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Stability of supercurrents and condensates in type I superconductors

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Abstract. Excitations of Cooper pairs into non-condensed bound pairs are similar to excitations of true bosons out of the Bose-Einstein condensate. Using the Landau criterion of superfluidity we evaluate the critical current above which these pair-excitations would lead to a finite resistivity. The predicted value strongly depends on the chosen approximation. The Thouless approach based on the Galitskii T-matrix and the Kadanoff-Martin theory which is in many aspects equivalent to the BCS theory, both lead to zero critical velocity, what is in conflict with the mere existence of supercurrents. In contrast, the T-matrix with multiple scattering corrections provides the critical velocity of pair excitation which is $\sqrt{3}$-times larger than the critical velocity of pair breaking. This agrees with the experimentally well established fact that supercurrents in type I superconductors are limited by pair breaking, not by pair excitation.

1 Introduction

Superconductivity results from Bose-Einstein condensation (BEC) of electron pairs and the coherence of all pairs in the condensate. Coherence is best known from the laser light, but there is one essential difference. Two or more mutually incoherent laser beams can share the same space while a superconducting condensate is always single-valued. The first question we ask here is by which mechanism two parallel condensates are excluded. We restrict here to conventional superconductors of type I and do not consider recent multi-gapped materials [1] which have two gaps but in different channels. Similarly one has to be cautious using the analogy of the superconductor with the BEC of stable bosons because not all bosons are in the condensate. Within the Bardeen-Cooper-Schrieffer (BCS) theory there are no bounded electron pairs out of the condensate. The second question we ask here is why bounded non-condensed pairs can be neglected in weakly coupled superconductors.

Bounded non-condensed pairs appear in the strong coupling regime, e.g. in ultracold Fermi gases interacting strongly enough to form diatomic molecules which eventually undergo the BEC. Their presence cannot be ignored, because the critical velocity of a superfluid flow is given by excitations of bound pairs out of the condensate [2].

The success of the BCS theory shows that such mechanism is not present in the weak coupling regime. Tuning the interaction through the BEC-BCS crossover, it was observed that the critical velocity of a superfluid flow changes its character, which documents that different decay mechanisms of the superfluid flow act at each side of the crossover [3]. In the BCS regime the flow reaches the critical velocity when Cooper pairs break into single-particle excitations. Here we derive the energy dispersion of non-condensed bounded pairs and show that the critical velocity of their excitation is higher than the critical velocity of pair breaking. Briefly, if we increase the supercurrent, the condensate is destroyed by the pair breaking before bounded pairs are excited.

1.1 Collective versus single-particle motion

The non-condensed bounded electron pairs belong to bosonic excitations of the system. Let us compare bosonic excitations of the superfluid and the superconducting system.

Even in weakly interacting Bose condensates, the interaction is dominant at low momenta, where any single-boson excitation becomes a collective motion with the velocity of sound. This reconstructed branch of excitations known as the Bogoliubov mode is indispensable to obtain a finite critical velocity of superflow. According to the Landau criterion [2], interacting bosons with acoustic dispersion can be excited out of the condensate by a flow around an obstacle only if the velocity of flow exceeds the velocity of sound. Above this critical velocity such excitations transfer a momentum to the Bose gas stopping

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its flow, while at lower velocities we observe superfluidity. Ideal bosons with free-particle parabolic dispersion can be excited out of the condensate by any slow flow, therefore the BEC in the ideal system does not lead to superfluidity.

In superconductors of BCS type, there is an analogous collective motion called the Anderson-Bogoliubov mode. For the Fermi gas with tunable interaction Combescot et al. [2] have studied how the Bogoliubov mode develops into the Anderson-Bogoliubov mode as the interaction strength is tuned across the BEC-BCS crossover. To this end they have employed the BCS theory at zero temperature to find that the critical velocity due to excitations of the Anderson-Bogoliubov mode is higher than the critical velocity due to pair breaking. In contrast, for the two-dimensional Hubbard model and zero temperature, Yunomae et al. [4] have shown that the Anderson-Bogoliubov mode has a roton-like minimum which is below the pair breaking energy and which leads to the lowest critical current even at the BCS side.

One cannot simply transfer the experience with bosons to the superconductors, because in the theory of superconductivity the focus is on interactions bounding electrons into Cooper pairs while the interaction among Cooper pairs themselves is mostly ignored. Let us inspect the BCS theory of superconductivity viewing bound electron pairs as bosons. The pairs in the condensate we call Cooper pairs while bound pairs out of the condensate we call excited pairs. The Cooper pairs form an ideal gas as they do not interact among themselves. This should lead to a zero critical velocity due to parabolic dispersion of excited pairs. Such a problem was unintentionally avoided in the BCS theory since excited pairs have been excluded from the wave function, see the discussion on page 1180 of the BCS paper [5].

Within the BCS theory one can perturb the condensate in two ways: to rise the collective motion of the condensate (e.g. create moving vortices or oscillations of Carlson-Goldman mode) or to break the Cooper pair into two electrons. The critical velocity due to the breaking of a Cooper pair into two quasiparticles is given by the gap divided by the Fermi momentum, $v_{\rm pb} = \Delta/k_{\rm F}$ [6]. There are no simple criteria of stability for collective excitations of the condensate, however. For each particular deformation of the order parameter, one has to solve the dynamics of the condensate from the BCS gap equation. The character of fluctuations depend on the material. In type II superconductors the lowest critical velocity results from pinning of vortices. In neutral systems the Goldstone theorem guarantees the existence of a gapless mode (Anderson-Bogoliubov mode) related to density fluctuations which lets move the condensate as a whole. In charged systems, the coupled dynamics of a condensate with quasiparticles leads to the plasma oscillation and the Carlson-Goldman mode which seem to be unimportant for the value of the critical velocity.

Pairs excited from the condensate into non-condensed bounded pairs represent another family of fluctuations. Their dominant role at the BEC side of the BEC-BCS crossover is well understood, but less is known for the BCS side. Adhikari et al. [7] have evaluated an energy dispersion of bounded pairs using the original Cooper treatment of a single interacting pair on the background of a non-interacting Fermi sea. Like the Cooper problem, this approach indicates properties of bounded pairs but lacks the effect of the condensate on bound pairs. Andrenacci et al. [8] studied the internal structure of excited pairs both in the normal state and in the superconducting state. In spite of these studies, in the BCS limit bounded non-condensed pairs are not much understood.

