Brine Channel Formation in Sea Ice

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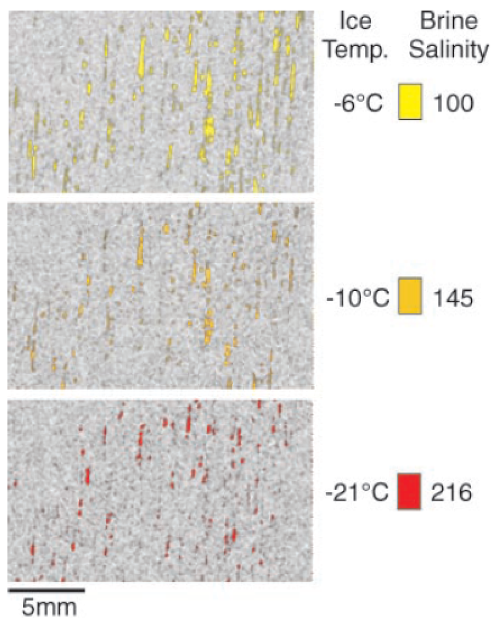
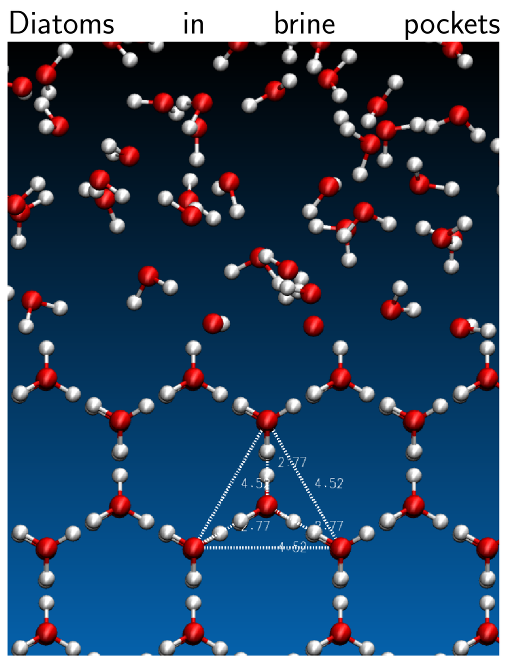
Introduction

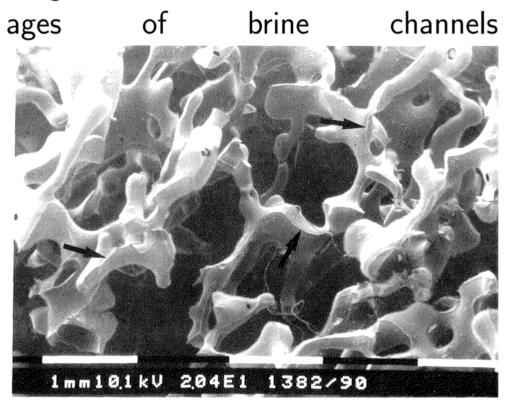
Diatoms in brine pockets



Magnetic-resonance images of brine channels



Molecular structure of water and ice



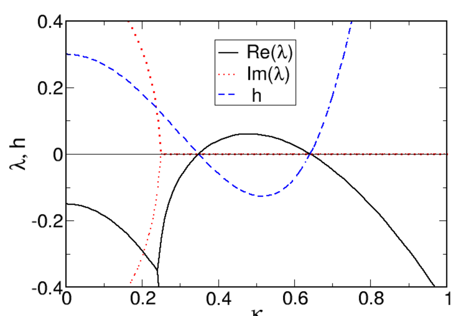
Cast of brine channels (Weissenberger)

Reaction-Diffusion Model (Turing)

$$\frac{\partial\psi(\xi,\tau)}{\partial\tau} = \alpha_1\psi - \psi^3 + \delta\psi^5 + \rho + \frac{\partial^2\psi(\xi,\tau)}{\partial\xi^2}$$
$$\frac{\partial\rho(\xi,\tau)}{\partial\tau} = -\alpha_2\rho - \psi + D\frac{\partial^2\rho(\xi,\tau)}{\partial\xi^2}$$

