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Coexistence of phase transitions and hysteresis near BEC

TT 40.84

CHARLES UNIVERSITY PRAGUE
faculty of mathematics and physics

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Abstract

Multiple phases occurring in a Bose gas with finite-range interaction are investigated. In the vicinity of the onset of Bose-Einstein condensation (BEC) the chemical potential and the pressure show a van-der-Waals like behavior indicating a first-order phase transition although there is no long-range attraction. Furthermore the equation of state becomes multivalued near the BEC transition. For a Hartree-Fock or Popov (Hartree-Fock-Bogoliubov) approximation such a multivalued region can be avoided by the Maxwell construction. For sufficiently weak interaction the multivalued region can also be removed using a many-body T-matrix approximation. However, for strong interactions there remains a multivalued region even for the T-matrix approximation and after the Maxwell construction, what is interpreted as a density hysteresis. This unified treatment of normal and condensed phases becomes possible due to the recently found scheme to eliminate self-interaction in the T-matrix approximation [1–3].

T-matrix approximation

A homogeneous interacting Bose gas in equilibrium is considered with the Hamiltonian

$$\hat{H} = \sum_{\boldsymbol{k}} \frac{\hbar^2 k^2}{2m} \hat{a}^\dagger_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}} + \frac{\lambda}{2\Omega} \sum_{\boldsymbol{q},\boldsymbol{p},\boldsymbol{k}} \hat{a}^\dagger_{\boldsymbol{p}} \hat{a}^\dagger_{\boldsymbol{q}-\boldsymbol{p}}\, g_{\boldsymbol{p}-\frac{\boldsymbol{q}}{2}}\; g_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2}}\, \hat{a}_{\boldsymbol{k}} \hat{a}_{\boldsymbol{q}-\boldsymbol{k}}$$

and a separable interaction is assumed with Yamaguchi form factors $g_{\boldsymbol{p}} = \left(1 + \frac{p^2}{\gamma^2}\right)^{-1}$ [4].

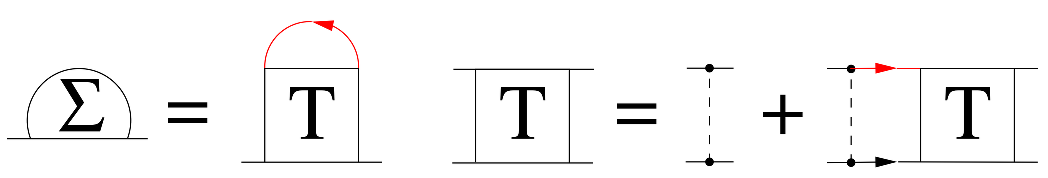

FIG. 1. Diagrams for the self energy and T-matrix [5], red arrows mark reduced propagators, to avoid self-interaction.

The resulting Green functions have the same structure as those obtained with anomalous propagators [1–3]. The dispersion in the BEC phase is

$$E_{\boldsymbol{k}} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} + n_{\boldsymbol{0}} \mathcal{T}(\boldsymbol{0})\right)^2 - n_{\boldsymbol{0}}^2 \mathcal{T}^2(\boldsymbol{k})}\ .$$

