ELSEVIER

Nuclear Instruments and Methods in Physics Research A 441 (2000) 40–43

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

www.elsevier.nl/locate/nima

The influence of electron–electron collisions on the stopping power within dielectric theory

A. Selchow*, G. Röpke, K. Morawetz

Department of Physics, University of Rostock, Universitätsplatz 3, D-18051 Rostock, Germany

Abstract

A possible approach to describe electron cooling is the dielectric theory. For the case of a (longitudinal) cold electron beam the contribution of electron–electron collisions is considered. Within a generalized linear response theory (Zubarev-approach) we have derived a generalized dielectric function (DF) including collisions. We discuss the importance of collisions in the dielectric function on different stages of approximation and their influence on ion stopping in an electron gas. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 52.25.Mq; 29.20.Dh; 71.45.Gm

Keywords: Dielectric function; Storage rings; Electron gas

Superimposing a cold electron beam on a stored ion beam with a broader thermal distribution leads to drag or brake forces onto these ions and hence to a narrowing (cooling) of the ion distribution. Besides the binary collision approach [1,2] an alternative theory for modeling cooling is based on the dielectric theory describing the forces on an ion by exciting or absorbing plasmons [3]. Following this way, we have to consider the polarizability of the cooling electron gas.

The polarizability $\Pi(k,\omega)$ is expressed by the DF by means of $\varepsilon(k,\omega) = 1 - V(k)\Pi(k,\omega)$, where $V(k)$ is the Coulomb potential. The general connection between DF and stopping power on an ion with mass m_i and velocity v is given by [4]

$$\frac{\mathrm{d}E}{\mathrm{d}s} = \frac{2e}{\pi\varepsilon_0}\frac{1}{v(t)^2}\int_0^\infty \frac{\mathrm{d}k}{k} \times \int_{-v(t)k+\hbar k^2/2m_i}^{v(t)k+\hbar k^2/2m_i} \mathrm{d}\omega\, \omega n_{\mathrm{B}}(\omega)\, \mathrm{Im}\,\varepsilon^{-1}(k,\omega), \qquad (1)$$

representing the stopping power as an integral over the response function $\mathrm{Im}\,\varepsilon^{-1}(\vec{k},\omega)$ and the bosonic plasmon distribution $n_{\mathrm{B}}(\omega)$.

The main objective is to discuss the DF occurring in Eq. (1) which describes the behavior of the electron gas in the cooler. In the present paper we restrict our investigations to the effect of electron–electron collisions on the stopping power. The relevant parameter for the nonideality is given by

$$\Gamma = \frac{e^2}{4\pi\varepsilon_0 k_{\mathrm{B}} T}\left(\frac{4\pi n}{3}\right)^{1/3}. \qquad (2)$$

* Corresponding author. Tel.: + 49-0-381-498-2858; fax: + 49-0-381-498-2857.

E-mail address: selchow@darss.mpg.uni-rostock.de (A. Selchow)

0168-9002/00/$ - see front matter © 2000 Elsevier Science B.V. All rights reserved.
PII: S0168-9002(99)01105-5

Under the condition $\Gamma \ll 1$ collisions can be neglected leading to the Random Phase Approximation (RPA) [5]

$$\Pi^{\mathrm{RPA}}(\vec{k}, z) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 p \frac{f_{p+k/2} - f_{p-k/2}}{E_{p+k/2} - E_{p-k/2} - \hbar z} \quad (3)$$

with $z = \omega + \mathrm{i}0$. Results for Eq. (1) will be discussed below.

Employing experimental values for the longitudinal temperature of the electron beam (TSR: $T_{\|} = 0.06$ meV, $n = 2.8 \times 10^{13}$ m^{-3} [6], ESR: $T_{\|} = 0.1$ meV, $n = 1.0 \times 10^{12}$ m^{-3} [7]) the nonideality parameter Γ amounts to 0.66 resp. 0.23. Thus, considering the longitudinal direction, in some experiments the electron plasma is intermediately strongly coupled. The RPA is not sufficient for $\Gamma \sim 1$ and collisions have to be taken into account.

