Surface potential in superconductors

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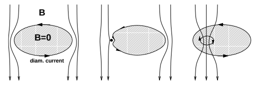
Electric fields ?

Meissner-Ochsenfeld effect: magnetic field expulsion → diamagnetic currents and Lorentz forces



- With increasing field magnetic field penetrates in form of vortices (type II superconductors) → inertial forces
- Internal fields to keep charge distribution close to neutrality, to compensate inhomogeneity due to diamagnetic currents
- Corresponding electrostatic potential has the form of a Bernoulli potential and compensate inertial and Lorentz forces

Hope:
Measurement of electrostatic potentials to access material parameter:

- Surface (Bok&Klein '68): no information (puzzle)
- Charge of vortices (Kumagai& Nozaki& Matsuda'01): bulk properties

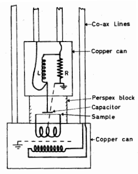
History of Bernoulli potential in SC - Experiment

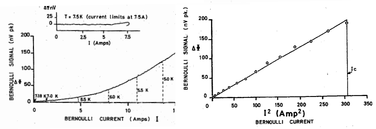
Ohmic contacts: *Null results*
H. W. Levis, 1953 , T. K. Hunt 1966 [1], due to constant electrochemical potential

Capacitive coupling:
No thermodynamic corrections
J. Bok and J. Klein 1968 [2], T. D. Morris, J. B. Brown 1971 [3]

$$e\varphi = -\frac{n_s}{n}\frac{1}{2}mv^2 = -\frac{1}{n}\frac{B^2}{2\mu_0}$$

and not (BCS for Pb at $T = 7$K)
$\left(\frac{n_s}{n} - 4\frac{n_n}{n}\frac{\partial \ln T_c}{\partial \ln n}\right)\frac{m}{2}v^2 = (0.1 + 3.2)\frac{m}{2}v^2$





Charged vortices in HTSC probed by NMR [4]

- NMR frequency depends on B, γ and number N of holes per Cu/plane
- Polarization of Cu, coupling of spin with electrical field gradient leads to splitting of quadrupole resonance

$$\nu_Q^{NQR} = E_{\pm 3/2} - E_{\pm 1/2} = AN + C.$$

Therefore the combined Zeeman plus quadrupole splitting

$$\nu_1(B) = E_{\frac{1}{2}} - E_{-\frac{1}{2}} = \gamma B$$
$$\nu_2(B) = E_{-\frac{1}{2}} - E_{-\frac{3}{2}} = \gamma B - \nu_Q^{NQR}$$
$$\nu_3(B) = E_{\frac{3}{2}} - E_{\frac{1}{2}} = \gamma B + \nu_Q^{NQR}$$

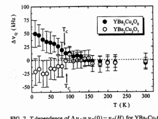
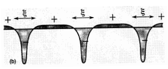


FIG. 7. T dependence of $\Delta\nu_Q = \nu_Q(0) - \nu_Q(H)$ for $YBa_2Cu_3O_7$ and $YBa_2Cu_4O_8$. In both materials nonzero $\Delta\nu_Q$ is clearly observed below T_c, showing that the electron density outside the core differs from that in zero field.

from which follows
$N_B - N_0 = \frac{1}{2A}(\nu_3(B) - \nu_2(B) - 2\nu_3(0))$
Problems:



Charge accumulated in vortex core per layer (BCS)

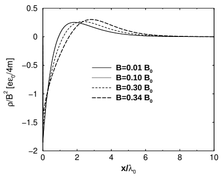
$$Q \approx \frac{2ek_F s}{\pi^3}\left(\frac{\lambda_{TF}}{\xi}\right)^2\left(\frac{d\ln T_c}{d\ln\mu}\right) \sim 10^{-5}e \quad \text{exp}: 10^{-2}e$$

underdoped regime: $Q > 0$, overdoped: $Q < 0$, contrast to experiment

Solution of Landau-Ginzburg equation [5, 10]

