Squeezing Mode in Nuclear Collisions

Klaus Morawetz[1] and Pavel Lipavský[2]

[1]Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Straße 38, 01187 Dresden
[2]Institute of Physics, Academy of Sciences, Cukrovarnická 10, 16200 Praha 6, Czech Republic

Nonequilibrium Mott effect

Equilibrium

Pressure to system with bound states - decomposition (ionisation)
Realised also with increase of density [1–4]:

- Threshold decrease with increasing density
- Bound level lowered slower than threshold: bound state breaks off (ionisation)

Nonequilibrium

Common threshold needs time to be established !!
Example: Collision of two nuclei at early stage

- Nonequilibrium particle emission [5, 6]
- Pauli principle forbid over-occupation of states, Fermi gap
- Before equilibration an outer nucleon feels rapidly increasing force due to other nuclei and Pauli - forbidden areas in phase space
- If speed high enough, shrinking of phase space before dissipation
- Diabatic model [7–9]: upper limit of projectile energy, longitudinal emission
- Squeezed mode here: lower limit of energy, transversal distribution

Analytical Model: Time Dependent Walls

Squeezing walls:

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\Psi_t(x) = 0,$$

$$\Psi_t(0)=\Psi_t(b-vt) = 0$$

Exactly solvable [10]:

$$\Psi_t(x)=\sqrt{\frac{2}{b-vt}}\sin\frac{k_n x b}{b-vt}\exp\left(-i\frac{m^2vx^2+k_n^2\hbar^2 bt}{2m\hbar(b-vt)}\right)$$

Energy levels

$$E_n(t)=\frac{mv^2}{12n^2\pi^2}(2n^2\pi^2-3)+\frac{n^2\pi^2\hbar^2}{2m(b-vt)^2}$$

Increase above ground state

$$\Delta E_n(t)=E_n(t)-E_n^0=E_\phi+E_n^0\left(\frac{b^2}{(b-vt)^2}-1\right)$$

Even at time $t=0$ the initial binding energy $E_n^0=\frac{\hbar^2\pi^2n^2}{2mb^2}$ is shifted by

$$E_\phi=\frac{E_{\rm proj}}{3}\left(1-\frac{3}{2n^2\pi^2}\right)$$

with the projectile energy $E_{\rm proj}=mv^2/2$ due to the finite velocity v.

Discussion

Ionisation happens when energy above threshold E_c

$$\frac{b^2}{(b-vt)^2}\ge\frac{E_c-E_\phi}{E_n^0}.$$

Immediate ionisation if phase kinetic energy E_ϕ exceeds threshold E_c

$$E_{\rm proj}\ge\frac{3}{1-\frac{3}{2n^2\pi^2}}E_c.$$

For projectile kinetic energies below, reaction time restriction

$$\frac{b}{v}\ge t\ge\frac{b}{v}\left(1-\sqrt{\frac{E_n^0}{E_c-E_\phi}}\right)$$

Geometrical estimate, radii $R_1\approx R_2\approx R$, impact parameter B

Time between first touching and closest approach

$$t_m=\frac{\sqrt{(2R)^2-B^2}}{v_{\rm proj}}\approx\frac{R-B/2}{v}$$

Ionisation if

$$B\le 2R\sqrt{\frac{E_n^0}{E_c-E_\phi}}\le 2R$$

means geometrically restricted ionisation

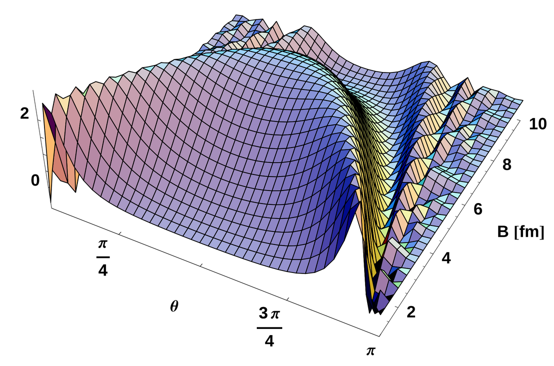
$$\frac{3}{1-\frac{3}{2n^2\pi^2}}E_c\ge E_{\rm proj}\ge\frac{3}{1-\frac{3}{2n^2\pi^2}}\left(E_c-E_n^0\left(\frac{2R}{B}\right)^2\right).$$