In the following, we are interested in the derivation of a DF including collisions. Other conditions unique to the situation in an electron cooler just as the anisotropy (expressed in a flattened electron distribution) and the confining magnetic field are neglected here.

In an earlier work [8] we have calculated the stopping power making use of the Mermin DF $\varepsilon^{\mathrm{M}}(\vec{k},\omega)$ [9] which accounts for collisions and respects particle conservation inside a plasma

$$\varepsilon^{\mathrm{M}}(k,\omega) = 1 + \frac{e^2}{\varepsilon_0 k^2} \frac{\Pi^{\mathrm{RPA}}(k,\omega + \mathrm{i}/\tau)}{1 - \frac{1}{1 - \mathrm{i}\omega\tau}\left[1 - \frac{\Pi^{\mathrm{RPA}}(k,\omega + \mathrm{i}/\tau)}{\Pi^{\mathrm{RPA}}(k,0)}\right]}. \quad (4)$$

The RPA $\Pi^{\mathrm{RPA}}(\vec{k}, z)$ appears now with a complex frequency $z = \omega + \mathrm{i}/\tau$ characterizing collisions. The parameter τ is not derived within the Mermin approach. Here τ is chosen as relaxation time which is given for a classical and nondegenerated plasma by the Faber–Ziman formula

$$\tau = \frac{(4\pi\varepsilon_0)^2}{e^4} \frac{(k_{\mathrm{B}} T)^{3/2} m_{\mathrm{e}}^{1/2}}{n} \frac{3}{4(2\pi)^{1/2}} \times \left(\int_0^\infty \mathrm{d}p\, p \exp[-p^2] \left(\ln\frac{\lambda - 1}{\lambda + 1} + \frac{2}{\lambda + 1} \right) \right)^{-1} \quad (5)$$

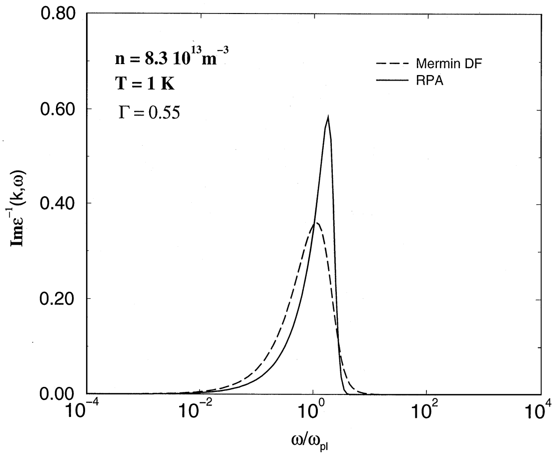

Fig. 1. Response functions for an electron gas in RPA and Mermin approximation. ω_{pl} denotes the plasma frequency.

$\lambda = (\hbar^2\kappa^2)/(4m_{\mathrm{e}}k_{\mathrm{B}}Tp^2) + 1$, κ denotes the inverse Debye length, m_{e} is the electron mass.

In Fig. 1 the response functions $\mathrm{Im}\,\varepsilon^{-1}(\vec{k},\omega)$ within RPA and Mermin theory are compared for a set of typical density and temperature parameters. Obviously, taking into account collisions leads to a strong width broadening and a shift to lower frequencies of the plasmon.

Results for the stopping power, Eq. (1), using the Mermin DF are compared in Fig. 2 with the pure RPA result, T-matrix calculations [4], and simulations [10], which reproduce the experimental data already quite well. A first conclusion is that the T-matrix calculations do much better agree with the simulations than the RPA results. The dielectric theory is improved if collisions are considered, but the changes due to the use of the Mermin function are small. To analyze the question of collisions we improve the Mermin DF and investigate the change in the response function $\mathrm{Im}\,\varepsilon^{-1}(k,\omega)$.

As pointed out above the Mermin theory considers only particle number conservation. The main question to be discussed here is what influence invoking further conservation laws has and how to derive the Mermin DF in a more systematic way in the context of nonequilibrium theory.

