbf{B})$

Stationary solution: $\hat{\rho}(\hat{\varepsilon}) = \sum_\pm\hat{P}_\pm f_\pm = \frac{f_++f_-}{2} + \sigma\cdot\mathbf{e}\,\frac{f_+-f_-}{2} = f + \sigma\cdot\mathbf{g}$
with effective splitting $f_\pm = f_0(\epsilon_k\pm|\mathbf{\Sigma}|)$
and selfconsistent meanfield $\epsilon_k(r) = \frac{k^2}{2m} + \Sigma_0(k,r)$
and selfconsistent precession $\mathbf{e}(k,r) = \mathbf{\Sigma}/|\mathbf{\Sigma}|$

Anomalous currents

due to Pauli structure $\hat{H}_{\rm eff} = H + \sigma\cdot\mathbf{\Sigma}$
with $\mathbf{\Sigma} = \mathbf{\Sigma}^H(\mathbf{k},\mathbf{r},t) + \mathbf{b}(\mathbf{k},\mathbf{r},t) + \mu_B\mathbf{B}$, particle and pseudospin current densities:

$$\hat{j}_\alpha = \sum_p[\hat{\rho}, v_\alpha]_+ = 2\sum_p[f\partial_{p_\alpha}\epsilon + \mathbf{g}\cdot\partial_{p_\alpha}\mathbf{\Sigma} + \sigma\cdot(\partial_{p_\alpha}\epsilon\mathbf{g} + f\partial_{p_\alpha}\mathbf{\Sigma})]$$
$$= j^n_\alpha + j^a_\alpha + \sigma\cdot\mathbf{S}_\alpha$$

scalar: normal and anomaly current, vector: pseudospin current S_{ij}

Dirac particles by the limit $\epsilon_\pm = \frac{p^2}{2m}\pm|\mathbf{\Sigma}| \to \pm vp$
• graphene only possess an anomalous particle current
• normal pseudospin current possesses unexpected finite $m\to\infty$ limit

Anomalous particle conductivities: universal limits

anomaly particle current $J_\alpha = (\sigma^{\rm Hall}_{\alpha\beta} + \sigma^{\rm inter}_{\alpha\beta} + \sigma^{\rm intra}_{\alpha\beta})E_\beta$
with $\mathbf{e} = \mathbf{\Sigma}/|\Sigma|$, $g = (f_+ - f_-)/2$

$$\sigma^{\rm Hall}_{\alpha\beta} = 2e^2\sum_p\frac{g}{1-\frac{\omega^2}{4|\Sigma|^2}}\mathbf{e}\cdot(\partial_{p_\alpha}\mathbf{e}\times\partial_{p_\beta}\mathbf{e}) \to \frac{e^2}{8\pi\hbar}\begin{cases}\frac{\Sigma_n}{\mu}+o(\tau_\omega^{-1}), & \mu>\Sigma_n\\ 1+o(\tau_\omega^{-1}), & \mu<\Sigma_n\\ \frac{\Sigma_n\tau}{\hbar}\left(\pi-\frac{4\tau\mu}{\hbar}+o(\mu^2)\right), & \mu>\Sigma_n\end{cases}$$

$$\sigma^{\rm inter}_{\alpha\beta} = 2e^2\sum_p\frac{g}{1-\frac{\omega^2}{4|\Sigma|^2}}\frac{i\omega}{2|\Sigma|}\partial_{p_\alpha}\mathbf{e}\cdot\partial_{p_\beta}\mathbf{e} \to \zeta\frac{e^2}{16\hbar}, \qquad (n\to 0)$$