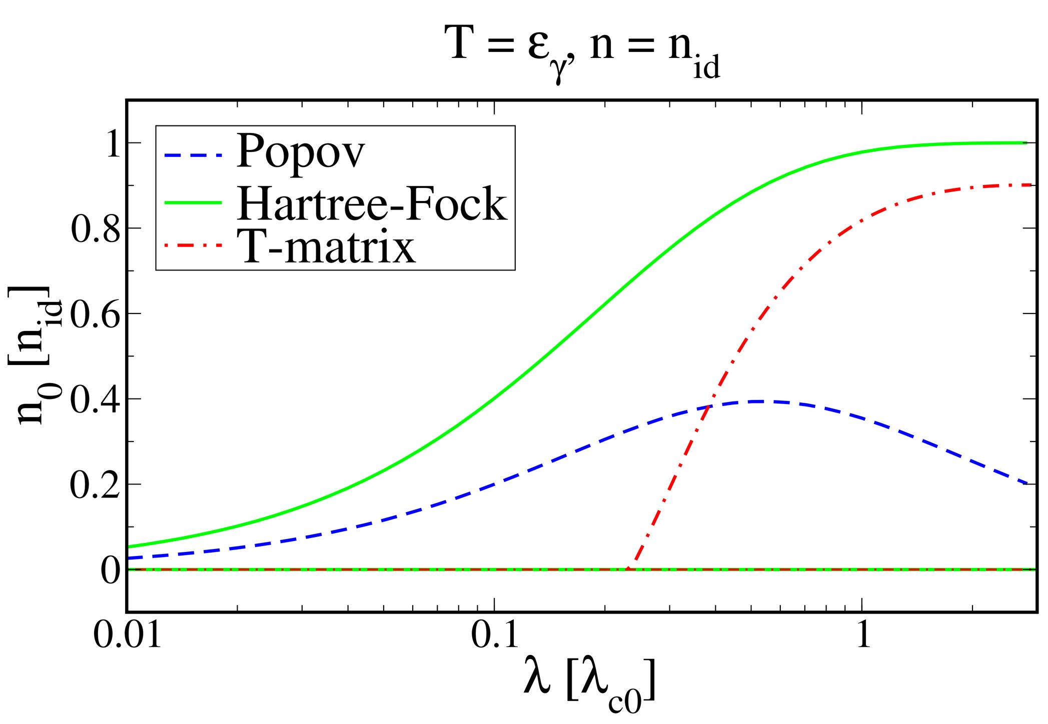

FIG. 2. Solutions for the condensate density at $n = n_{id}$.

The Popov and Hartree-Fock approximation always have two solutions for the condensate density $n_{\boldsymbol{0}}$ at the ideal critical density n_{id}. For the T-matrix approximation the second solution appears only above a critical interaction strength.

Weak repulsive interaction

The total energy in T-matrix approximation is

$$U = \underbrace{\sum_{\boldsymbol{k}\neq\boldsymbol{0}} E_{\boldsymbol{k}} f_{\rm B}(E_{\boldsymbol{k}})}_{\text{quasi particles}} + \underbrace{\sum_{\boldsymbol{k}\neq\boldsymbol{0}} \frac{n_0^2 \mathcal{T}^2(\boldsymbol{k})}{4E_{\boldsymbol{k}}} (1 + 2 f_{\rm B}(E_{\boldsymbol{k}}))}_{\text{two-particle contribution}}$$

$$- \underbrace{\sum_{\boldsymbol{k}\neq\boldsymbol{0}} E_{\boldsymbol{k}} v_{\boldsymbol{k}}^2}_{\text{depletion}} + \underbrace{\frac{\mathcal{T}(\boldsymbol{0})}{\Omega}\left(N^2 - N N_{\boldsymbol{0}} + \tfrac{1}{2} N_{\boldsymbol{0}}^2\right)}_{\text{attraction in momentum space}}$$

and the total number of particles is

$$N = N_{\boldsymbol{0}} + \underbrace{\sum_{\boldsymbol{k}\neq\boldsymbol{0}} f_{\rm B}(E_{\boldsymbol{k}})}_{\text{quasi particles}} + \underbrace{\sum_{\boldsymbol{k}\neq\boldsymbol{0}} v_{\boldsymbol{k}}^2}_{\text{depletion}}$$

$$+ \sum_{\boldsymbol{k}\neq\boldsymbol{0}} 2 v_{\boldsymbol{k}}^2 f_{\rm B}(E_{\boldsymbol{k}})\ .$$