α_1 = temperature-dependent rate
α_2 = desalinization rate
δ = measure for specific heat

$$\begin{pmatrix}\tilde\psi\\ \tilde\rho\end{pmatrix} = \begin{pmatrix}\tilde\psi_i\\ \tilde\rho_i\end{pmatrix} exp(\lambda(\kappa)\tau + i\kappa\xi)$$



Dispersion of the linear stability versus the dimensionless wave number κ for $\alpha_1 = 0.7$, $\alpha_2 = 1$, $D = 6$ together with the function $h(\kappa)$

steady state

$$\lambda(\kappa)^2 + [\kappa^2(1+D) + \alpha_2 - \alpha_1]\lambda(\kappa) + h(\kappa^2) = 0$$

with $h(\kappa^2) = D\kappa^4 + (\alpha_2 - \alpha_1 D)\kappa^2 - \alpha_1\alpha_2 + 1$

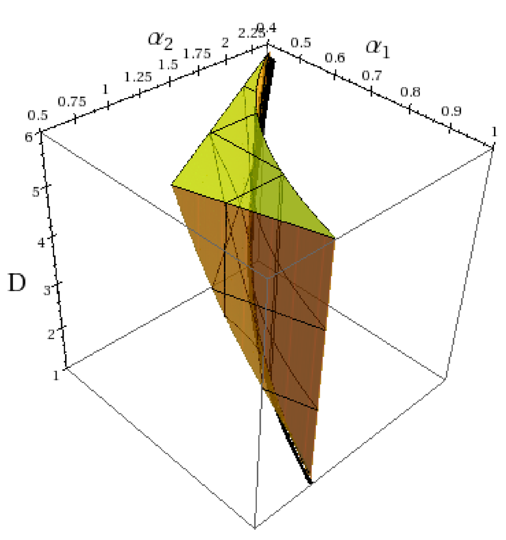
Turing Space

homogeneous phase stable if eigenvalues negative

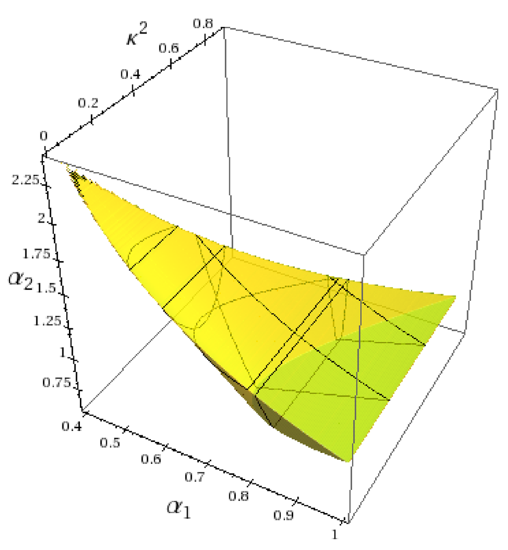
$$\text{condition I}: \quad \alpha_2 > \alpha_1 \text{ and } \alpha_1\alpha_2 < 1.$$

spatial inhomogeneous case, $\kappa^2 > 0$ some spatial fluctuations may be amplified and form macroscopic structures, i.e. the Turing structure, modes growing in time$\mathrm{Re}\lambda(\kappa) > 0$

$$\text{condition II}: \quad D > \frac{(1+\sqrt{1-\alpha_1\alpha_2})^2}{\alpha_1^2}.$$
$$\text{cond. III}: \quad \kappa^2 \in \frac{1}{2D}\left(\alpha_1 D - \alpha_2 \pm \sqrt{(\alpha_1 D + \alpha_2)^2 - 4D}\right)$$



The Turing space where spatial structures can occur, lower limiting line, $\alpha_2 = 1/\alpha_1$, $D = 1/\alpha_1^2$ (thick)



The possible wave numbers κ^2 where spatial structures can occur for $D = 6$ in dependence on α_1 and α_2

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Lit.: B. Kutschan, K. Morawetz, S. Gemming: *Modeling the morphogenesis of brine channels in sea ice*, Phys. Rev. E 81 (2010) 036106-1-10