This extended (or generalized) Mermin DF is given by a systematic treatment using an ansatz for the nonequilibrium statistical operator (Zubarev theory) [11]. The idea is to characterize the

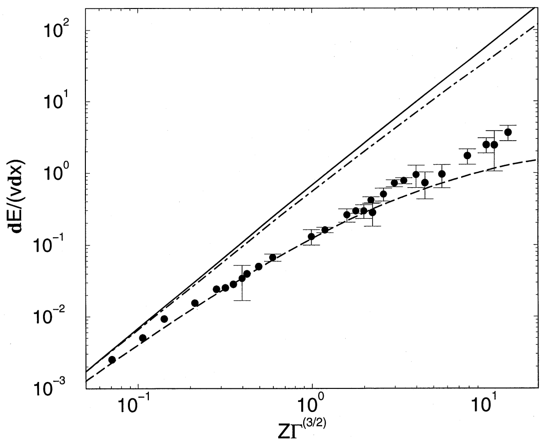

Fig. 2. Comparison between simulations (circles with error bars) and calculations for the stopping power in RPA (solid line), Mermin (dot-dashed line) and T-matrix approximation (dashed line).

nonequilibrium situation using a generalized Gibbs ensemble including relevant observables and respective Lagrange parameters. Maximizing the entropy leads to a new statistical operator $\rho_{\mathrm{rel}}(t)$ called the relevant one. For the present problem, we have chosen the local density $n(\vec{r})$ and local kinetic energy $E(\vec{r})$ as relevant observables respecting particle and energy conservation locally. The balance equations for $n(\vec{r})$ and $E(\vec{r})$ are fulfilled.

A basic demand on the relevant observables within this framework is the validity of the self-consistency condition. This means that the time-dependent averages of the relevant observables are always depicted by the relevant statistical operator. Expanding $\rho_{\mathrm{rel}}(t)$ up to first order in the external field one obtains from this condition the time-dependent average of any quantity. According to the definition of the polarizability we have to determine the averaged-induced density $\langle \delta n(\vec{r}) \rangle^{t}$ which can now be expressed in terms of equilibrium correlation functions (Kubo products).

Assigned to the relevant observables are appropriate Lagrange parameters (response parameters) which are identifiable as shifts in inverse temperature and chemical potential. They are ensuring the conservation laws locally as mentioned above. The response parameters are determined by a system of linear response equations. Eliminating them by solving this set the induced density can be figured out. For detailed calculations we refer to Refs. [12,13].

We present here only the result for the generalized Mermin DF including energy conservation

$$\varepsilon^{\mathrm{M2}}(k,\omega) = 1 - \frac{e^2}{(\varepsilon_0 k^2)} \frac{N}{D}, \tag{6}$$

$$\begin{aligned} N = {} & 4\beta^2 k^4 m n^2 (\eta - \mathrm{i}\omega)(\eta \Pi_0 - \mathrm{i}\omega \Pi) \\ & - 16k^4 m(\eta - \mathrm{i}\omega)(\eta\Pi - \mathrm{i}\omega\Pi_0)\Pi\Pi_0 \\ & + n\beta k^2(\eta - \mathrm{i}\omega)^2(\beta\hbar^2 k^4 + 24k^2 m \\ & + 12\beta m^2 \eta(\eta - \mathrm{i}\omega))\Pi\Pi_0 \\ & - 4\beta^2\hbar^2 k^4 m\eta(\eta - \mathrm{i}\omega)^3 \Pi^2 \Pi_0, \\ D = {} & - 16mk^4(\eta\Pi - \mathrm{i}\omega\Pi_0)^2 \\ & - 4m\beta^2(\eta - \mathrm{i}\omega)^2 \eta\Pi(\hbar^2 k^4(\eta\Pi - \mathrm{i}\omega\Pi_0) \\ & - m^2(\eta - \mathrm{i}\omega)^2 \mathrm{i}\omega\Pi_0) \\ & + n\beta k^2(\eta - \mathrm{i}\omega)(\beta\hbar^2 k^4 + 24k^2 m)(\eta\Pi - \mathrm{i}\omega\Pi_0) \\ & + 4n\beta^2 m^2 k^2(\eta - \mathrm{i}\omega)^2((\eta + 2(\eta - \mathrm{i}\omega))\eta\Pi \\ & - \eta(\eta - \mathrm{i}\omega)\mathrm{i}\omega\Pi_0) + 4n^2\beta^2 m k^4(\eta - \mathrm{i}\omega)^2. \end{aligned}$$

The abbreviation Π denotes $\Pi^{\mathrm{RPA}}(\vec{k},z)$ and Π_0 is the static limit $\Pi^{\mathrm{RPA}}(\vec{k},0)$, $z = \omega + \mathrm{i}\eta$, $\eta = 1/\tau$.