order of limits					$\sigma^{\rm inter}_{xx}=\zeta\frac{e^2}{16\hbar}$
5.	4.	3.	2.	1.	ζ
$\mu\to 0$	$\omega\to 0$	$\Sigma\to 0$	$m\to\infty$	$\tau\to\infty$	-1
$\mu\to 0$	$\tau\to\infty$	$\Sigma\to 0$	$m\to\infty$	$\omega\to 0$	0
$\tau\to\infty$	$\mu\to 0$	$\Sigma\to 0$	$m\to\infty$	$\omega\to 0$	1
$\mu\to 0$	$\Sigma\to 0$	$\tau\to\infty$	$m\to\infty$	$\omega\to 0$	1
$\mu\to 0$	$\tau\to\infty$	$m\to\infty$	$\Sigma\to 0$	$\omega\to 0$	1
$\mu\to 0$	$\tau\to\infty$	$\omega\to 0$	$m\to\infty$	$\Sigma\to 0$	1
$\mu\to 0$	$\Sigma\to 0$	$m\to\infty$	$\tau\to\infty$	$\omega\to 0$	0
$\mu\to 0$	$m\to\infty$	$\tau\to\infty$	$\Sigma\to 0$	$\omega\to 0$	0
$\mu\to 0$	$m\to\infty$	$\Sigma\to 0$	$\omega\to 0$	$\tau\to\infty$	0
$\mu\to 0$	$\Sigma\to 0$	$\omega\to 0$	$m\to\infty$	$\tau\to\infty$	0

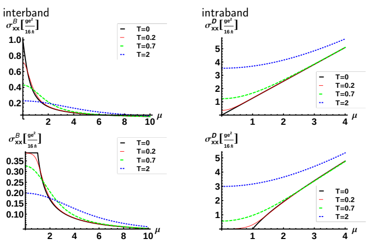
• chiral nature of charge carriers leads to minimal finite conductivity even with vanishing density of scatterers
• field has to create first electron-hole pairs before they can be accelerated

$$\sigma^{\rm intra}_{\alpha\beta} = i2e^2\sum_p\partial_{p_\alpha}\partial_{p_\beta}\Sigma\frac{g}{\omega} = \frac{i\epsilon_0\omega_p^2(n,T,\Sigma_n)}{\omega+\frac{i}{\tau}}$$

Intra- and interband conductivities

interband

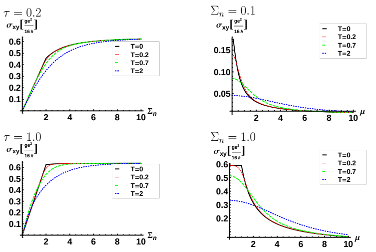
intraband



$\Sigma_n = 0$ (above) and $\Sigma_n = 1EU$ (below) with $\tau = 1\hbar/EU$ vs chemical potential

• intraband conductivity has a threshold at the effective Zeeman field

Hall conductivity



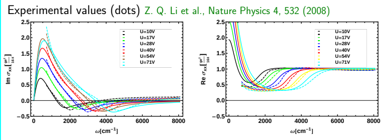
vs effective Zeeman field with $\mu = 2$

vs chemical potential with $\tau = 1$

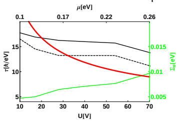
• independent of density for large Zeeman field, for vanishing scattering universal value
• threshold at chemical potentials about Zeeman energy, below constant

Optical conductivity

Experimental values (dots) Z. Q. Li et al., Nature Physics 4, 532 (2008)



• if Zeeman field larger than chemical potential conductivity exclusively by intraband transitions and independent of density (universal)



relaxation time for screened Coulomb interaction, fitted effective Zeeman field, only with relaxation time (dashed)

Pseudospin conductivity

pseudospin current $\mathbf{S}_\alpha = 2\sum_p(\partial_{p_\alpha}\epsilon\delta\mathbf{g} + \delta f\partial_{p_\alpha}\mathbf{b})$, normal spin-Hall

$$\left.\begin{matrix}\sigma^{\rm as}_{\alpha\beta}\\ \sigma^{\rm sym}_{\alpha\beta}\end{matrix}\right\} = \frac{e}{m_e\omega}\sum_p\frac{p_\alpha g}{1-\frac{\omega^2}{4|\Sigma|^2}}\left\{\begin{matrix}\frac{i\omega}{2|\Sigma|}\mathbf{e}\times\partial_{p_\beta}\mathbf{e}\\ i\partial_{p_\beta}\mathbf{e}\end{matrix}\right.$$