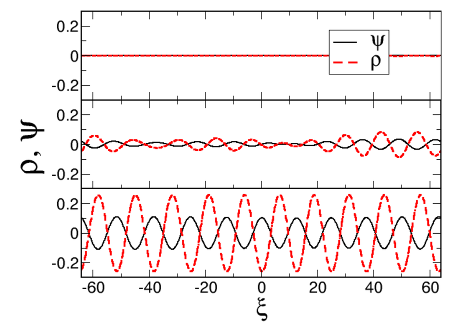
Critical Modes (Turing)

critical wavenumber from largest modes

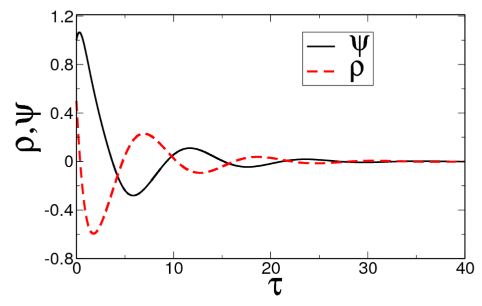
$$D_c = \frac{(1+\sqrt{1-\alpha_1\alpha_2})^2}{\alpha_1^2}$$

with the critical wavenumber determining size of structure $\frac{2\pi}{\kappa_c}$,

$$\kappa_c^2 = \frac{D_c f_\psi + g_\rho}{2D_c} = \frac{D_c\alpha_1 - \alpha_2}{2D_c}$$