Comparing the response function of the generalized DF and the Mermin-DF under various conditions we found that the differences between them are in the bound of few percent (see Fig. 3, same conditions as Fig. 1). Also, it has to be mentioned that the generalized DF as well as the Mermin-DF fulfills basic requirements such as sum rules and classical limits (e.g. static screening).[1]

Generally speaking, our theory makes a systematic approach to the DF possible. Using as relevant observable only density and taking into consideration only particle conservation the Mermin DF is recovered [15].

Since the differences between the generalized DF and the Mermin DF are small the collisions are already well considered employing the Mermin DF. Hence no essential modification of the

[1] The DF derived by using a Fokker–Planck collision integral in [14] is also applicable to the cooling electrons. The results are up to few percents close to the Mermin-DF.

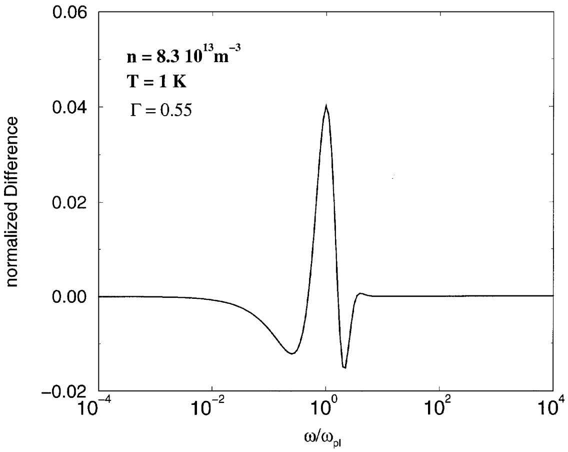

Fig. 3. Differences between the response functions for an electron gas in Mermin and generalized Mermin approximation in percent. ω_{pl} denotes the plasma frequency.

stopping power is expected, compared with the result using the Mermin DF. Thus other phenomena should be considered. First, electron–ion collisions has to be included in the relaxation time τ, where in contrast to the Born approximation strong collisions should be taken into account. Furthermore, Eq. (1) describes the coupling to the plasma degrees of freedom only in Born approximation and has to be generalized accounting also for strong collisions.

References

[1] Ya.S. Derbenev, A.N. Skrinsky, Part. Acc. 8 (1977) 1.
[2] Ya.S. Derbenev, A.N. Skrinsky, Part. Acc. 8 (1978) 235.
[3] H. Poth, Phys. Rep. 196 (1990) 135.
[4] K. Morawetz, G. Röpke, Phys. Rev. E 54 (1996) 4134.
[5] J. Lindhard, Dan. Mat. Fys. Medd. 28 (8) (1954) 1.
[6] M. Beutelspacher, private communication.
[7] T. Winkler et al., Hyperfine Interact. 99 (1996) 277.
[8] A. Selchow, K. Morawetz, Phys. Rev. E 59 (1999) 1015.
[9] N.D. Mermin, Phys. Rev. B 1 (1970) 2362.
[10] G. Zwicknagel, Q. Spreiter, C. Toepffer, Hyperfine Interact. 108 (1997) 131.
[11] D.N. Zubarev, V. Morozov, G. Röpke, Statistical Mechanics of Nonequilibrium Processes, Akademie, Berlin, 1997.
[12] K. Morawetz, U. Fuhrmann, Phys. Rev. E, in press.
[13] A. Selchow, G. Röpke, Mermin-like dielectric function including energy conservation, in preparation.
[14] V. Parkhomchuk, D. Pestrikov, Sov. Phys. Tech. Phys. 25 (1980) 818.
[15] G. Röpke, A. Selchow, A. Wierling, H. Reinholz, Phys. Lett. A, 260 (1999) 365.