Formation of Bloch and Plasma Oscillations

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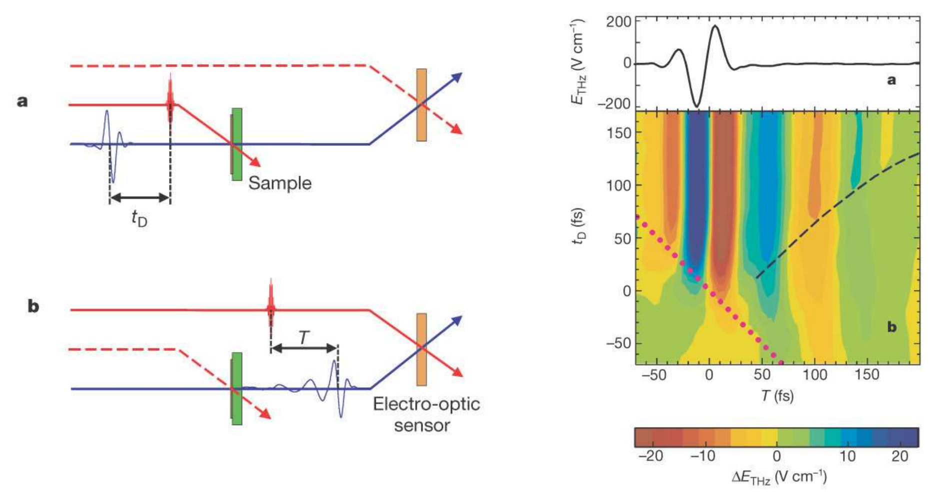
MAX-PLANCK-GESELLSCHAFT

Femtosecond laser irradiation



pump pulse creating charge carriers at time t_0

probe pulse at time t_D later

A. Leitenstorfer, Nature 426(2003)23, R. Huber et al. Nature 414(2001)286

Time and frequency–dependent dynamical response

$$\frac{1}{\epsilon(\omega,t)}=1+\frac{e^2}{\epsilon_0 q^2}\int\limits_0^{t-t_0} dT e^{i\omega T}\chi(t,t-T)$$

time delay after probe pulse $T=t-t_D-t_0$

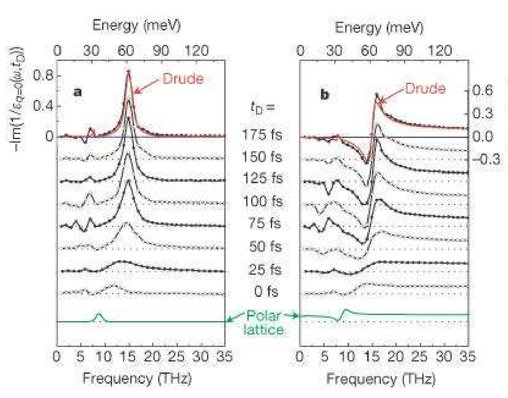
Fourier transformed into frequency

Quantum kinetic equations:

Gartner et al. PRB 66(2002) 075205

Vu and Haug PRB62 (2000) 7179

Kira and Koch PRL 93 (2004) 076402



Solution of meanfield equation

- Linearize kinetic equation to obtain density response

$$\delta n(q,t)=\int\limits_{t_0}^{t} dt'\chi(t,t')V_q^{\rm ext}(t')$$

- one-particle reduced density matrix $\hat\rho$ obeys

$$\dot{\hat\rho}+i[\hat{\mathcal{E}}+\hat V^{\rm ind}+\hat V^{\rm ext},\hat\rho]=\frac{\hat\rho^{\rm l.e.}-\hat\rho}{\tau}$$

- local equilibrium density matrix $\hat\rho^{\rm l.e.}$ deviates by chemical potential linked to $\delta n(q,t)$ by density conservation
- linearization leads to

$$\chi(t,t')=\Pi(t,t')+\int\limits_{t'}^{t} d\bar t\left[\Pi(t,\bar t)\frac{e^2}{\epsilon_0 q^2}+I(t,\bar t)\right]\chi(\bar t,t')$$

- large-time limit $t+t'\to\infty$ approaches familiar form $\chi=\frac{\Pi^{\rm M}}{1-\frac{e^2}{\epsilon_0 q^2}\Pi^{\rm M}}$ with