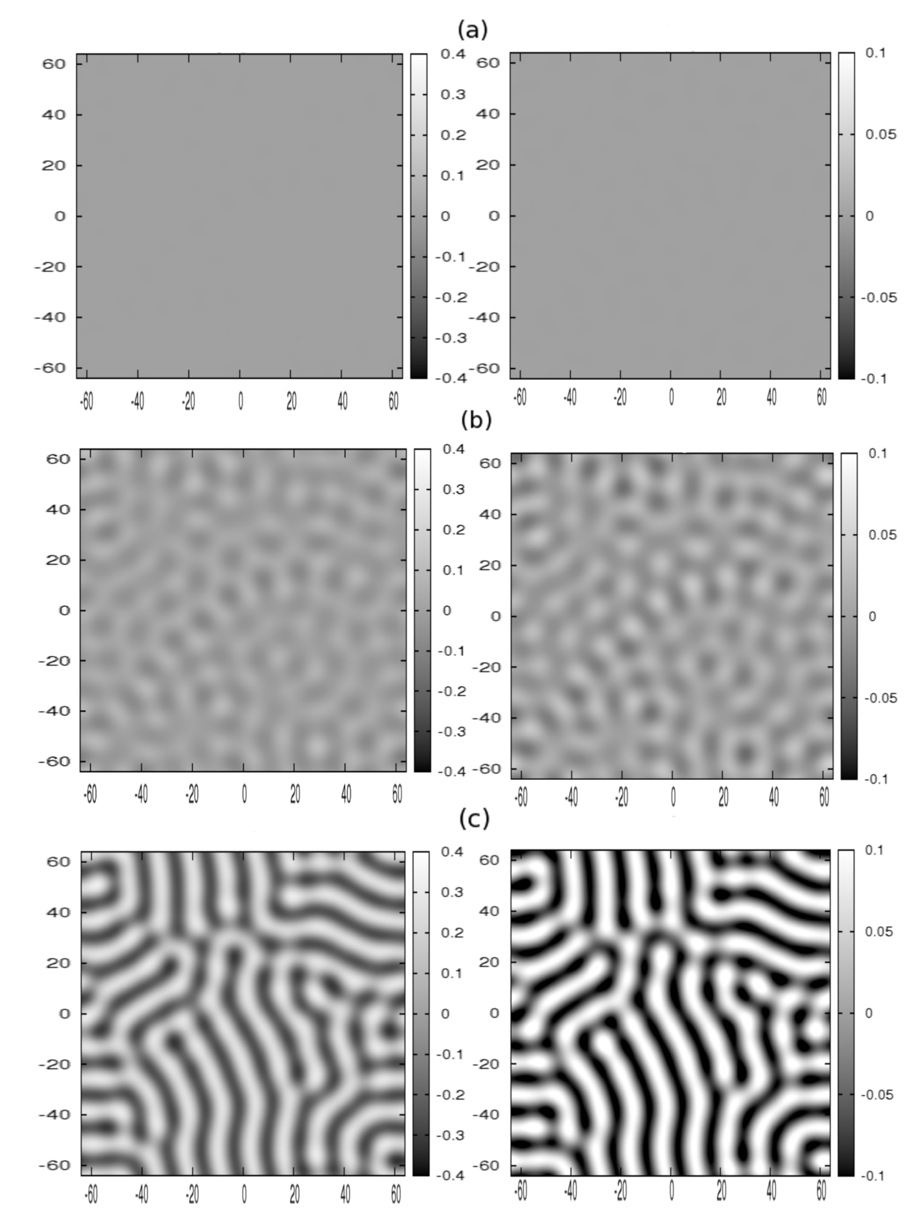


Time evolution of the order parameter ψ and salinity ρ versus spatial coordinates for $\tau = 100, 170, 400$ (from above to below)for $\alpha_1 = 0.7$, $\alpha_2 = 1$, $\delta = \frac{3}{16\alpha_1}$, $D = 6$ with the initial condition $\rho(\tau = 0) = 0.5 \pm 0.01N(0,1)$ and periodic boundary conditions



Time evolution of the order parameter ψ and the salinity ρ for $\alpha_1 = 0.7$, $\alpha_2 = 1$, $\delta = \frac{3}{16\alpha_1}$ and the initial order parameter $\psi(\tau = 0) = 1$ and the dimensionsless salinity $\rho(\tau = 0) = 0.5$

Time Evolution (Turing)



Structure formation for 3 time steps $\tau = 100, 170, 400$ (from top to bottom, a-c) for the order parameter Ψ (left) and the salinity ρ (right). The parameters are $\alpha_1 = 0.7$, $\alpha_2 = 1$, $\delta = \frac{3}{16\alpha_1}$, $D = 6$ with the initial condition $\rho(\tau = 0) = 0.5 \pm 0.01N(0,1)$ and periodic boundary conditions

Link to Experimental Data (Turing)

Experimental data	critical wave number	model parameter
$D_1 = 10^{-5}\frac{cm^2}{s}$	$\frac{2\pi}{\kappa_c} = 12.6 = \frac{2\pi}{k_c}\frac{\sqrt[4]{b_1 b_2}}{\sqrt{D_1}}$	$b_1 b_2 = 2.5 \times 10^6 s^{-2}$ $a_1 = \sqrt{b_1 b_2}\alpha_1 = 1111 s^{-1}$ $a_2 = \sqrt{b_1 b_2}\alpha_2 = 1587 s^{-1}$ $D_2 = D_1 D = 6 \times 10^{-5}\frac{cm^2}{s}$
$\tau_d = 10^5 s^{-1}$	$a_1 \sim \frac{T_c - T}{T_c}\frac{1}{\tau_d}$	$a_1 \sim 1107 s^{-1}$