1.2 Plan of the paper

In this paper we discuss the last type of excitations: bound pairs which are not in the condensate. Using the two-particle T-matrix we evaluate their energy dispersion and according to the Landau criterion we establish the velocity needed to excite a pair out of the condensate.

It will be seen that the prediction is sensitive to the details of the T-matrix approximation. For the Thouless approach based on the Galitskii T-matrix and for the Kadanoff-Martin theory which is in many aspects equivalent to the BCS theory, the critical velocity of pair excitations is zero, what is in clear conflict with the existence of stable supercurrents. For the T-matrix with multiple scattering corrections we show that the critical velocity of pair excitation is higher than the critical velocity of pair breaking. The pair breaking [9] is thus the mechanism which controls the stability of supercurrents and the excited pairs need not to be assumed. This justifies the neglect of non-condensed bound pairs by which the T-matrix approach simplifies to the BCS theory.

The paper is organized as follows. In Section 2 we discuss the standard treatments of Cooper pairs and suggest to avoid the use of anomalous functions. In Section 3 equations for the T-matrix with multiple scattering corrections are presented. In Section 4 we focus on the condensate and derive the BCS equation for the gap and the GL equation as the asymptotic near the critical temperature. In Section 5 we prove that the condensate is single-valued and apply the Landau criterion of superfluidity to derive the critical velocity of the excitation of Cooper pairs out of the condensate. We also indicate why the Galitskii and the KM theories result in zero critical velocity of pair excitation. Section 6 contains the summary.

2 Approaches to Cooper pairs

2.1 Excited pairs and anomalous functions

When a Cooper pair is scattered from the condensate into an excited state, its energy increases. To evaluate this difference, it is desirable to treat initial and final states with the same approximation. Unfortunately, such demand excludes theories based on anomalous Green functions.

Within anomalous Green functions the interaction of electrons forming the Cooper pair is included up to infinite order as it is necessary to describe bound states. To see how the infinite-order approximation is achieved, we express the gap Δ from the anomalous Gor'kov function F by the Fock-like relation,

$$\Delta = V(k_B T/\Omega) \sum_{\omega,k} F(\omega, k). \tag{1}$$

Here V is the interaction potential and the sum over momenta is restricted by a cutoff [10]. The anomalous function is proportional to the gap

$$F(\omega, k) = G_0(-\omega, -k)\Delta G(\omega, k), \tag{2}$$

which by iteration yields an infinite ladder covering all orders of the interaction potential. In contrast, the interaction of non-condensed electrons is on the level of the Hartree-Fock approximation, therefore electrons cannot form bound pairs out of the condensate.

The unequal treatment of condensed and non-condensed pairs is the major advantage of the anomalous Green function approach as it tremendously simplifies the calculations. At the same time it disregards this approach from studying the excited pairs such that we suggest to avoid anomalous functions.

2.2 Superconductivity without anomalous functions

Theories of superconductivity which do not benefit from anomalous functions or closely related trial wave functions of indefinite number of particles are very seldom. Let us remind their roots.

The existence of anomalous functions follows from the BCS wave function [5]

$$|\mathrm{BCS}\rangle = \prod_k \left(u_k + \mathrm{e}^{2i\phi} v_k \psi^\dagger_{k\uparrow} \psi^\dagger_{-k\downarrow} \right) |\mathrm{vac}\rangle, \tag{3}$$

which implies that $F = \langle \mathrm{BCS}|\psi_{k\uparrow}\psi_{-k\downarrow}|\mathrm{BCS}\rangle \neq 0$. In this sense, the use of wave functions of indefinite number of particles is equivalent to the use of the anomalous functions.

The possibility to project the BCS wave function on the state of fixed number of particles has been discussed already by Bardeen et al. [5]. A simple projection was introduced by Rickayzen [11], who used the fact that the energy is independent of the phase ϕ. Since the states of different ϕ are degenerate, their linear combination

$$|\mathrm{PBCS}\rangle = \int_0^{2\pi} d\phi \; \mathrm{e}^{-N\phi} \prod_k \left(u_k + \mathrm{e}^{2i\phi} v_k \psi^\dagger_{k\uparrow} \psi^\dagger_{-k\downarrow} \right) |\mathrm{vac}\rangle \tag{4}$$

has a fixed number of N particles and the same ground-state energy as the BCS function. This projected wave function implies zero anomalous functions,

$$\langle \mathrm{PBCS}|\psi_{k\uparrow}\psi_{-k\downarrow}|\mathrm{PBCS}\rangle = 0, \tag{5}$$

nevertheless it gives the BCS gap in the single-particle spectrum.

One can link the two approaches on the level of anomalous functions. The phase ϕ enters the anomalous function $F = \mathrm{e}^{2i\phi}|F|$ and thus the gap function $\Delta = |\Delta|\mathrm{e}^{2i\phi}$. By averaging over a random phase the mean value of the anomalous function vanishes $\langle F\rangle = 0$ and also $\langle \Delta\rangle = 0$, while the magnitude remains finite, $\langle |F|^2\rangle = |F|^2$ and $\langle |\Delta|^2\rangle = |\Delta|^2$. This determines the theory of superconductivity without anomalous functions where the two-particle Green function has to be treated instead of the product $\bar{F}F$. The corresponding product $\bar{\Delta}\Delta$ appears then from the two-particle T-matrix, which has to be evaluated up to infinite order, i.e., at least in ladder approximation.

2.3 T-matrix approaches to superconductivity

The ladder approximation sums the two-particle interactions up to infinite order. Accordingly, it has been formulated within all many-body machineries and one can find it in many versions. In spite of similar basic features, the individual approaches essentially differ when applied to superconductivity.

T-matrix approaches to superconductivity date back to the late 1950 and early 1961 [12–14]. These theories are rooted in the perturbation expansion for the normal state of the system and yield the superconducting phase transition as a bonus. We will compare three versions of the T-matrix theory: the original Galitskii theory [15,16] applied to superconductivity by Thouless [13]; the Kadanoff-Martin (KM) theory [14]; and the T-matrix with multiple scattering corrections [17,18].

