Conductivity in quasi two-dimensional systems

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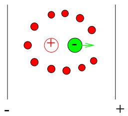
Introduction

1. One–particle properties in fields
   - Quantum kinetic equation (spectral function, gauge invariance, localization)
   - Debye–Onsager relaxation effect
2. Nonlinear current and conductivity in quasi two-D systems (cylindrical Fermi sphere)
   - Mott transition Metal-Insulator transition
   - Interference effect: Debye Onsager Relaxation = Localization

Debye-Onsager-Relaxation-Effect in plasmas



Screening cloud retarded- dipole force

Debye '23 [1] $E\,(1-\frac{\kappa e^2}{6\epsilon_0 T})$

Onsager '27 [2] $E\,(1-\frac{\kappa e^2}{3(2+\sqrt{2})\epsilon_0 T})$

Treatment within quantum kinetic theory [3–6]

$$\frac{\partial}{\partial t}f - e\,\mathbf{E}\frac{\partial}{\partial \mathbf{k}}f = I[f(\mathbf{k}-\mathbf{p}),\mathbf{E}]$$

Balance of current $\mathbf{J} = n\frac{e}{m}\mathbf{p}$

$$\frac{\partial}{\partial t}\mathbf{p} + en\mathbf{E} = R(E,t)en\,\mathbf{J} + en\frac{\delta E(E,t)}{E}\mathbf{E}$$

Conductivity beyond linear response [6] $\sigma(E) = \frac{1-\frac{\delta E(E)}{E}}{R(E)}$

Relaxation effect for dynamical screening

For single charged ions and big mass differences, twice the Debye result

$$\frac{\delta E_a}{E} = -\frac{\kappa e_a^2}{6T}\mathcal{F}[\frac{eE}{\kappa T}] = -\frac{e^2\kappa_e}{6T}\begin{cases} 2+o(E) \\ \frac{3\kappa T}{\sqrt{2}eE}+o(1/E)^2 \end{cases}$$

Approximative treatment: Replace dynamical screening $\overline{\epsilon(\omega,q)^{-2}}$ by thermal averaging $(1+\kappa^2 V_{aa}(q)/4\pi)^{-1}$, replace V_sV_s by V_sV, Debye result:

$$\frac{\delta E_a}{E} = -\frac{\kappa e_a^2}{6T}\mathcal{F}_{\rm dyn}[\frac{eE}{\kappa T}] = -\frac{e^2\kappa_e}{6T}\begin{cases} 1+o(E) \\ \frac{3\kappa T}{eE}+o(1/E)^2 \end{cases}$$

Asymmetric dynamical screening

Particles b (ions with charge Z) dynamically screened, a statically screened (Onsager treatment-background), for electron-ion plasma $q=1/(Z+1)$:

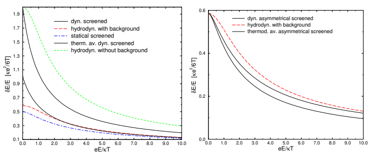
$$\frac{\delta E_a}{E} = \frac{\delta E^{\rm dyn}}{E}\frac{Zq}{\sqrt{q}+1} = -\frac{\kappa e^2}{3T}\frac{Zq}{\sqrt{q}+1}+o(E)$$

The low field limit agrees with the hydrodynamical result exactly.

Approximative treatment

$$\frac{\delta E_a}{E} = -\frac{\kappa e_a^2}{6T}\mathcal{F}^{\rm dyn}_{\rm asy}[\frac{eE}{\kappa T}] = -\frac{e^2\kappa_e}{6T}\begin{cases} 2-\sqrt{2}+o(E) \\ \frac{3\kappa T}{2eE}+o(1/E)^2 \end{cases}$$

The thermally averaged approximation agrees with the hydrodynamical approximation in the low and high field limit.



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Metal-Insulator transition

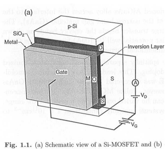
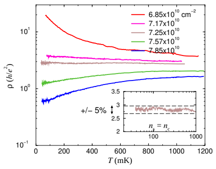


Fig. 1.1. (a) Schematic view of a Si-MOSFET and (b)

From [7]



Resistivity vs. temperature for different electron densities in Si MOSFET [8]

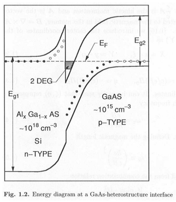


Fig. 1.2. Energy diagram at a GaAs-heterostructure interface

Model

- Quasi-2D electron gas (cylindrical Fermi surface)
- Localization (Insulator) – Delocalization (Metal) transition as Mott transition
- Electron interacts with charged and neutral impurities