Landau-Ginzburg equation extended to low temperatures Bardeen '56

$$-\frac{\hbar^2\nabla^2}{2m^*}\psi + \frac{e^{*2}\mathbf{A}^2}{2m^*}\psi - \frac{\gamma T_c^2}{2n}\left(1 - \frac{t^2}{\sqrt{1-\frac{2}{n}\psi^2}}\right)\psi = 0$$



$$B_0 = \frac{\Phi_0}{2\pi\lambda_0^2} = e\mu_0 n\frac{\hbar}{2m}$$
$$t = T/T_c = 0.9$$
$$\lambda = 1.7\lambda_0$$
$$\kappa_{\rm GL} = \sqrt{2}$$

Linearization for low magnetic fields $\psi = \sqrt{n(1-t^4)/2} + \delta\psi$

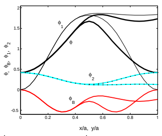
$$\delta\psi = \frac{\psi_\infty}{2-\kappa^2}\frac{e^{*2}\lambda^4 B^2}{2\hbar^2}\left(e^{-2x/\lambda} - \frac{\sqrt{2}}{\kappa}e^{-\sqrt{2}x/\xi}\right)$$

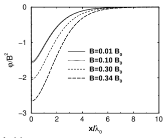
Electrostatic potentials

$$e\varphi - \lambda_{TF}^2\nabla^2 e\varphi = = \chi\frac{|\psi|^2}{n} + \frac{\partial\varepsilon_{\rm con}}{\partial n}\frac{2|\psi|^2}{n} + \frac{T^2}{2}\frac{\partial\gamma}{\partial n}\sqrt{1-\frac{2|\psi|^2}{n}}$$

Example Nb (hexagonal) $t = \frac{T}{T_c}, \omega = \frac{2|\psi|^2}{n(1-t^4)}$

$$\frac{en}{\frac{1}{4}\gamma T_c^2(1-t^2)(1-t^4)}\varphi = \phi_B + C_1\omega + C_2\sqrt{1-(1-t^4)\omega}$$





Linearization with respect to magnetic field

$$e\delta\varphi = -\frac{B^2}{2\mu_0 n}\left(1 + \frac{4t^4}{1-t^4}\frac{\partial\ln T_c}{\partial\ln n}\right)\frac{1}{1-\frac{2}{\kappa^2}}\left(e^{-2x/\lambda} - \frac{\sqrt{2}}{\kappa}e^{-\sqrt{2}x/\xi}\right)$$

- For $\kappa > \sqrt{2}$ Bernoulli potential extends over $\lambda/2$
- For $\kappa < \sqrt{2}$ Bernoulli potential extends on the scale $\xi/\sqrt{2}$

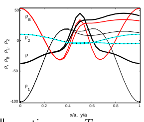
$$e\delta\varphi = -\frac{1}{2}mv^2\left(\frac{n_s}{n} + 4\frac{n_n}{n}\frac{\partial\ln T_c}{\partial\ln n}\right)\frac{1}{1+\frac{\sqrt{2}}{\kappa}}\left(1 + \frac{e^{\left(\frac{2}{\lambda}-\frac{\sqrt{2}}{\xi}\right)x} - 1}{1 - \frac{\kappa}{\sqrt{2}}}\right)$$

- Rickayzen's result for $\kappa \to \infty$ (low temperatures)
- For $\kappa \to \sqrt{2}$ potential approaches $e\delta\varphi = ...\frac{1}{2}\left(1 + \frac{2x}{\lambda}\right)$

Charge profile

From Poisson equation

Example Nb:
$\tilde{\rho} = \frac{\rho}{en}$
$= 9.5\,10^{-11}\cdot(1-t^2)(1-t^4)^2\tilde{\nabla}^2\phi$
charge density is small compared to total charge density
$\frac{\rho}{en} \approx \frac{\epsilon_0 B^2}{mn} = \frac{B^2}{\mu_0 n}\frac{1}{mc^2} \sim 10^{-10}$