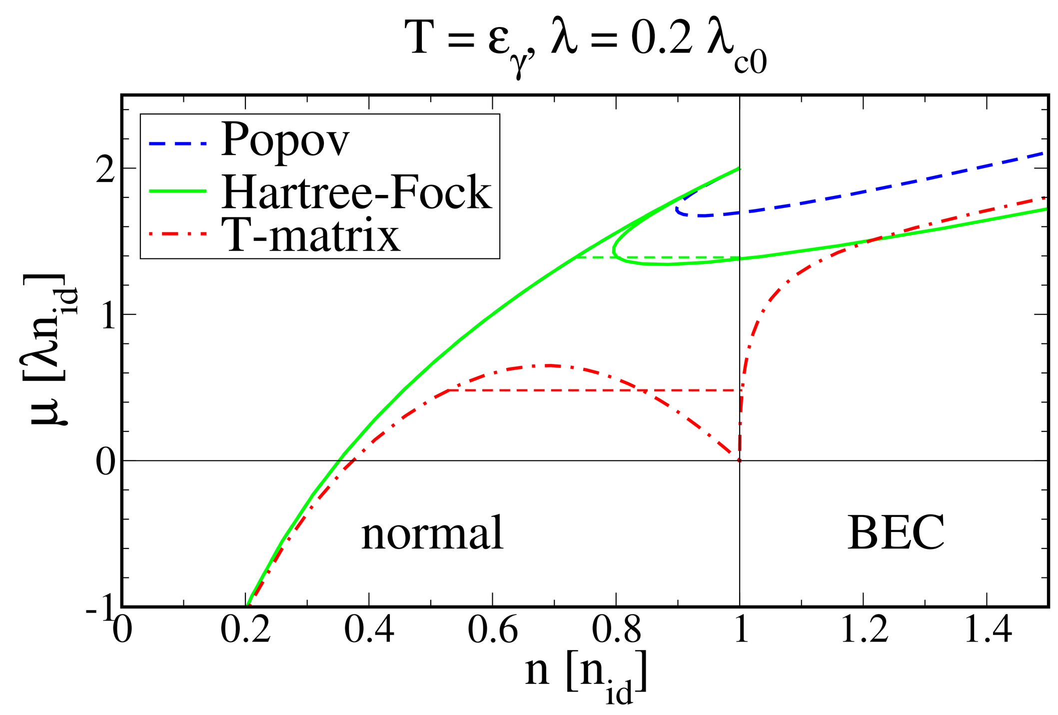

FIG. 3. Chemical potential in Popov, Hartree-Fock and T-matrix approximation, $n_{id} \approx 0.059\gamma^3$ is the ideal critical density for Bose condensation, the constant quantities are given above the diagram, $\varepsilon_\gamma = \hbar^2\gamma^2/2m$, $\lambda_{c0} = 8\pi\hbar^2/m\gamma$.

The Popov and Hartree-Fock approximation [6–8] show an unphysical behavior of the chemical potential due to an overestimation of the attraction in momentum space. Furthermore there is a drop of the chemical potential when BEC sets in signaling an instability of the system and a first-order phase-transition. The multivalued region can be avoided with the Maxwell construction. For the T-matrix approximation medium effects compensate the repulsive interaction near the onset of BEC and therefore the attraction in momentum space. The effective vanishing of the interaction leads again to an instability and a first-order phase-transition.