All these three versions of the T-matrix become singular at the critical temperature, what signals the onset of Cooper pairing [13,14,19]. The predicted critical temperatures differs, however, due to different approximations of single-particle Green functions from which the T-matrix is constructed. While the KM prediction is close to the BCS result, Haussmann [20] has shown that the Galitskii theory yields a lower critical temperature with the difference becoming essential for strong interactions. Since the T-matrix with multiple scattering corrections in the normal state is identical to the Galitskii theory [17], it gives identical critical temperatures.

Below the critical temperature these three versions of T-matrices significantly differ in the excitation spectra. The corresponding differences of the critical velocities are compared in Table 1.

The Galitskii theory fails to reproduce the gap in the single-particle spectrum [12]. Since the critical velocity of pair breaking equals the gap divided by the Fermi momentum k_F, the pair-breaking critical velocity is zero. Such system cannot have permanent supercurrents. The critical velocity of pair excitations is also zero as it follows from the parabolic dispersion obtained e.g. by Haussmann [20,21], see also derivation in Section 5.1.2.

The single-particle energy spectrum of the KM theory is very similar to the BCS theory [12,14]. In particular, it has a gap which guarantees the finite pair-breaking critical velocity. Unlike the BCS theory, the KM theory offers excited bound pairs. They have the energy spectrum of an

Table 1. Comparison of the critical velocities for different T-matrix approaches, Galitskii, Kadanoff-Martin (KM) and the T-matrix corrected by multiple-scattering (TMSC).

	Critical velocities of	
	pair breaking	pair excitation
Galitskii	0	0
KM	Δ/k_{F}	0
TMSC	Δ/k_{F}	$\sqrt{3}\Delta/k_{\mathrm{F}}$

ideal gas, therefore the critical velocity of pair excitation is zero (see Ref. [22] or Sect. 5.1.3).

Some approaches combine the anomalous Green functions with the ladder approximation. In the two-step procedure the T-matrix is used to derive normal-state properties which are than used to study superconductivity on the BCS level [23]. In the one-step procedure the T-matrix is constructed from Nambu-Gor'kov functions [21,24,25]. For the contact interaction the evaluation of the T-matrix is feasible even with the screening and exchange channels included, which is known as the fluctuation exchange (FLEX) approximation. The FLEX approximation extended by anomalous functions has been intensively studied for the Hubbard model [26–31]. In all these approaches excited pairs eventually appear as poles of the T-matrix, but they are treated differently from Cooper pairs which are covered by a rather complex set of relations for anomalous functions.

The T-matrix with multiple scattering corrections has a single-particle energy spectrum of BCS type [17,18]. In this paper we evaluate the energy spectrum of excited pairs and show that it is separated from the energy of Copper pairs by a small gap proportional to the square of the BCS gap divided by the Fermi energy. This tiny gap is sufficient to yield a critical velocity of pair excitation $\sqrt{3}$-times larger than the pair-breaking velocity which means that the condensate breaks into quasiparticles before it excites into bound states. Let us outline the main ingredients of this approach.

3 Corrected T-matrix by multiple scattering

3.1 Reduced selfconsistency form

The T-matrix with multiple scattering corrections includes the Galitskii as well as Kadanoff-Martin approach as specific simplifications. We thus introduce only the T-matrix with multiple-scattering corrections and eventually simplify it to the other theories.

It was shown recently [18] that multiple-scattering corrections to the T-matrix are equivalent to a reduced selfconsistency in the loop of the selfenergy and can be traced back to higher-order cluster-cluster correlations [32]. These rearrangements are possible due to the symmetry of the T-matrix with respect to the interchange of interacting particles [18]. We note that this symmetry is just the one required by the Baym-Kadanoff criterion for conserving approximations. Therefore the multiple-scattering corrected T-matrix conserves energy and momentum. Since the formulation in reduced selfenergies is simpler and its structure is closer to the familiar Kadanoff-Martin theory, we introduce the theory using the reduced selfconsistency form.

The formation of bound pairs and their dispersion parallels the formation of molecules in a gas. Unlike the collective motion, this process does not create a space charge or electrical currents, we thus do not include the electromagnetic field in our model. The Hamiltonian assumed has only a single electronic band with parabolic kinetic energy and a separable BCS interaction.

The Dyson equation

$$G_\uparrow = G^0 + G^0 \Sigma_\uparrow G_\uparrow, \tag{6}$$

expresses the propagator G of electrons with spin $\uparrow$ via the selfenergy

$$\Sigma_\uparrow = \frac{1}{L^3}\sum_{\mathbf{Q}} \Sigma_{\mathbf{Q}\uparrow}, \tag{7}$$

which is a sum over momentum $\mathbf{Q}$ of interacting pairs with discrete values corresponding to the sample volume L^3. We call $\Sigma_{\mathbf{Q}\uparrow}$ a $\mathbf{Q}$-part for brevity. In the case of condensation mode, $\mathbf{Q}$ is the momentum of the Cooper pair and we will denote it by $\mathbf{C}$.

To avoid self-interactions mediated by the condensate, the internal lines of the $\mathbf{Q}$-part of the selfenergy should not include processes related to the $\mathbf{Q}$-mode itself [18]. To this end we introduce the $\mathbf{Q}$-reduced propagator

$$G_{\not{\mathbf{Q}}\downarrow} = G^0 + G^0 \Sigma_{\not{\mathbf{Q}}\downarrow} G_{\not{\mathbf{Q}}\downarrow}, \tag{8}$$

which is dressed by all but the $\mathbf{Q}$-part of the selfenergy

$$\Sigma_{\not{\mathbf{Q}}\downarrow} = \frac{1}{L^3}\sum_{\mathbf{Q}'\neq\mathbf{Q}} \Sigma_{\mathbf{Q}'\downarrow}. \tag{9}$$

The $\mathbf{Q}$-part of the $\uparrow$ selfenergy is obtained by closing the loop of $\downarrow$ line of the T-matrix by the $\mathbf{Q}$-reduced propagator:

$$\Sigma_{\mathbf{Q}\uparrow}(k) = k_{\mathrm{B}}T\sum_{\Omega} \mathcal{T}_{\uparrow\downarrow}(k,Q-k;k,Q-k)G_{\not{\mathbf{Q}}\downarrow}(Q-k), \tag{10}$$

where we have used the convention that fermionic four-momenta $k \equiv (\omega,\mathbf{k})$ are in lower cases and bosonic four-momenta $Q \equiv (\Omega,\mathbf{Q})$ are in upper cases. The sum runs over bosonic Matsubara's frequencies Ω. Functions of opposite spin are readily obtained reversing all spins in the equations.