→ neglect of charge profile in Maxwell equation, γ, n, T_c
Linearization with respect to magnetic field

$$\rho = \frac{2e\epsilon_0 B^2}{m\left(1-\frac{2}{\kappa^2}\right)}\left(\frac{n_s}{n} + 4\frac{n_n}{n}\frac{\partial\ln T_c}{\partial\ln n}\right)\left(e^{-2x/\lambda} - \frac{\kappa}{\sqrt{2}}e^{-\sqrt{2}x/\xi}\right)$$

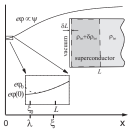
Surface value $x = 0$ always negative, bulk value positive
width of the surface $\rho(w) = 0$

$$w = \frac{\lambda}{2}\frac{\ln\frac{\kappa}{\sqrt{2}}}{\frac{\kappa}{\sqrt{2}} - 1}$$

- $\kappa > \sqrt{2}$ surface charge is formed by the contribution on the scale of the GL coherence length ξ
$\kappa \to \infty$ width $w \to \frac{\xi}{\sqrt{2}}\ln\frac{\kappa}{\sqrt{2}}$ bulk charge extends on scale of λ
- $\kappa < \sqrt{2}$ surface charge is formed by the contribution on the scale of London penetration depth λ
$\kappa \ll \sqrt{2}$ width $w \to \frac{\lambda}{2}\ln\frac{\sqrt{2}}{\kappa}$ bulk charge extends on scale of ξ

Budd-Vannimenus theorem for superconductors

Electrostatic potential induced by diamagnetic currents extends on λ, ξ
Close to T_c, $\lambda, \xi \gg$ BCS coherence length ξ_0, introduce intermediate scale $\xi_0 \ll L \ll \xi, \lambda$



- On scale $0 < x < L$ GL wave function $\psi(x) \approx \psi(x \to 0) \equiv \psi(0)$
- $\xi_0 \ll x < L$ GL-value $\varphi(x)$
- $\varphi(x) \approx \varphi(x \to 0) \equiv \varphi(0) \neq \varphi_0$ by surface dipole

virtual compression of crystal lattice, $0 < x < \delta L$: $\delta\rho_{\rm lat} = -\rho_{\rm lat}$

work on system with area S $\quad \delta\mathcal{W} = S\int dx\delta\rho_{\rm lat}\frac{\partial f}{\partial\rho_{\rm lat}} = S\int dx\delta\rho_{\rm lat}\varphi$

$0 < x < \delta L$: $\varphi(x) \approx \varphi_0$
$\delta L < x < \xi_0$: $\int \sim \xi_0/L \quad = S\delta L\rho_{\rm lat}(\varphi(0) - \varphi_0)$
$\xi_0 < x < L$: $\varphi(x) \approx \varphi(0)$
Increase of free energy $\quad = -\frac{\partial(f_{\rm el}SL)}{\partial(SL)}S\delta L = \left(-f_{\rm el} + n\frac{\partial f_{\rm el}}{\partial n}\right)S\delta L$

Obtain modification of Budd-Vannimenus theorem [6]

$$\rho_{\rm lat}\left(\varphi_0 - \varphi(0)\right) = f_{\rm el} - n\frac{\partial f_{\rm el}}{\partial n}$$

Potential step at surface due to surface dipole in terms of free energy with no regards of potential inside
$e\varphi = -\frac{\partial f_{\rm el}}{\partial n} \to -e\varphi(0)$ for $x = 0$ (includes no surface dipole), to get:

$$\rho_{\rm lat}\varphi_0 = f_{\rm el} + \varphi(0)\left(\rho_{\rm lat} + en\right)$$