Mermin-Das polarization (Mermin PRB 1 (1970) 2362)

Solution of transient time response by one-sided Fourier transform

$$\frac{1}{\epsilon(\omega,t)}=1+\frac{e^2}{\epsilon_0 q^2}\int\limits_0^{t-t_0} dT e^{i\omega T}\chi(t,t-T)$$

- limit of long wave lengths $q\to 0$

appreciable simplification $\Pi(t,t')\approx\frac{q^2 n(t')}{m}(t'-t)e^{\frac{t'-t}{\tau}}$ and $I(t,t')\approx\frac{1}{\tau}e^{\frac{t'-t}{\tau}}$

- integral equation into differential one with $n(t)\approx n_0\Theta(t-t_0)$
- Analytic solution K. Morawetz, P. Lipavský, M. Schreiber, Phys. Rev. B 72 (2005) 233203

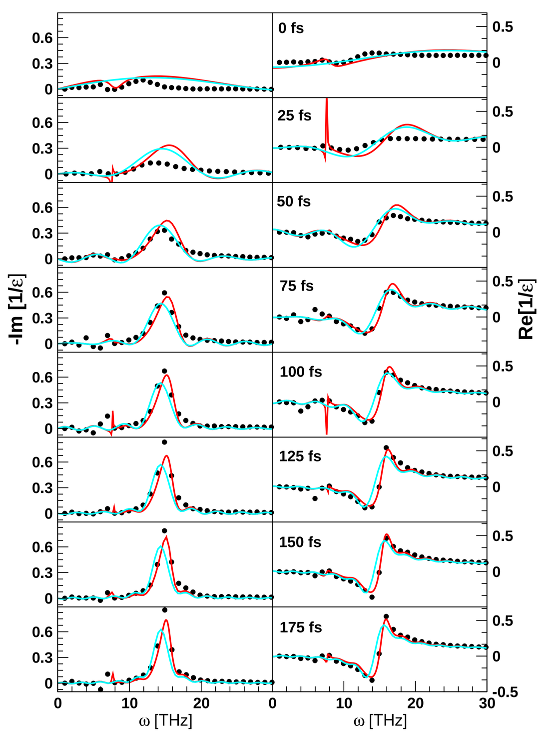
$$\boxed{\frac{1}{\epsilon(\omega,t)}=1-\frac{\omega_p^2}{\sqrt{\omega_p^2-\frac{1}{4\tau^2}}}\int\limits_0^{t-t_0} dt'\, e^{(i\omega-\frac{1}{2\tau})t'}\sin\left(\sqrt{\omega_p^2-\frac{1}{4\tau^2}}\,t'\right)}$$

Achievement:

long-time limit yields the Drude formula $\lim\limits_{t\to\infty}\frac{1}{\epsilon}=1-\frac{\omega_p^2}{\omega_p^2-\omega(\omega+\frac{i}{\tau})}$

This limit is not so easy to achieve within short-time expansions, e.g. approximate result [ElSayed et al, PRB 49 (1994) 7337] gives $1-\frac{\omega_p^2}{\omega_p^2-(\omega+i/\tau)^2}$.

Comparison with experiment



GaAs

Resonance about 8 THz due to optical phonons, adding intrinsic contribution of crystal lattice

$$\epsilon_{\rm GaAs}=\epsilon_\infty\left(\epsilon+\frac{\omega_{\rm LO}^2-\omega_{\rm TO}^2}{\omega_{\rm TO}^2-\omega^2-i\gamma\omega}\right)$$

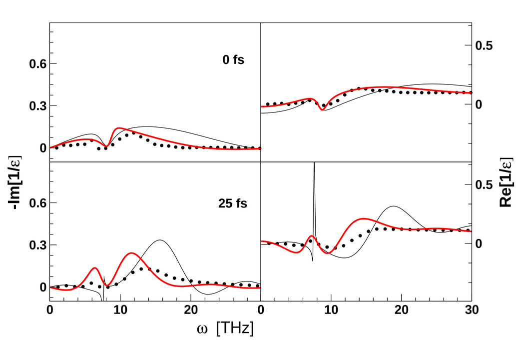
$\frac{\omega_{\rm LO}}{2\pi}=8.8$ THz, $\frac{\omega_{\rm TO}}{2\pi}=8.1$ THz lattice damping γ =0.2 ps^{-1} and background polarizability of ion lattice $\epsilon_\infty=11.0$