Mott-Effect

Saha equation

Thermal equilibrium $e+i\leftrightarrow a$ $\quad \mu_e+\mu_i\leftrightarrow\mu_a$ with $\mu = \Sigma + T\ln(\exp n\lambda^2-1)$, $\lambda^2 = 2\pi\hbar^2/mT$, $n_i\approx n_e$ and $m_e\ll m_i\sim m_a$

Mass action law $\quad \frac{n_a}{n_i}\propto {\rm e}^{\beta(E_b+\Sigma_e)}$

Pressure ionization if $E_b+\Sigma_e(n_T)=0$

Doped semiconductor

Donor density $N_D = N_D^0+N_D^+$ $\quad N_D^0 = \frac{N_D}{\frac{1}{2}{\rm e}^{\beta(E_D-E_F)}+1}$

Electron density $n\approx N_D^+$ $\quad n = 2\lambda^2{\rm e}^{\beta(E_F-E_c)}$

Elimination of Fermi-level needed: selfenergy $\quad \frac{N_D^0}{n} = \frac{n\lambda^2}{4}{\rm e}^{\beta(E_c-E_D+\Sigma_e)}$

Self -energy

Dynamical Screening $V(q)=2\pi e_a e_b\hbar/q$ - charged impurities

$$\Sigma^{\gtrless}(q,\omega) = \int\frac{dq}{(2\pi\hbar)^2}V(q)^2 f^{\gtrless}_{k-q}\int d\omega'\delta(\epsilon_{k-q}-\hbar\omega-\hbar\omega')\frac{\Pi^{\gtrless}(q,\omega')}{|\mathcal{E}(q,\omega')|^2}$$

Classical fluctuations $\Pi^{\gtrless}(q,\omega) = \pm 2g(\pm\hbar\omega){\rm Im}\Pi(q,\omega)\approx\frac{2T}{\hbar\omega}{\rm Im}\Pi(q,\omega)$

$$\Sigma^{\gtrless}(q,\omega) = -2\pi T\int\frac{dq}{(2\pi\hbar)^2}V_{aa}(q){\rm Re}\left(1-\frac{1}{\mathcal{E}(q,0)}\right)f^{\gtrless}_{k-q}\delta(\epsilon_{k-q}-\hbar\omega)$$

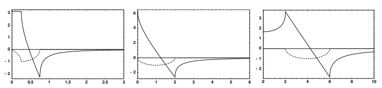
Result $[\kappa_a = \frac{\hbar\kappa}{2p_f}]$

$$\Sigma(\omega,k) = \frac{2\pi e_a^2 T_b m_a}{\hbar p_f(x+\sqrt{x_0})}\mathcal{K}\left[\left(\frac{\sqrt{x_0}-x}{\sqrt{x_0}+x}\right)^2\right]\to 2\pi^2 T\kappa_a \quad {\rm for}\quad \hbar\omega=\epsilon_f$$

Statical Screening $V_s(q)^2 = \frac{V(q)^2}{|\mathcal{E}(q,0)|^2} = \frac{2\pi g_{ea}\hbar}{q+\hbar/r_0}$ - Neutral impurities

$$\Sigma_s^{\gtrless}(k,\omega) = 2\pi n_i\int\frac{dq}{(2\pi\hbar)^2}V_s(q)^2 f^{\gtrless}_{k-q}\delta(\hbar\omega-\epsilon_{k-q})$$

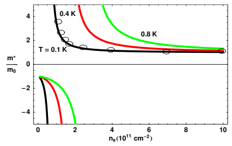
$$\Sigma_s(k,\frac{k^2}{2m}) = -\frac{g_{ea}^2 m_a s_a}{4\hbar x^2}\frac{n_i}{n_a}\left(\frac{1}{4-\kappa_i^2}-\frac{\kappa_i\arccos\frac{\kappa_i}{2}}{(4-\kappa_i^2)^{3/2}}\right) \to -\frac{8}{s_a}\frac{n_i}{n_a}(\ln\kappa_i-1)\epsilon_f + o(\frac{1}{\kappa_i^3}) \quad {\rm for}\quad \omega=\epsilon_f$$



The real (solid) and imaginary (dashed) part of the $4\pi\hbar^2/m\Pi$, versus $x_0=\frac{\hbar\omega}{4\epsilon_f}$ for $x=\frac{k}{2p_f}=0.5$, $x=1$ and $x=2$

Effective mass

$$\frac{m}{m^*} = \frac{1+\frac{1}{k}\frac{\partial\Sigma}{\partial k}}{1-\frac{\partial\Sigma}{\partial\omega}} = \frac{1-\frac{1}{4\epsilon_{fa}}\Sigma(p_f,\epsilon_f)}{1+\frac{1}{4\epsilon_f}\Sigma(p_f,\epsilon_f)}\approx 1-\frac{1}{2\epsilon_f}\Sigma(p_f,\epsilon_f)$$



The effective mass for temperatures $0.1, 0.4, 0.8$K from left to right . The open circles are experimental values of [9].