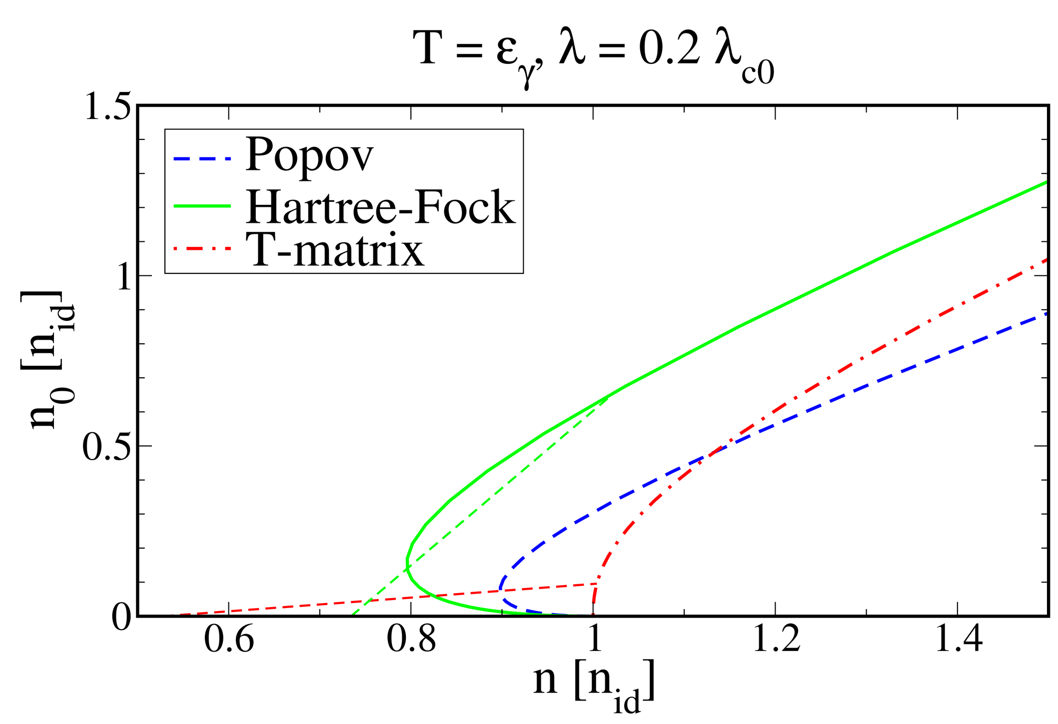

FIG. 4. Condensate density in Popov, Hartree-Fock and T-matrix approximation.

In the coexistence region the condensate density changes linearly.