The T-matrix is constructed from the $\mathbf{Q}$-reduced propagator in the $\downarrow$ line and the full propagator in the $\uparrow$ line as:

$$\begin{aligned}\mathcal{T}_{\uparrow\downarrow}(k,Q-k;p,Q-p) &= V(\mathbf{k},\mathbf{Q}-\mathbf{k};\mathbf{p},\mathbf{Q}-\mathbf{p})\\ &+\frac{k_{\mathrm{B}}T}{L^3}\sum_{k'} V(\mathbf{k},\mathbf{Q}-\mathbf{k};\mathbf{k}',\mathbf{Q}-\mathbf{k}')\\ &\times G_\uparrow(k')G_{\not{\mathbf{Q}}\downarrow}(Q-k')\\ &\times \mathcal{T}_{\uparrow\downarrow}(k',Q-k';p,Q-p).\end{aligned} \tag{11}$$

Except for the reduced propagator, this is the standard ladder approximation [16]. The frequency sum runs over fermionic Matsubara's frequencies ω. The set of equations is complete.

3.2 Advantages of the multiple-scattering corrected T-matrix

By additional approximations of the **Q**-reduced propagator one can convert the above theory either into the Galitskii approximation or the Kadanoff-Martin theory. The **Q**-reduced propagators eliminate non-physical repeated collisions in the spirit of multiple scattering expansion [17,18]. If repeated collisions are not eliminated, which is achieved by the approximation $G_{\not{\mathbf{Q}}\downarrow} \approx G_\downarrow$, one arrives at the Galitskii T-matrix. If the repeated collisions are eliminated by a radical neglect of all interactions in the loop, $G_{\not{\mathbf{Q}}\downarrow} \approx G^0$, one recovers the Kadanoff-Martin theory.

One may argue that the exclusion of a single mode is negligible in the thermodynamic limit since each mode contributes with a weight of inverse volume. This is true only for normal modes. The condensation mode, however, contributes with a weight proportional to the macroscopic number of particles times the inverse volume which shows that this mode leads to a finite contribution even in the thermodynamic limit. This corresponds to the finite contribution of the single condensation mode in the BEC.

The multiple scattering corrections as well as the eliminated self-interaction lead to the fact that the T-matrix and thus the selfenergy depend on the evaluated process. This parallels the Hartree approximation in which each electron experiences a different electrostatic potential given by all electrons but itself. Using the same mean potential for all electrons one introduces a self-interaction which might destroy features given by non-identical potentials.

3.3 Separable interaction

For the discussion in this paper we employ the simple BCS interaction

$$\begin{aligned} V(\mathbf{k}, \mathbf{Q}-\mathbf{k}; \mathbf{p}, \mathbf{Q}-\mathbf{p}) = -V\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{k})|\right) \\ \times\,\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{Q}-\mathbf{k})|\right)\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{p})|\right) \\ \times\,\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{Q}-\mathbf{p})|\right). \end{aligned} \quad (12)$$

From equation (11) one can see that this separable potential implies the separable T-matrix

$$\begin{aligned} \mathcal{T}_{\uparrow\downarrow}(k, Q-k; p, Q-p) = -\mathcal{T}_Q\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{k})|\right) \\ \times\,\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{Q}-\mathbf{k})|\right)\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{p})|\right) \\ \times\,\theta\left(\omega_{\mathrm{D}} - |\epsilon(\mathbf{Q}-\mathbf{p})|\right) \end{aligned} \quad (13)$$

and equation (11) simplifies to a scalar equation

$$\mathcal{T}_Q = V - V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_\uparrow(k) G_{\not{\mathbf{Q}}\downarrow}(Q-k)\mathcal{T}_Q. \quad (14)$$

The sum over $\mathbf{k}$ is restricted by cutoffs of the BCS model. The **Q**-part of the selfenergy simplifies to:

$$\Sigma_{\mathbf{Q}\uparrow}(k) = -k_{\mathrm{B}}T\sum_{\Omega}\mathcal{T}_Q G_{\not{\mathbf{Q}}\downarrow}(Q-k). \quad (15)$$

This applies to momenta $\mathbf{k}$ inside the cutoff region $|\epsilon(\mathbf{k})| < \omega_{\mathrm{D}}$. Out of this region the selfenergy is zero. The outer region can be ignored since it does not contribute to the condensation. Equations (6)–(9), (14) and (15) form a closed set.

4 Condensation mode

4.1 Energy gap

The poles of the T-matrix describe the long-living bosonic excitations of the system controlled by strong interactions. In case these excitations occur at negative frequencies they are excited bound states (**Q**-mode) and in case that they appear at zero frequency these are Cooper pairs (**C**-mode). The T-matrix of the **C**-mode diverges at $\Omega = 0$ reaching values proportional to the volume L^3 (see Ref. [17]).

To make a link with the standard notation of the BCS theory we introduce a function $|\Delta|$ from the singular term of the T-matrix

$$|\Delta| = \sqrt{\frac{k_{\mathrm{B}}T}{L^3}\mathcal{T}_{0,\mathbf{C}}}, \quad (16)$$

where the subscript denotes $Q = (0, \mathbf{C})$. The T-matrix depends on $|\Delta|^2$ being averaged over all thermodynamically allowed phases. On the other hand, its value does not change if we select one preferential phase $\Delta = |\Delta|\mathrm{e}^{2i\phi}$ and express the singular element of the T-matrix as:

$$\mathcal{T}_{0,\mathbf{C}} = \frac{L^3}{k_{\mathrm{B}}T}\bar{\Delta}\Delta. \quad (17)$$