Conductivity and Relaxation Effect

$$\sigma(E) = \frac{n_a e_a^2}{m_a}\tau\left(1-\frac{\delta E}{E}\right)$$

Statical Screening - Neutral impurities

the low density expansion $a_0 = -2m_a g_{ai}r_0^2/\hbar^2$

$$\tau_i^{-1}\frac{m_a e_a^2}{n_a e_a^2\hbar} = \frac{2^{3/2}s_i n_i}{s_a n_a}\left(\frac{-a_0}{2r_0}\right)^2\left\{1-\frac{\pi}{2\kappa_i}-\frac{\pi^2T^2}{24\,\epsilon_f^2}\frac{\pi^3}{16\kappa_i}+o(\kappa_i^{-4})\right\}$$

$$\frac{\delta E}{E} = \frac{n_i}{n_a}\left(\frac{-a_0}{2r_0}\right)^2\left\{1+\frac{4-3\ln 2\kappa_i}{\kappa_i^2}+o(\kappa_i^{-4})\right\}$$

Dynamical Screening - charged impurities

$$\tau_{ab}^{-1}\frac{m_a e_a^2}{n_a e_a^2\hbar} = \frac{8s_b}{s_a^3}\left(\frac{T_a}{\epsilon_f}+\frac{T_b}{\epsilon_f}\right)\kappa_a\left(1-\frac{\pi}{4\kappa_a}+\frac{2}{3\kappa_a^2}+o(\kappa_a^{-3})\right)$$

$$\frac{\delta E}{E} = -\frac{2s_b}{s_a^2}\left(\frac{T_a}{\epsilon_f}+\frac{T_b}{\epsilon_f}\right)\left(-1+(\ln(2\kappa_a)-1)\frac{1}{\kappa_a^2}+o(\kappa_a^{-4})\right)$$

Since

$$\kappa_a = \frac{\hbar\kappa}{2p_f} = \sqrt{\frac{\hbar}{\tau_0\epsilon_f}} \qquad \frac{1}{\tau_0} = \frac{me_a^2e_b^2s^2}{8\hbar^3}$$

we have

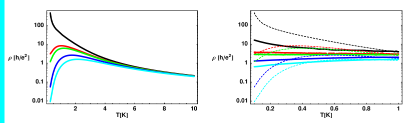
$$1-\frac{\delta E}{E} = 1-\frac{T}{\epsilon_f}\left(1+\frac{\tau_0\epsilon_f}{2\hbar}\ln\left(\frac{\tau_0\epsilon_f}{4\hbar}\right)\right)$$

Relaxation effect corresponds to weak localization corrections !

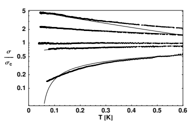
Conductivity Result

Bloch-Grüneisen analog formula: $\quad \sigma = \frac{e^2}{h}\frac{1-\frac{\delta E_i}{E}(n)-T\frac{\delta E}{E}(n)}{R_i(n)+TR(n)}$

Case A: Exponential Binding energy of impurities $E_b\approx 2$meV and $(a_0/r_0)^2\approx 0.05$



Case B: Assume trapping by $n_i/n_a = 1-T/T_c$ (Three particle clustering)



E. Nakhmedov and K. Morawetz, cond-mat/0201077

$$\frac{\sigma}{\sigma_0} = 1+a\left(1-\frac{T}{T_c}\right)-\frac{\hbar}{\epsilon_f\tau}\ln\frac{\hbar}{4T\tau}$$

Summary

1. Nonlinear current or conductivity is lowered by relaxation (interference) effect describing the dynamics of screening cloud
   - Statical screening: half Debye result dynamical screening: twice Debye result, thermally averaged dynamical screening: Debye result, asymmetrically dynamical screening or background friction: Onsager result
   - Relaxation effect describes weak localization at low temperature
2. Metal-insulator transition as Mott effect
   - Effective mass by dynamically screened selfenergy describes data
   - Fit of trapping by three-particle clustering better than impurity one

paper available at http://www.mpipks-dresden.mpg.de/∼morawetz