Simplification:
relative charge deviation $(\rho_{\rm lat} + en)/\rho_{\rm lat} \sim 10^{-8} - 10^{-4}$
small corrections in the GL equation, with same accuracy from $\quad \varphi_0 = \frac{f_{\rm el}}{\rho_{\rm lat}}$

Magnetic pressure

Consider $L \gg \xi, \lambda$ as sample thickness, no restriction on temperature
Budd-Vannimenus theorem becomes with magnetic energy

$$\rho_{\rm lat}(\varphi_0 - \varphi_\infty) = \frac{B_0^2}{2\mu_0} + f_{\rm el}^\infty + n_\infty\frac{\partial f_{\rm el}^\infty}{\partial n}$$

charge neutrality, $\rho_{\rm lat} = -en_\infty$ and $e\varphi_\infty = -\frac{\partial f_{\rm el}^\infty}{\partial n}$

$$\varphi_0 = -\frac{B_0^2}{2\mu_0 en_\infty} - \frac{f_{\rm el}^\infty}{en_\infty}. \qquad (1)$$

Measurement by Morris and Brown gives only magnetic pressure

$$\rho_{\rm lat}(\varphi_0^L - \varphi_0^R) = \frac{1}{2\mu_0}(B_L^2 - B_R^2)$$

electrostatic force on lattice $F_{\rm elst} = \int_L^R dx\,E\rho_{\rm lat} = \rho_{\rm lat}(\varphi_0^L - \varphi_0^R)$ compensates Lorentz force $F_{\rm Lor} = BJ$ with mean magnetic field $B = \frac{1}{2}(B_L + B_R)$ and net current $B_L - B_R = \mu_0 J$

General proof for Slab geometry by integral of motion

If $-\nabla\frac{\partial f}{\partial\nabla\nu} + \frac{\partial f}{\partial\nu} = 0$ then Legendre transformation $g = f - \sum_\nu\frac{\partial f}{\partial\nabla\nu}\nabla\nu$ has no gradients $\nabla g = 0$, i.e. $g = \text{const} = f_{\rm el}^\infty$ deep in bulk and

$$f_{\rm el} = f_{\rm el}^\infty + \frac{\hbar^2}{m^*}|\nabla\psi|^2 - \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} - \varphi(\rho_{\rm lat} + en)$$
$$\to f_{\rm el}^\infty + \frac{B^2}{2\mu_0} - \varphi(0)(\rho_{\rm lat} + en) \quad \text{for } x = 0$$

at the surface where $\nabla\psi = 0$ what implies $E = 0$
Together with (1) follows surface relation

$$\rho_{\rm lat}\varphi_0 = f_{\rm el} + \varphi(0)\left(\rho_{\rm lat} + en\right)$$

valid for any temperature provided $f \equiv f[\psi, \nabla\psi]$

Summary

1. Surface charge
   - Bernoulli potential in superconductors discussed by charge build up in surface region $max[\xi/\sqrt{2}, \lambda/2]$
   - Surface charge extends over a range $L = min[\xi/\sqrt{2}, \lambda/2]$
   - Contrast to former theories: surface charge is not localized on Thomas-Fermi screening length $\lambda_{\rm TF}$ since negligible small amplitude $\sim \lambda_{\rm TF}^2/L^2$
2. Surface dipole
   - Budd-Vannimenus theorem generalized to superconductors by:
     - asymptotic extension of internal potential towards surface, $\varphi(0)$, appears instead of bulk potential
     - finite temperature instead of the ground-state
     - density of electronic charge differs locally from the lattice charge density due to charge depleted at the surface
   - electrostatic potential of surface from free energy and bulk electrostatic potential $\phi_0 = \frac{f_{\rm el}}{\rho_{\rm lat}}$
   - Relation between surface potential and magnetic pressure shows that potential across the surface cannot reveal pairing forces
   - Can be measured directly in bulk, first observation of charge transfer with dominant contribution of pairing forces [4]
   Alternative explanations: [7–9]

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