Comparision between both Models

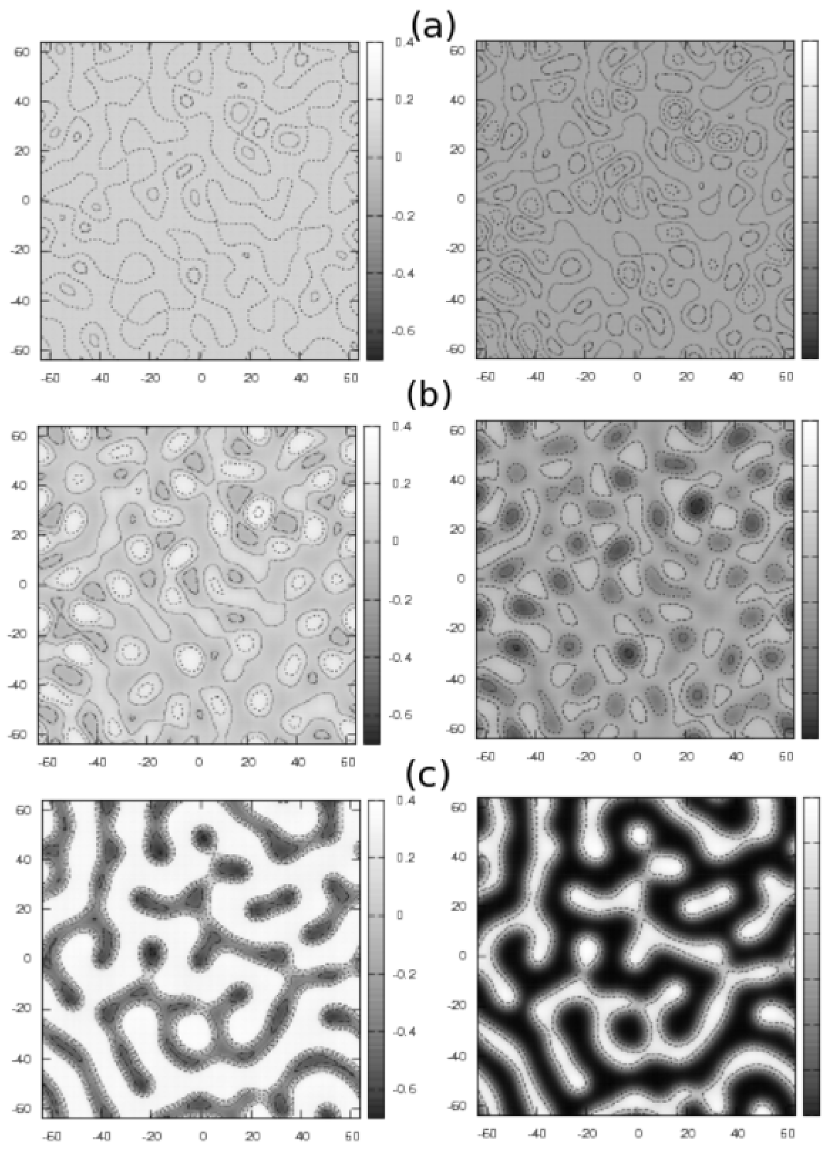
Turing-Ginzburg-Landau	Phase field
salinity is not preserved	salinity is preserved (conservative quantity)
brine channel formation is a result of the kinetic nonlinear feedback	brine channel formation is a consequence of a exact free energy functional and the conservation condition of salinity

Summary

1. reaction diffusion system which connects the basic ideas both of Ginzburg and of Turing can describe the formation of brine channels with realistic parameters
2. phase field model (Cahn-Hilliard-Equation) seems to be more realistic

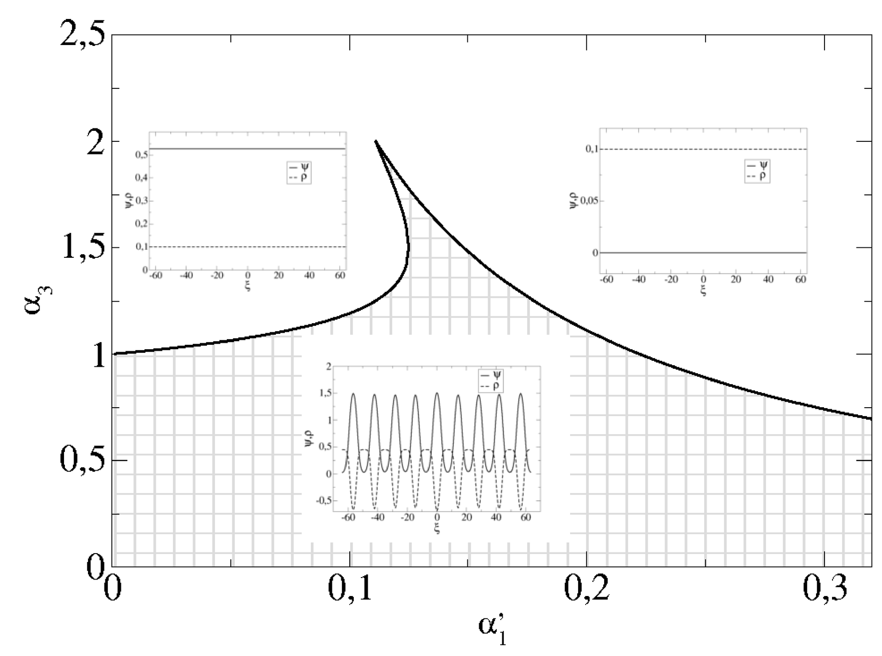
Time Evolution (Phase Field)

$$\frac{\partial\psi(\xi,\tau)}{\partial\tau} = -\alpha_1\psi + \psi^2 - \alpha_3\psi^3 - \psi\rho + D\frac{\partial^2\psi(\xi,\tau)}{\partial\xi^2}$$
$$\frac{\partial\rho(\xi,\tau)}{\partial\tau} = \frac{1}{2}\frac{\partial^2\psi^2(\xi,\tau)}{\partial\xi^2} + \frac{\partial^2\rho(\xi,\tau)}{\partial\xi^2}$$