Huber et al, phys.stat.sol.(b) 234 (2002) 207

pumb pulse at $t_0=-40$fs, probe pulse full-width at half-maximum $27fs$, plasma frequency ω_p =14.4 THz, relaxation time $\tau=85$ fs

Improvement by realistic shape

- too fast build-up of the collective mode at the time $t=25$ fs
- just the time duration of the experimental pulse
- we have approximated this by the instant jump
- more realistic smooth populating can be modeled by an arctan-function and numerical solution improves the description



First two time evolution steps comparing an instant population of conduction band with a smooth transition.

Hybridization of plasma and Bloch oscillations

Electrons in periodic potential under electric field E show Bloch [Z. Phys. **52** (1928) 555] oscillations given by period of lattice d $\omega_B=\frac{eEd}{\hbar}$, experimentally observed K. Leo, Semicond. Sci. Technol. **13** (1998) 249 .

Question: How do Bloch oscillation and plasma oscillation influence each other and how do they forme time-dependently

Polarization: $\Pi(t,t')=[\frac{nq^2}{m}+o(q^4)]e^{\frac{t'-t}{\tau}-i\frac{\Omega_q^2}{2}(t'-t)^2}$

Mermin correction:$I(t,t')=[\frac{1}{\tau}+o(q^2)]e^{\frac{t'-t}{\tau}-i\frac{\Omega_q^2}{2}(t'-t)^2}$

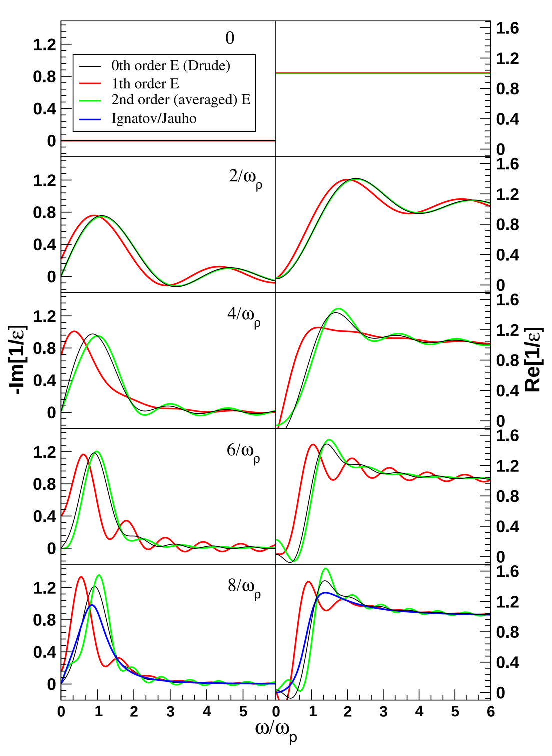
with field and wave vector dependence

$$\Omega_q^2=\frac{e\mathbf{Eq}}{m\hbar}=\omega_B\omega_p\frac{q_{||}v_0}{\hbar\omega_p}$$

plasma frequency: $\omega_p^2=4\pi e^2 n/T$, maximal electron velocity: $v_0=\hbar/md$

If electrons in resonance with plasma frequency, $q_{||}v_0\to\hbar\omega_p$

Results



Time evolution of response function

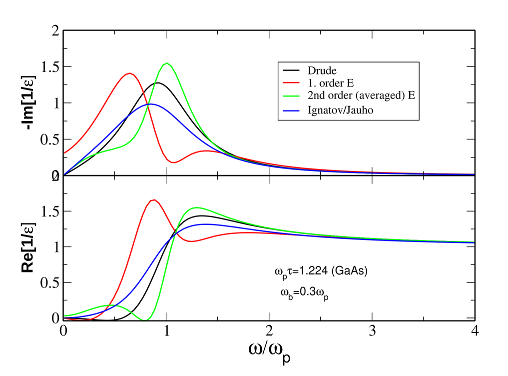
Large times (equilibrium):

$$\epsilon(\omega,\infty)=1-\frac{\omega_p^2}{\omega(\omega+\frac{i}{\tau})}\left[1+i\Omega_q^2\frac{3\omega+\frac{i}{\tau}}{\omega(\omega+\frac{i}{\tau})^2}+\Omega_q^4\frac{\frac{1}{\tau^2}-i\frac{6}{\tau}\omega-15\omega^2}{\omega^2(\omega+\frac{i}{\tau})^4}\right]+\mathcal{O}(\Omega_q^6)+\mathcal{O}(q^2)$$