The **C**-part of the selfenergy is dominated by the condensation at $\Omega = 0$:

$$\begin{aligned} \Sigma_{\mathbf{C}\uparrow}(k) &= -k_{\mathrm{B}}T\sum_{\Omega}\mathcal{T}_{\Omega,\mathbf{C}} G_{\not{\mathbf{C}}\downarrow}(\Omega-\omega, \mathbf{C}-\mathbf{k}) \\ &= -L^3\bar{\Delta}G_{\not{\mathbf{C}}\downarrow}(-\omega, \mathbf{C}-\mathbf{k})\Delta \\ &\quad -k_{\mathrm{B}}T\sum_{\Omega\neq 0}\mathcal{T}_{\Omega,\mathbf{C}} G_{\not{\mathbf{C}}\downarrow}(\Omega-\omega, \mathbf{C}-\mathbf{k}) \\ &\approx -L^3\bar{\Delta}G_{\not{\mathbf{C}}\downarrow}(-\omega, \mathbf{C}-\mathbf{k})\Delta. \end{aligned} \quad (18)$$

The contributions of $\Omega \neq 0$ are regular and become negligible in the thermodynamic limit $L^3 \to \infty$. The selfenergy thus splits into a singular contribution of the **C**-mode and a regular reminder due to all other modes:

$$\Sigma_\uparrow(k) = -\bar{\Delta}G_{\not{\mathbf{C}}\downarrow}(-\omega, \mathbf{C}-\mathbf{k})\Delta + \Sigma_{\not{\mathbf{C}}\uparrow}(k). \quad (19)$$

The **C**-reduced propagator $G_{\not{\mathbf{C}}} = G^0 + G^0\Sigma_{\not{\mathbf{C}}}G_{\not{\mathbf{C}}}$ does not depend on the singular selfenergy, therefore it has no gap in the energy spectrum. It depends on Δ only indirectly via internal lines of $\Sigma_{\not{C}}$ and plays the role of the normal state propagator. The energy gap in the full propagator is given by the Gor'kov-type equation

$$G_{\downarrow}(k) = G_{\not{C}\downarrow}(k) - G_{\downarrow}(k)\bar{\Delta}G_{\not{C}\uparrow}(-\omega, \mathbf{C}-\mathbf{k})\Delta G_{\not{C}\downarrow}(k), \tag{20}$$

which directly follows from equations (6), (8) and (18).

From equation (20) we can identify the anomalous function $\bar{F}(k) = G_{\downarrow}(k)\bar{\Delta}G_{\not{C}\uparrow}(-\omega, \mathbf{C}-\mathbf{k})$, which corresponds to the complex conjugate of Gor'kov relation (2). Briefly, fixing the phase we arrive at the algebra analogous to the Gor'kov equations. Note that we have not introduced the anomalous functions as a prerequisite but have obtained them as a result of the algebra in which all interacting pairs were treated equally [33].

To be exact, our Δ is rather the off-diagonal selfenergy of the Eliashberg approach and the regular part of the selfenergy is Eliashberg's diagonal selfenergy. As known from the Eliashberg theory, the energy gap in the single-particle spectrum differs from $2|\Delta|$ by a renormalization factor due to the energy dependence of the regular part of the selfenergy (see Ref. [18]). This renormalization is not essential for our discussion, we thus call Δ the gap for simplicity.

4.2 Gap equation

The value of the gap is given by the T-matrix of the condensation mode

$$\mathcal{T}_{0,\mathbf{C}} = V - V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_{\uparrow}(\omega,\mathbf{k})G_{\not{C}\downarrow}(-\omega,\mathbf{C}-\mathbf{k})\mathcal{T}_{0,\mathbf{C}}. \tag{21}$$

Since the T-matrix diverges, it becomes much larger than the interaction potential, $\mathcal{T}_{0,\mathbf{C}} \gg V$, which allows us to neglect the first term in the right hand side of equation (21). Substituting $\mathcal{T}_{0,\mathbf{C}}$ from (17) and dividing by $L^3\Delta/(k_{\mathrm{B}}T)$ we obtain the gap equation

$$\bar{\Delta} = -V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_{\uparrow}(\omega,\mathbf{k})G_{\not{C}\downarrow}(-\omega,\mathbf{C}-\mathbf{k})\bar{\Delta}. \tag{22}$$

In the notation of the T-matrix, the gap equation (22) reads $V\mathcal{T}^{-1}_{0,\mathbf{C}}\bar{\Delta} = 0$. Ignoring the normal state solution $\bar{\Delta} = 0$, we can divide equation (22) by $\bar{\Delta}$ and V so that the gap equation reads $\mathcal{T}^{-1}_{0,\mathbf{C}} = 0$. Such form is independent of phase ϕ needed to define Δ. We will use $\mathcal{T}^{-1}_{0,\mathbf{C}} = 0$ in Section 5 (see e.g. Eq. (33)). In this section we keep $\bar{\Delta}$ to introduce approximations valid near the critical temperature.

4.3 Ginzburg-Landau theory

Near the critical temperature T_c the gap becomes small, $\Delta \to 0$, therefore we can expand the propagator to the lowest order in Δ. For the systematic expansion of the gap equation into the GL equation, see reference [34]. From the Gor'kov equation (20) follows:

$$G_{\downarrow}(k) \approx G_{\not{C}\downarrow}(k) - G_{\not{C}\downarrow}(k)\bar{\Delta}G_{\not{C}\uparrow}(-\omega,\mathbf{C}-\mathbf{k})\Delta G_{\not{C}\downarrow}(k). \tag{23}$$

At the same time, the critical velocity of the condensate becomes small, therefore the momentum of the condensate $\mathbf{C}$ has to be small. We can thus also expand the sum in the gap equation (22) to the second order in $\mathbf{C}$. As result one obtains the GL equation

$$\frac{\hbar^2|\mathbf{C}|^2}{2m^*}\bar{\Delta} + \alpha\bar{\Delta} + \beta|\Delta|^2\bar{\Delta} = 0 \tag{24}$$

with the GL parameters

$$\alpha = \chi + \chi V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_{\not{C}\uparrow}(\omega,\mathbf{k})G_{\not{C}\downarrow}(-\omega,-\mathbf{k}), \tag{25}$$

$$\beta = -\chi V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G^2_{\not{C}\uparrow}(\omega,\mathbf{k})G^2_{\not{C}\downarrow}(-\omega,-\mathbf{k}) \tag{26}$$

and

$$\frac{\hbar^2}{2m^*} = \chi V\frac{\partial^2}{\partial\mathbf{C}^2}\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_{\not{C}\uparrow}(\omega,\mathbf{k})G_{\not{C}\downarrow}(-\omega,\mathbf{C}-\mathbf{k})\bigg|_{\mathbf{C}=0}. \tag{27}$$

Replacing $\mathbf{C}$ with the covariant derivative $i\hbar\nabla - e^*\mathbf{A}$ one obtains the customary form of the GL equation [35].