with $\omega\to\omega+i/\tau$, for zero temperature and linear Rashba coupling

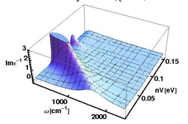
$$\sigma^z_{yx} = \frac{e}{8\pi\hbar}\left[1-\frac{\hbar^2+4\Sigma_n^2\tau_\omega^2}{4\epsilon_v\tau_\omega\hbar}\arctan\left(\frac{4\hbar\epsilon_v\tau_\omega}{\hbar^2+4\tau_\omega^2(2\epsilon_v\mu+\Sigma_n^2)}\right)\right]$$
$$\sigma^z_{xx} = \frac{2}{\hbar}\Sigma_n\tau\sigma^z_{yx}$$

with $\tau_\omega = \tau/(1-i\omega\tau)$ and $\epsilon_v = mv^2$

for graphene limit of infinite mass universal value $\lim_{m\to\infty}\sigma^z_{yx} = \frac{e}{8\pi\hbar}$ contrary to expectation vanishing normal parts of pseudospin current

Density response function

induced density $\delta n = \chi\Phi^{\rm ext}$, inverse dielectric function $\frac{1}{\epsilon} = \frac{V^s}{\Phi^{\rm ext}} = 1 + V_0\chi$



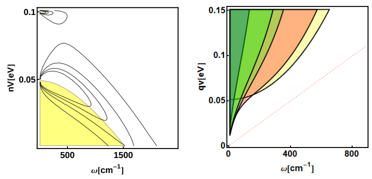
vs frequency and domain interactions, $vq = 1eV$ and density corresponding to $28V$

• with increasing magnetic domain strength V collective peak sharpened and shifted towards lower frequencies, second peak around $V = 0.1$ is at the line $\mu \le \Sigma_n$
• small parameter $\eta = \frac{vq}{\hbar\omega} = \frac{1}{300}$ in the optical regime cannot lead to any sign change

$$\frac{1}{\epsilon} = 1 - \frac{\omega_p^2}{\omega^2}\eta^2 - i\frac{\omega_p^2}{\omega^2}\frac{\pi\alpha}{16}\eta^3 + o(\eta^4)$$

Sign change of dielectric function

but non-optical regime, where $\hbar\omega \neq cq$ $\quad \epsilon = 1 + i\frac{\alpha}{1+\frac{\Sigma_n V^2 c^2 q^2}{\omega^2 \dots}}\frac{\pi\alpha}{16} + o(\alpha^2)$



range where the real part of dielectric function becomes negative (yellow)

range of sign change for magnetic domain from left to right: $nV = 0, 0.02, 0.06, 0.07, 0.09$eV, straight line optical limit $\hbar\omega = qv$

• sign change is a prerequisite for Cooper pairing

Summary

• Coupled quantum kinetic equation for systems with SU(2) structure:
  – mean field interaction (scalar+vector), suited for magnetized impurities, spin-flip, ..
  – arbitrary magnetic and electric fields, spin-orbit interaction
• anomalous currents in graphene as infinite mass limit of spin-orbit coupling
• influence of magnetic domain puddles and meanfields recast into effective Zeeman field on intra-, interband longitudinal and Hall conductivities
• density-independent universal conductivity for large Zeeman fields or small densities
• experimental optical conductivity well reproduced by intrinsic effective Zeeman field
• pseudospin current non-trivially universal value though quasiparticle velocity vanishes
• effective Zeeman field leads to frequency and wavelength range where screened interaction changes sign allowing Cooper pairs

• Europhys. Lett., 104 (2013) 27005: *Terrahertz out-of-plane pulses due to spin-orbit coupling*
• *Quantum kinetic theory of spin-polarized systems in electric and magnetic fields with spin-orbit coupling:*
Phys. Rev. B 92 (2015) 245425: *I. Kinetic equation and anomalous Hall and spin-Hall effects,*
Phys. Rev. B 92 (2015) 245426: *II. RPA response functions and collective modes*
• Phys. Rev. B 94 (2016) 165415: *Dynamical charge and pseudospin currents in graphene and possible Cooper pair formation*