Wigner function

$$f(p,R,t) = \int\limits_{-(b-vt)}^{b-vt} dr e^{-irp}\Psi^+(R-\frac{r}{2},t)\Psi(R+\frac{r}{2},t)$$
$$= 2\frac{\sin k}{k}\left(\frac{k^2\cos(n\pi)}{k^2-n^2\pi^2}-\cos(\frac{2\pi nR}{b-vt})\right)$$

and $\hbar k=p(b-vt)+mvR$

Angular distribution of emitted nonequilibrium particles



The Wigner function integrated over the spatial coordinates and zero momentum versus angle and impact parameter. The emission angle was approximated by the ratio of impact parameter to the distance of nuclei according to $b-vt = B/\sin(\alpha)$. The projectile energy is around Fermi energy. For nonzero momenta the transversal maximum is shifted to lower impact parameter.

Kinetic simulation

Solve numerically the nonlocal kinetic equation [11–13]

$$\frac{\partial f_1}{\partial t}+\frac{\partial\varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r}+\frac{\partial\varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k}=$$
$$\int\mathcal{P}^{\pm}\{f_3^-f_4^-(1-f_1)(1-f_2^-)-f_1f_2^{\pm}(1-f_3^{\pm})(1-f_4^{\pm})\}$$

- Enskog-type shifts of arguments [14, 15]:

$$f_1 \equiv f_a(\mathbf{k},\mathbf{r},t) \qquad f_3 \equiv f_a(\mathbf{k}-\mathbf{q}-\Delta_K,\mathbf{r}-\Delta_3,t-\Delta_t),$$
$$f_2 \equiv f_b(\mathbf{p},\mathbf{r}-\Delta_2,t) \qquad f_4 \equiv f_b(\mathbf{p}+\mathbf{q}-\Delta_K,\mathbf{r}-\Delta_4,t-\Delta_t)$$

- effective scattering measure $\mathcal{P}\propto|\mathcal{T}|^2$, the $\mathcal{T}$-matrix
- quasiparticle energy ε contains mean field and correlated self energy
- Delays are derivatives of scattering phase shift $\phi={\rm Im}\ln\mathcal{T}$ given by realistic potentials [16].

References

[1] W. D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, *Quantum Statistics of Charged Particle Systems* (Akademie Verlag, Berlin, 1986).

[2] M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. (NY) **202**, 57 (1990).

[3] H. Stein, A. Schnell, T. Alm, and G. Röpke, Zeit. Phys. A **351**, 295 (1995).

[4] P. Bozek, P. Danielewicz, K. Gudima, and M. Ploszajczak, Phys. Lett. B **421**, 31 (1998).

[5] J. Tõke and et. al., Phys. Rev. Lett. **77**, 3514 (1996).

[6] R. Yanez and et al., Phys. Rev. Lett. **82**, 3585 (1999).

[7] W. Cassing and W. Nörenberg, Nucl. Phys. A **431**, 558 (1984).

[8] L. X. Ge and W. Nörenberg, Z. Phys. A **323**, 293 (1986).

[9] W. Cassing, Z. Phys. A **326**, 21 (1987).

[10] V. V. Dodonov, A. B. Klimov, and D. E. Nikonov, J. Math. Phys. **34**, 3391 (1993).

[11] V. Špička, P. Lipavský, and K. Morawetz, Phys. Rev. E (1998), sub. nucl-th/0012040.