In the strict sense, the above GL equation (24) does not include the magnetic field and gradients of the amplitude of the gap. This follows from our assumption that the system is homogeneous which made it possible to express all formulas in momentum representation. Since the gradient terms of the GL theory are not essential to our discussion, we refer the reader interested in details of the gradient expansion to reference [36] in which the gap equation expressed in terms of the T-matrix is analyzed in the space-time representation under non-equilibrium conditions.

Gor'kov has evaluated the GL coefficients (25)–(27) using the quasi-particle approximation of the normal-state propagator $G^{-1}_{\not{C}\downarrow}(k) \approx \omega - \epsilon(\mathbf{k})$. For the parabolic approximation of the energy dispersion $\epsilon(\mathbf{k}) = \hbar^2|\mathbf{k}|^2/2m$ he obtained $m^* = 2m$,

$$\beta = \frac{3}{2E_{\mathrm{F}}},\ \alpha = -\frac{6\pi^2k_{\mathrm{B}}^2T_c}{7\zeta_{[3]}E_{\mathrm{F}}}(T_c - T),\ \chi = \frac{8\pi^2k_{\mathrm{B}}^2T_c^2}{7\zeta_{[3]}n} \tag{28}$$

where E_{F} is the Fermi energy and $\zeta_{[3]} = 1.202$ is the Riemann zeta function. The norm χ cancels in the GL equation, with n being the electron density.

We restrict our attention to the vicinity of the critical temperature. The limiting form (24) will be thus sufficient to our discussion.

5 Stability of the supercurrent

5.1 Exclusion of parallel condensation

Now we are ready to solve the central problem of this paper. First we show that once the condensate is formed in the **C**-mode, a parallel condensation in another **Q**-mode is excluded. Second we show that the critical velocity for breaking a condensed Cooper pair into two quasiparticles is lower than the critical velocity of excitation of Cooper pairs into bound states out of the condensate.

These results apply only to the T-matrix with multiple scattering corrections. We will show that the Galitskii T-matrix and Kadanoff-Martin theory lead to unphysical zero critical velocity of the excitations of Cooper pairs out of the condensate.

5.1.1 Multiple-scattering corrected T-matrix

Assuming a single condensate, we have in fact assumed that none of the **Q**-terms of the T-matrix diverges with the volume for $\mathbf{Q} \neq \mathbf{C}$. We will show that if the **Q**-term was not divergent in the beginning, the presence of the condensate in the **C**-mode eliminates its chances to become singular.

For the regular mode the **Q**-reduced propagator approaches the full one except for terms $\propto L^{-3}$,

$$G_{\not{\mathbf{Q}}\uparrow} = G_{\uparrow}. \tag{29}$$

The zero frequency component of the T-matrix of the **Q**-mode thus satisfies the equation

$$\mathcal{T}_{0,\mathbf{Q}} = V - V\frac{k_{\rm B}T}{L^3}\sum_k G_{\uparrow}(\omega,\mathbf{k})G_{\downarrow}(-\omega,\mathbf{Q}-\mathbf{k})\mathcal{T}_{0,\mathbf{Q}}. \tag{30}$$

It is more convenient to write it in the inverse form

$$\frac{1}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{1}{V} + \frac{k_{\rm B}T}{L^3}\sum_k G_{\uparrow}(\omega,\mathbf{k})G_{\downarrow}(-\omega,\mathbf{Q}-\mathbf{k}). \tag{31}$$

Near the critical temperature it is sufficient to keep terms quadratic in Δ and **Q**. From the expansion (23) we then find

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*} + \alpha + 2\beta|\Delta|^2. \tag{32}$$

The factor of two in front of β follows from the fact that for non-condensed pairs both propagators depend on the gap.

It would be interesting to compare this two-particle excitation spectrum with other approaches based on the T-matrix [21,24–31]. Unfortunately, to our knowledge, the energy spectrum of bounded two-particle excitations in the presence of the gap has not been published. This is in part due to the focus of these studies on the d-symmetry of the gap and its roots in the spin-channel of the T-matrix.

The values of Δ are given by the inverse T-matrix in the condensation mode

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^*} + \alpha + \beta|\Delta|^2 = 0 \tag{33}$$

which equals zero because of the divergence of $\mathcal{T}_{0,\mathbf{C}}$. This equation is identical to the gap equation in the GL approximation (24).

When equation (33) holds, the inverse T-matrix of the **Q**-mode remains non-zero. To see this, let us substitute the gap from equation (33) into the inverse T-matrix (32) to get:

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^*} - \alpha - \frac{|\mathbf{C}|^2}{m^*}. \tag{34}$$

The right hand side can reach zero only if $|\mathbf{C}|^2$ is sufficiently large to compensate $-\alpha$.

Values of the pair momentum **C** are limited by the critical current, $|\mathbf{C}|^2 < Q_c^2$. The current is proportional to the square of the gap times the momentum, $\mathbf{j} \propto \mathbf{C}|\Delta|^2$. Using equation (24) one finds $\mathbf{j} \propto \mathbf{C}\left(-\alpha - |\mathbf{C}|^2/2m^*\right)$. The critical current is obtained as the maximum one, $\partial\mathbf{j}/\partial\mathbf{C}|_{C=Q_c} = 0$, for $Q_c^2 = 2m^*|\alpha|/3$ (see Ref. [37]). Accordingly

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} > \frac{|\mathbf{Q}|^2}{2m^*} - \alpha - \frac{Q_c^2}{m^*} = \frac{|\mathbf{Q}|^2}{2m^*} - \frac{\alpha}{3}. \tag{35}$$

Since α is negative below the critical temperature, inequality (35) implies that the T-matrix in the **Q**-mode remains finite. This mode thus cannot become singular once the condensation develops in the **C**-mode. Therefore, a parallel condensation in two competitive modes is excluded. Briefly, there is only a single condensate, as it is tacitly assumed in the BCS theory.