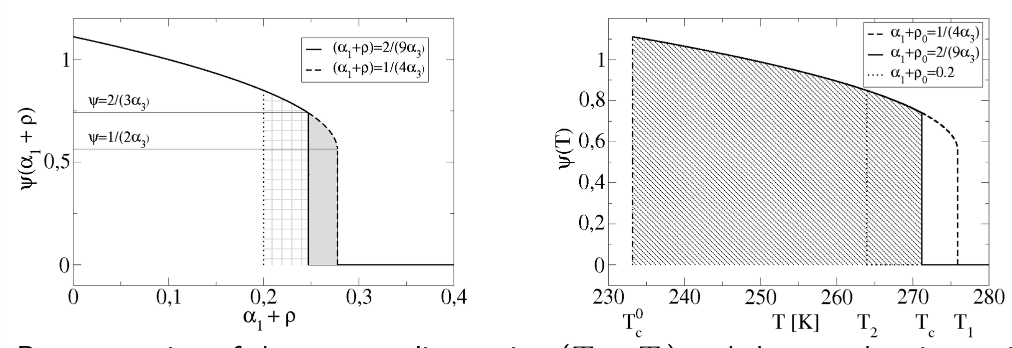
Structure formation for 3 time steps $\tau = 0, 175, 500$ (from top to bottom, a-c) for the order parameter Ψ (left) and the salinity ρ (right). The parameters are $\alpha_1 = 0.1$, $\alpha_3 = 1$, $D = 0.5$ with the initial condition $\rho(\tau = 0) = 0.1 \pm 0.001N(0,1)$ and periodic boundary conditions

Phase Diagram (Phase Field)



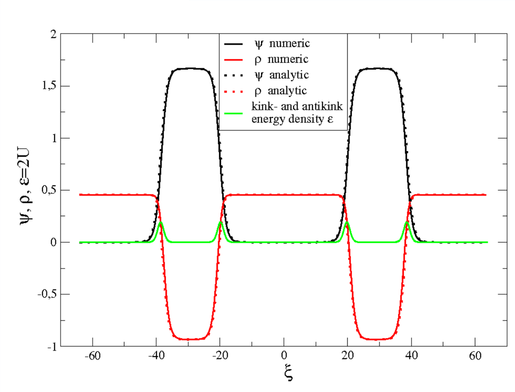
The instability region of the fixed point $\psi_0 = \psi_0^+ = \frac{1}{2\alpha_3} + \frac{1}{2\alpha_3}\sqrt{1 - 4\alpha_1'\alpha_3}$ and $\rho_0 = const$ as phase diagram. In the checked region spatial structures can occur. (left above: $\alpha_1 = 0.01$, $\rho = 0.1$, $\alpha_3 = 1.5$, center: $\alpha_1 = 0.1$, $\rho = 0.1$, $\alpha_3 = 0.9$, right above: $\alpha_1 = 0.2$, $\rho = 0.1$, $\alpha_3 = 0.9$)

Phase Transition (Phase Field)

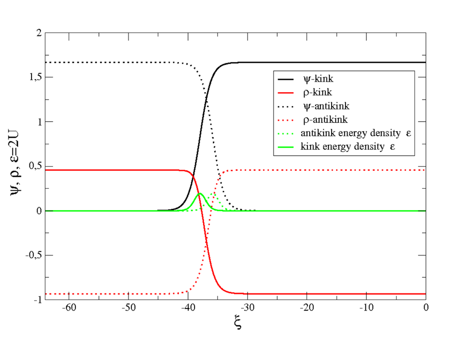


Representation of the supercooling region ($T < T_c$) and the superheating region ($T > T_c$)

Long Time Solution (Phase Field)



Instanton-like solution for the time $\tau = 5 \cdot 10^6$ with $\alpha_1 = 0,1$, $\alpha_3 = 0.9$, $D = 0.5$, $\psi_B = 0$ and $\rho_B = 0.45555555$



Kink- and Antikink solution for $D = 0.5$, $\alpha_1 = 0.1$, $\alpha_3 = 0.5$ and $\xi_0 = -36.9$