Strong repulsive interaction

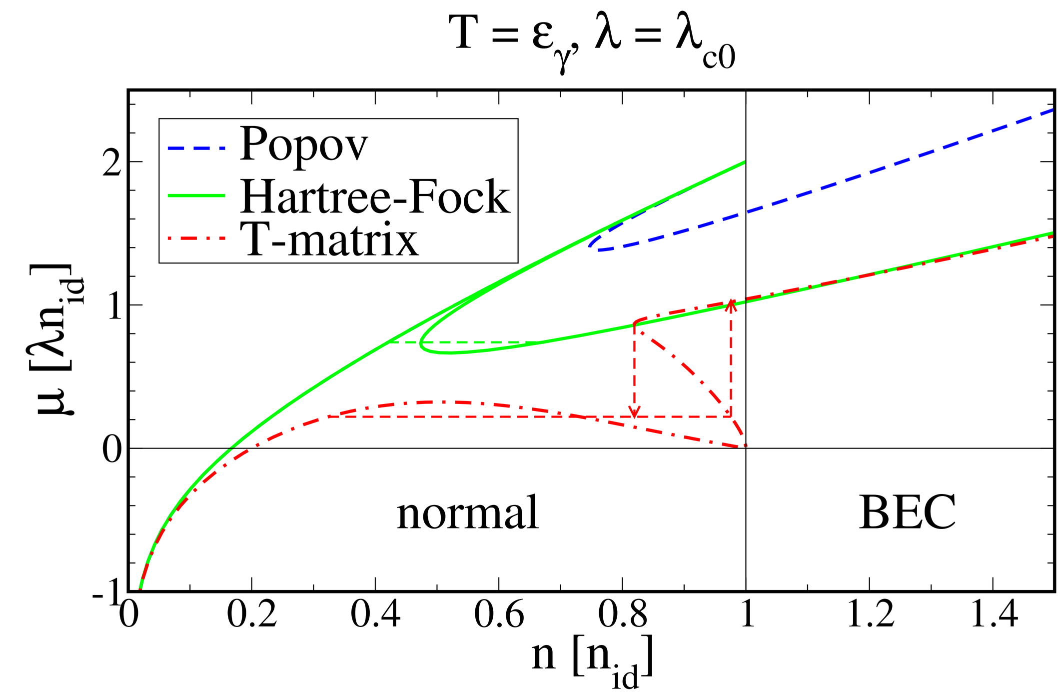

FIG. 5. Chemical potential, the vertical arrows mark the density hysteresis.

For a strong repulsive interaction the attraction in momentum space is too strong to be compensated by medium effects, therefore also the T-matrix approximation yields a multivalued region, which cannot be avoided by the Maxwell construction. Therefore a true physical relevance is attributed to this behavior and it is interpreted as appearance of a hysteresis.

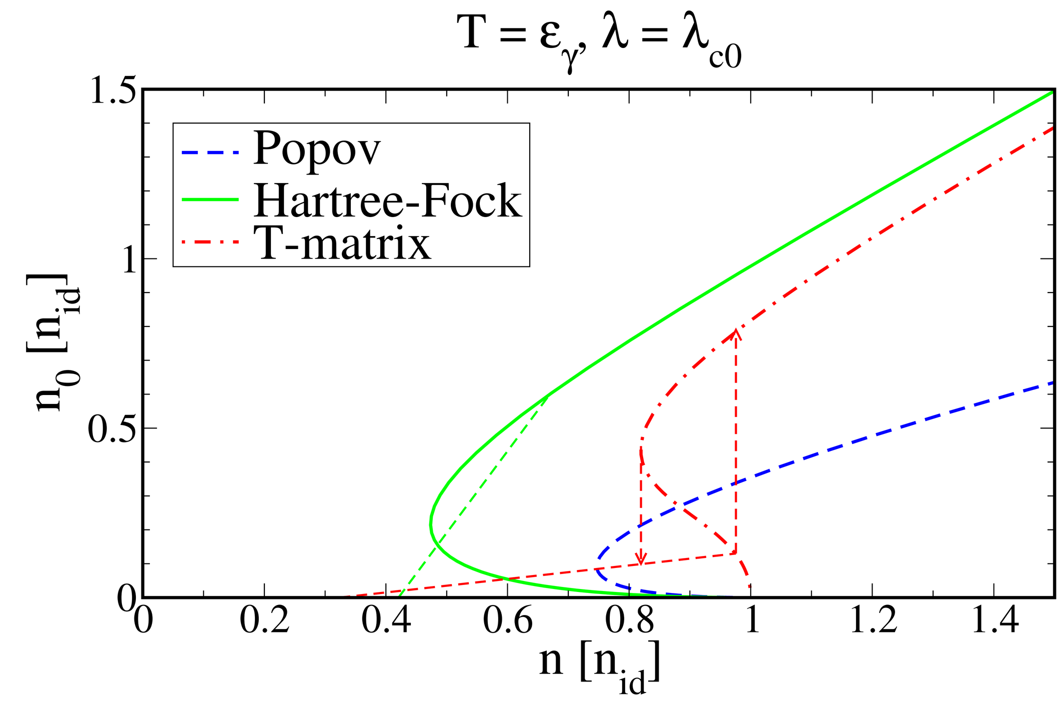

FIG. 6. Condensate density, the vertical arrows mark the density hysteresis.

Summary and conclusions

The appearance of BEC destabilizes the repulsive Bose gas due to the attraction in momentum space or medium effects, leading to a first-order phase transition. The BEC sets in with the first-order phase transition, therefore the critical density for BEC is decreased. During the first-order phase transition the condensate density increases linearly. In the case of strong interactions multiple solutions appear besides the coexistence region of the first-order phase transition, what can be interpreted as density hysteresis.

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