5.1.2 Galitskii T-matrix

By omitting the multiple-scattering corrections the present theory simplifies to the Galitskii T-matrix. This is achieved by approximating $G_{\not{\mathbf{C}}\downarrow} \approx G_{\downarrow}$ in equations (25)–(27) with the help of which one can derive GL parameters. The inverse Galitskii T-matrix in the condensation mode thus reads

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^{*\rm Gal}} + \alpha^{\rm Gal} + 2\frac{k_{\rm B}T}{L^3}\beta^{\rm Gal}\mathcal{T}_{0,\mathbf{C}} \to 0 \tag{36}$$

with

$$\alpha^{\rm Gal} = \chi + \chi V\frac{k_{\rm B}T}{L^3}\sum_k G_{\uparrow}(\omega,\mathbf{k})G_{\downarrow}(-\omega,-\mathbf{k}), \tag{37}$$

$$\beta^{\rm Gal} = -\chi V\frac{k_{\rm B}T}{L^3}\sum_k G_{\uparrow}^2(\omega,\mathbf{k})G_{\downarrow}^2(-\omega,-\mathbf{k}) \tag{38}$$

and

$$\frac{\hbar^2}{2m^{*\rm Gal}} = \chi V\frac{\partial^2}{\partial\mathbf{C}^2}\frac{k_{\rm B}T}{L^3}\sum_k G_{\uparrow}(\omega,\mathbf{k})G_{\downarrow}(-\omega,\mathbf{C}-\mathbf{k})\bigg|_{\mathbf{C}=0}. \tag{39}$$

Coefficients (38) and (39) are evaluated at the critical temperature, $T = T_c$, while the linear dependence of $\alpha^{\rm Gal}$ on $T_c - T$ has to be kept. For these temperatures the GL coefficients are given by normal-state Green functions.

One can see that $\mathcal{T}_{0,\mathbf{C}}$ diverges as L^3 so that the left hand side of (36) goes to zero as indicated by the limit $\to 0$. Due to the approximation $G_{\mathbf{C}\downarrow} \approx G_\downarrow$ in the selfenergy loop (15), the divergence of the T-matrix does not result in the BCS gap (see Ref. [12]). Apparently, the divergence of the T-matrix is a necessary but not sufficient condition for the BCS gap.

Out of the condensation mode, $\mathbf{Q} \neq \mathbf{C}$, the Galitskii T-matrix is also constructed from the full Green functions, therefore

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\mathrm{Gal}}} + \alpha^{\mathrm{Gal}} + 2\frac{k_{\mathrm{B}}T}{L^3}\beta^{\mathrm{Gal}}\mathcal{T}_{0,\mathbf{C}}. \tag{40}$$

Subtracting equation (36) we find

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\mathrm{Gal}}} - \frac{|\mathbf{C}|^2}{2m^{*\mathrm{Gal}}}. \tag{41}$$

In contrast to the result for the multiple-scattering corrected T-matrix (34), the dispersion (41) supports a nucleation of the second condensate at the energy minimum $\mathbf{Q} = 0$ in the presence of the persistent current $\mathbf{C} \neq 0$.

5.1.3 Kadanoff-Martin theory

The analysis of the Kadanoff-Martin theory is very similar. This approximation is obtained from the present one using $G_{\mathbf{Q}\downarrow} \approx G^0$. The inverse T-matrix thus reads

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{C}}} = \frac{|\mathbf{C}|^2}{2m^{*\mathrm{KM}}} + \alpha^{\mathrm{KM}} + \frac{k_{\mathrm{B}}T}{L^3}\beta^{\mathrm{KM}}\mathcal{T}_{0,\mathbf{C}} \to 0 \tag{42}$$

with

$$\alpha^{\mathrm{KM}} = \chi + \chi V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_\uparrow(\omega,\mathbf{k})G^0(-\omega,-\mathbf{k}), \tag{43}$$

$$\beta^{\mathrm{KM}} = -\chi V\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_\uparrow^2(\omega,\mathbf{k}){G^0}^2(-\omega,-\mathbf{k}) \tag{44}$$

and

$$\frac{\hbar^2}{2m^{*\mathrm{KM}}} = \chi V\frac{\partial^2}{\partial\mathbf{C}^2}\frac{k_{\mathrm{B}}T}{L^3}\sum_k G_\uparrow(\omega,\mathbf{k})G^0(-\omega,\mathbf{C}-\mathbf{k})\bigg|_{\mathbf{C}=0}. \tag{45}$$

Out of the condensation mode we find

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\mathrm{KM}}} + \alpha^{\mathrm{KM}} + \frac{k_{\mathrm{B}}T}{L^3}\beta^{\mathrm{KM}}\mathcal{T}_{0,\mathbf{C}}, \tag{46}$$

therefore

$$\frac{\chi}{\mathcal{T}_{0,\mathbf{Q}}} = \frac{|\mathbf{Q}|^2}{2m^{*\mathrm{KM}}} - \frac{|\mathbf{C}|^2}{2m^{*\mathrm{KM}}}. \tag{47}$$

Again, like the Galitskii T-matrix (41) this dispersion supports a nucleation of the second condensate at the energy minimum $\mathbf{Q} = 0$ in the presence of the persistent current $\mathbf{C} \neq 0$.

To summarize we have found that only the T-matrix with multiple-scattering corrections excludes a condensation in a second mode which explains the single-valued condensate tacitly assumed in the BCS theory. In contrast, both the Thouless-like theory based on the Galitskii T-matrix and the Kadanoff-Martin theory allow for a condensation in two (or more) modes, which is in conflict with experimental findings.

5.2 Excitation of Cooper pairs from the condensate

Now we discuss the possibility to excite a Cooper pair out of the condensate by an object moving with velocity $\mathbf{v}$ in the static condensate. Going into the running coordinate system, this criterion is used to check the stability of the condensate flowing with velocity $-\mathbf{v}$ around a static obstacle.

5.2.1 Multiple-scattering corrected T-matrix

The right hand side of equation (32) represents the energy of a non-condensed pair of momentum $\mathbf{Q}$. In the static condensate, $\mathbf{C} = \mathbf{0}$, the gap is given by equation (33) as $|\Delta|^2 = -\alpha/\beta$. From the energy dispersion (32) one finds that a Cooper pair can be excited from the condensate into a non-condensed state with the energy cost

$$\mathcal{E}_{\mathbf{Q}} = |\mathbf{Q}|^2/2m^* + \alpha + 2\beta|\Delta|^2 = |\mathbf{Q}|^2/2m^* - \alpha.$$

Let us estimate under which conditions Cooper pairs can be excited by an external perturbation.