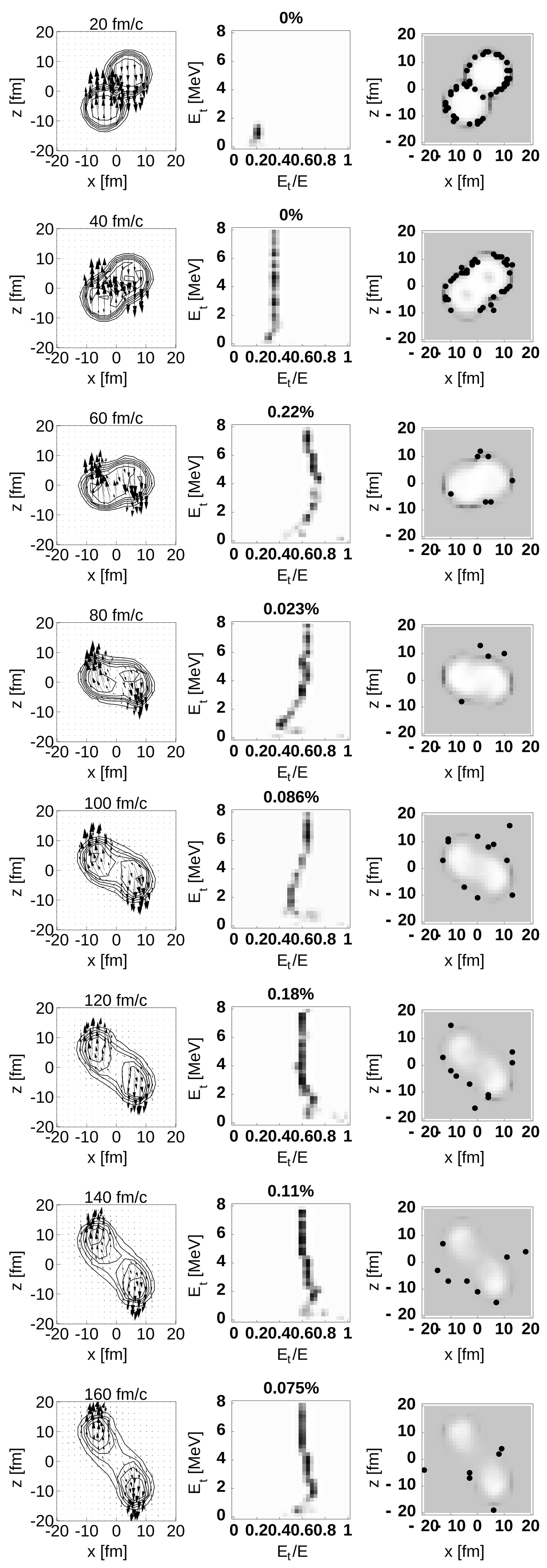
[12] K. Morawetz *et al.*, Phys. Rev. Lett. **82**, 3767 (1999).

[13] K. Morawetz, Phys. Rev. C **62**, 44606 (2000).

[14] V. Špička, P. Lipavský, and K. Morawetz, Phys. Lett. A **240**, 160 (1998).

[15] P. Lipavský, K. Morawetz, and V. Špička, (1999), book in press Annales de Physique, K. Morawetz, Habilitation University Rostock 1998.

[16] K. Morawetz, P. Lipavský, V. Špička, and N. Kwong, Phys. Rev. C **59**, 3052 (1999).



Evolution of a $Ta+Au$ collision at 33 MeV lab energy and 8 fm impact parameter. The left panel gives the spatial density contour-plot in the $(x,0,z)$ plane and the local currents as arrows. The middle panel shows the transversal energy distribution versus the ratio of transversal to total energy. The amount of emitted particles due to the squeezed mode which is located in the right lower corner is given as plot label. The right panel represents the contour-plots of the total energy of particles. The dark areas gives the positive total energy indicating areas where particle can escape and the lighter colour scales the deepness of negative total energy indicating bound states. The black dots are the position of test particles in the $(x,0,z)$ plane which contribute to the new squeezing mode.

Summary

- Time dependent Schroedinger equation is solved analytically
- Phase and energy is induced already at initial time due to finite velocity of moving walls
- Evidence for nonequilibrium Mott effect
- Immediate and geometrical restricted emission of nonequilibrium particles dependent on projectile energy
- Transversal distribution for special impact parameter
- Observed events in realistic simulation of heavy ion collision