No collisions $\tau\to\infty$ linear field effect reported in K. Morawetz, A. P. Jauho, Phys. Rev. E **50** (1994) 474

Resonant waves compare with: A. A. Ignatov, A. P. Jauho, J. Appl. Phys. **85** (1999) 3643, Y. A. Kosevich, Phys. Rev. B **63** (2001) 205313

Comparison with equilibrium



Response function for large times compared with the Drude response and Ignatov/Jauho result

$$\epsilon_{\rm IJ}(\omega)=1-\frac{\omega_p^2}{\omega(\omega+\frac{i}{\tau})}\left[1-\omega_B^2\frac{\omega^2\tau^2+3i\omega\tau-3}{(\omega+\frac{i}{\tau})^2}\right]+\mathcal{O}(\omega_b^4)$$

If no mean-fields ? $\longrightarrow$ Levinson equation

Time diagonal part of Kadanoff/Baym equation $\Delta_E=\frac{k^2+p^2-(k-q)^2-(p+q)^2}{2m}$

$$\frac{\partial\rho_k}{\partial t}=\frac{2}{\hbar^2}\sum_{pq}V_q^2\int\limits_0^t d\bar t\,\cos\left\{\frac{t-\bar t}{\hbar}\Delta_E\right\}\Big(\rho'_{k-q}\rho'_{p+q}(1-\rho_p)(1-\rho_k)-\rho_p\rho_k(1-\rho'_{k-q})(1-\rho'_{p+q})\Big)_{\bar t}$$

time-dependent Fermi's Golden Rule and memory effects

Properties:

- density conservation $\frac{\partial}{\partial t}n=0$ • momentum conservation $\frac{\partial}{\partial t}\langle p_1\rangle=0$
- Time variation of kinetic energy $\frac{\partial}{\partial t}(\langle\frac{p_1^2}{2m}\rangle+\langle V_{12}\rangle)\equiv 0$

Phys. Lett. A 199 (1995) 241 , Eur. Phys. J. A (1999) 291-305, Phys. Rev. C 64 (2001) 024613

The inclusion of memory effects produces the full energy conservation.

But, several effects hidden:

Off-shell tails in ρ, renormalization of scattering rates and wave function, quasiparticle energies, collision delays: Ann. Phys. (NY) 294 (2001) 135

Better: neglect memory effects, only time-dependent Fermi's Golden Rule:

e.g. Maxwellian plasma [Phys. Lett. A 246 (1996) 311; Phys. Rev. E 63 (2001) 20102]

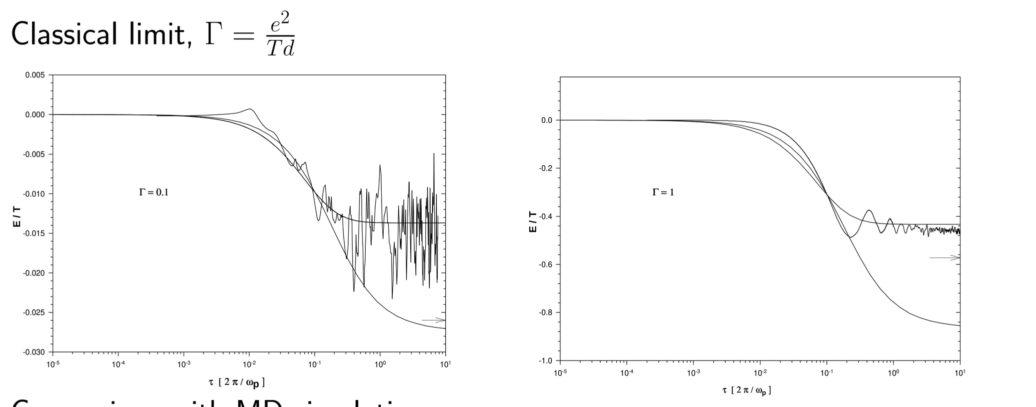
static Debye $z=\omega_p\sqrt{t^2-it\frac{\hbar}{T}}$:

$$\dot E_{\rm corr}(t)=-\frac{e^2 n\kappa T}{2\hbar}{\rm Im}\left[(1+2z^2)e^{z^2}(1-{\rm erf}(z))-\frac{2z}{\sqrt\pi}\right]$$

dynamical screening $z_1=\omega_p\sqrt{2t^2-it\frac{\hbar}{T}}$:

$$\dot E^{\rm dynam}_{\rm corr}(t)=-\frac{e^2\kappa T}{\hbar}{\rm Im}\left[e^{z_1^2}(1-{\rm erf}(z_1))\right]$$

Classical limit, $\Gamma=\frac{e^2}{Td}$



Comparison with MD simulations

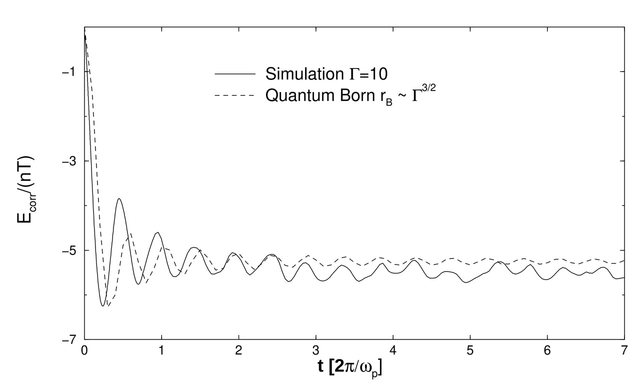
Dense classical plasma by quantum approximation

On a correspondence between classical and quantum particle systems, Phys. Rev. E 66 (2002) 022103

Quantum-Born [Eur. Phys. J. A (1997) 291]:

$$\frac{E^T_{\rm corr}(t)-E^0_{\rm corr}(t)}{nT}=\frac{1}{(36\pi^4)^{1/6}}\frac{{r_s}^3}{\Gamma}\left(\frac{\sin y\tau}{y\tau}-1\right)\left(\frac{1}{b_l}\arctan(\frac{1}{b_l})+\frac{1}{b_l^2+b_l^4}\right)$$

Bruckner parameter $r_s=\frac{d}{a_B}$, $b_l=\frac{\hbar\kappa}{2p_f}=\sqrt\Gamma(48\pi^2)^{-1/6}$, time in plasma periods $\tau=\frac{2\pi}{\omega_p}t$ or $y\tau=4\epsilon_f t/\hbar=(2)^{4/3}\pi^{5/3}3^{5/6}\tau/\sqrt{r_s}$



Simulation of dense classical plasma [G. Zwicknagel]

best fit ($\hbar\sim T$):

$$r_s^{\rm fit}=\frac{1}{2}\sqrt{\frac{3}{8}}\Gamma^{3/2}$$

$$\lambda^2=\frac{d}{\kappa}=\frac{d^2}{(3\Gamma)^{1/2}}\approx d^2$$

Summary

- collisions have no time to happen yet (only mean-fields are formed)
- separate gross feature of the formation of collective modes at transient times which are due to mean-field fluctuations
- simple analytic formula for time dependence of the dielectric function
- hybridization of plasma and Bloch oscillations, formed on same time scale

But: relaxation time needed and what if no mean-fields (quasi-neutrality) ?

- Short time behavior by time-dependent Fermi's golden rule ≡ Finite duration approximation of non-Markovian collision integrals
  - -low temperature value is universal; ratio $\sim\hbar/\epsilon_F$ (formation of quasi-particles), high temperature limit $\sim 1/\omega_p$
  - -analytical results for dynamical screening, static screening leads to 1/2 correlation energy
  - -good agreement with molecular dynamics
  - -strong coupling by equivalence: N classical particles $\leftrightarrow$ $N-1$ quantum particles