According to the Landau criterion [38] the external perturbation moving with velocity $\mathbf{v}$ can excite the Cooper pair of momentum $\mathbf{Q}$ if the Cherenkov-type condition $(\mathbf{vQ}) = \mathcal{E}_{\mathbf{Q}}$ is satisfied, i.e.,

$$(\mathbf{vQ}) = \frac{|\mathbf{Q}|^2}{2m^*} - \alpha. \tag{48}$$

This equation is solved by real $\mathbf{Q}$ only if

$$|\mathbf{v}| > v_{\mathrm{pe}} = \sqrt{\frac{2|\alpha|}{m^*}}. \tag{49}$$

The velocity v_{pe} is the critical velocity for excitation of the Cooper pair from the condensate into a bound pair out of the condensate.

Let us compare the critical velocity of the pair excitation with the critical velocity for the pair breaking $v_{\mathrm{pb}} = \Delta/k_{\mathrm{F}}$, where k_{F} is the Fermi momentum [6]. To this end we use GL coefficients derived by Gor'kov. From equation (24) follows $\Delta = \sqrt{|\alpha|/\beta} = \sqrt{|\alpha|k_{\mathrm{F}}^2/(3m)}$ so that

$$v_{\mathrm{pe}} = \sqrt{3}v_{\mathrm{pb}}. \tag{50}$$

Since the critical velocity of the pair breaking is lower than the critical velocity of pair excitation, the stability of the condensate is controlled by the pair breaking.

Although our analyses are justified only for clean superconductors, it is interesting to notice that using the above estimates for dirty superconductors with $m^* > 6m$ one finds that the pair excitation might be easier than the pair breaking, $v_{\mathrm{pe}} < v_{\mathrm{pb}}$. The dirty superconductors, however, are of type II and their observed critical currents are much lower due to the motion of Abrikosov vortices [6].

We have discussed only the vicinity of the critical temperature. In conventional superconductors, the extended Ginzburg-Landau formalism [34] shows a monotonic dependence of the GL potential on $|\Delta|^2$ at any temperature and the generalized kinetic energy of de-Gennes [35] is a monotonic digamma function of $|\mathbf{Q}|^{\mathbf{2}}$, therefore we can expect that the above analysis remains qualitatively correct also at lower temperatures.

5.2.2 Galitskii T-matrix and Kadanoff-Martin theory

Within the Galitskii approximation, for the system at rest, $\mathbf{C} = 0$, the Cooper pairs can be excited out of the condensate by any small velocity since the Cherenkov-type condition corresponding to dispersion (41)

$$(\mathbf{vQ}) = \frac{\mathbf{Q}^2}{2m^{*\mathrm{Gal}}}, \tag{51}$$

can be satisfied for any small velocity $\mathbf{v}$. The Galitskii T-matrix thus yields zero critical velocity from the Landau criterion (along with the zero critical velocity from the pair breaking).

For the Kadanoff-Martin approximation and the system at rest, $\mathbf{C} = 0$, the Cooper pairs can also be excited out of the condensate by any small velocity since the Cherenkov-type condition corresponding to dispersion (47),

$$(\mathbf{vQ}) = \frac{|\mathbf{Q}|^2}{2m^{*\mathrm{KM}}}, \tag{52}$$

can be satisfied for any small velocity $\mathbf{v}$. We have thus recovered the result of Chen et al. [22] that the Kadanoff-Martin theory fails to justify superconductivity providing zero critical velocity from the Landau criterion.

6 Conclusions and discussion

We have shown that the T-matrix approach can be used to justify two basic assumptions of the BCS theory: First, the condensate is single-valued, i.e., new condensate cannot nucleate even if the present condensate is driven out of the total energy minima and the new condensate would be thermodynamically favorable. Second, excitations of bound electron pairs can be neglected since the critical velocity of their Cherenkov-type generation is higher than the critical velocity of pair breaking.

These conclusions are not general for all T-matrix approaches. In fact, among three discussed approximations only the T-matrix with multiple-scattering corrections provides this result while the Galitskii T-matrix and the Kadanoff-Martin theory result in zero critical velocity of the Cherenkov-type generation of bounded excited pairs.

In the discussion we have assumed conventional metals and focused on the critical line, where one can benefit from the Ginzburg-Landau type limit with the Gor'kov expansion around large Fermi energy. We have found that for the Galitskii T-matrix and the Kadanoff-Martin theory the energy spectrum for the motion of the condensate as whole, i.e., the Ginzburg-Landau equation, is the same as the energy spectrum of excitations out of the condensate. This implies the possibility to nucleate a new condensate and a zero critical velocity for Cherenkov-type processes. Only for the T-matrix with multiple-scattering corrections these two energy spectra differ by the factor of two in the non-linear term (compare β-term in Eqs. (24) and (32)), what inhibits any new nucleation and gives the finite critical velocity of Cherenkov-type processes.

The factor of two in the non-linear term can be also interpreted in another manner. The non-condensed pairs feel the gap due to the condensate twice stronger than it is felt by Cooper pairs in the condensate. This reminds the factor of two by which the bosons out of the Bose-Einstein condensate interact stronger with the condensate than the condensed bosons interact among themselves (see Chap. 2.3 of Leggett [39]). In this sense the T-matrix with the multiple-scattering corrections seems to be the simplest theory suited to bridge the BCS and boson picture of the superconductivity.

The gap in the energy spectrum of non-condensed bounded pairs was unexpected. In parallel with the bosonic excitation spectrum we have expected an acoustic spectrum at low energies. In the theory of interacting true bosons the gap is known to result as an ill feature of some approximations [40]. Since the T-matrix does not deal with the interaction of bounded electron pairs – it results only as a byproduct of the selfconsistency – it is possible that this gap disappears in more advanced approximations. On the other hand, the acoustic dispersion appears in the BEC limit, while little is known about bounded pairs in the BCS regime.

In the present paper we have focused on the weak coupling limit applying the multiple scattering corrections to the electron propagation. In the strong coupling limit, the multiple scattering corrections have to be implemented to the composed Boson line. This leads to the expected acoustic spectrum at low energies (see [41,42]).

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