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In-medium relativistic kinetic theory and nucleon-meson systems

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Abstract. Within the $\sigma - \omega$ model of coupled nucleon-meson systems, a generalized relativistic Lennard-Balescu-equation is presented resulting from a relativistic random phase approximation (RRPA). This provides a systematic derivation of relativistic transport equations in the frame of nonequilibrium Green's function technique including medium effects as well as fluctuation effects. It contains all possible processes due to one-meson exchange and special attention is kept to the off-shell character of the particles. As a new feature of many-particle effects, processes are possible, which can be interpreted as particle creation and annihilation due to in-medium one-meson exchange. In-medium cross sections are obtained from the generalized derivation of collision integrals, which possess complete crossing symmetries.

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1. Introduction

Since Walecka [1, 2] has established his model of meson exchange already 20 years ago, field theoretical models were successfully applied for the description of heavy-ion collisions at intermediate energies [3–6]. Thereby the derivation of transport equations starting from a microscopic model was paid great attention [7–9]. A number of publications are devoted to the description of relativistic many-body theory of high density matter [10, 11].

The most powerful and appropriate method to describe quantum many particle systems under extreme conditions like nuclear matter or condensed stellar objects is the real-time-Green functions technique. Many excellent reviews are published concerning this method during the past decade [7, 8, 12–14]. Moreover there are many investigations in classical relativistic treatment [15, 16] and quantum relativistic treatments can be found in [17–19]. Several papers are devoted the formulation of transport equations in quantum field theory [7, 9, 20, 21].

In this paper we like to present briefly a derivation of relativistic transport equations for the $\sigma - \omega$ model. The formal development of relativistic transport models is well known and documented in the literature [22, 23]. Recently the relativistic kinetic equation are derived for spin-polarized nucleons [24]. The authors give the derivation of the selfenergies to the meanfield approximation and derive collision integrals including the quasi-particle energies on the meanfield level. Here in this paper we restrict to unpolarized nucleons interacting via a one-boson exchange and show how collision integrals can be derived on a consistent level with the used many particle approximation condensed in the quasi-particle energy. Special attention is kept to the description of fluctuations and the influence of medium effects on the kinetic equation. This demands an infinite sum of diagrams, which we have chosen as RPA like. Contrary, perturbative expansions due to the large coupling constants is not justified. Due to this reason we use the standard partial summation describing large scale excitations. The other possibility of partial summations in the s-channel will lead to the T-matrix approximation describing short range correlations.

Additional reaction channels are opened by the influence of many particle effects, which would be forbidden in uncorrelated systems. Further, we derive a decoupling between nucleon and meson-equations, which results in an effective squared one-meson exchange potential containing no mixed terms of meson contributions. This is established by the use of generalized optical theorems.

The outline of the paper is the following. In Chap. 2 we review the fundamental equations of the Yukawa-Lagrangian and introduce the nonequilibrium Green's function technique based on the very fundamental principle of weakening of initial correlation. The structure of equations are developed in the Schwinger formalism, generalized to the set of four nonequilibrium Green's functions as it was similar done by Bezzerides [25] and recently by Davis [23]. The spectral information and therefore the quasiparticle properties are discussed in Chap. 3 which establish a quite natural generalization of Bruckner theory by the aspect of complete particle–antiparticle symmetry of interacting quasiparticles [7].

To set up a consistent kinetic theory an equation for the Wigner distribution function should be available, which includes all features of relativistic quasiparticle behaviour like Pauli-blocking, screening and relativistic effects, such as scattering between particles and anti-particles and pair creation and destruction processes. The consequent treatment of many particle effects is adopted in Sect. 4 as it was done in the nonrelativistic case [26]. The generalization of collision integrals includes the effect of density fluctuations by an dynamical meson exchange potential and describes completely the particle-antiparticle symmetry. Particularly, it contains Pauli blocking and inelastic processes by the medium. A dynamical potential enters the equation due to density fluctuations and the equation can be considered therefore as a generalization of quantum-mechanical Boltzmann equation [27] to a Lennard-Balescu-type one. This is the main result of the present paper and should be the starting point to kinetic description of hot nuclear matter instead of the relativistic Vlassow treatment [28]. Similar equations have been obtained in T-matrix approximation but without pair creation and dynamically screening by De Boer [29], De Groot [16], Botermans [7] and Malfliet [30].

From the found collision integrals we derive in-medium cross-sections, which differ from ordinary Born approximation by two facts. Firstly, no mixed coupling terms occur between different kinds of meson contributions. This is due to the more general decoupling on the level of self energies instead of the decoupling normally used. Secondly, the medium effects are represented in the cross section by an energy shift from the vector-meson exchange and the renormalized effective mass by the scalar-meson contribution.

II. Model and basic equations

The Yukawa Lagrangian, which couples scalar and vector meson fields to fermion fields describes the simplest form of a model for nuclear matter [2]

$$\begin{aligned} L = & -\bar{\Psi}(-i\gamma^{\mu}\partial_{\mu} + \kappa_0)\Psi \\ & + \tfrac{1}{2}\partial_{\mu}\Phi\partial^{\mu}\Phi - \tfrac{1}{2}m_{s0}^2\Phi^2 + :g_s\bar{\Psi}\Phi\Psi: \\ & - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{v0}^2\varphi_{\lambda}\varphi^{\lambda} + :g_v\bar{\Psi}\gamma_{\lambda}\Psi\varphi^{\lambda}:. \end{aligned} \tag{1}$$

Here m_{s0}, m_{v0} are the bare scalar and vector meson mass and κ_0 is the bare nucleon one respectively. Further, :: denotes the normal ordering procedure to ensure the renormalization of baryon density [31]. After renormalization of the masses the coupling constants have to be chosen in such a way, that on one hand, the equilibrium ground state properties can be fitted, and on the other hand, the scattering data can be reproduced [5, 32]. Because of the renormalizability of the model Lagrangian we can choose for the occurring divergent vacuum terms a standard procedure resulting in physical masses. For our used selfconsistent Hartree-Fock and RPA approximation this is well known in literature [31]. In the following we use therefore already the renormalized fields and masses, which may be considered as an effective model.

From (1) the equations of motion read

$$(\Box + m_s^2)\Phi = g_s : \bar{\Psi}\Psi : \equiv j_{\Psi} \tag{2}$$

$$(\Box + m_v^2)\varphi_v = g_v : \bar{\Psi}\gamma_v\Psi : \equiv j_{\Psi v} \tag{3}$$

$$(i\gamma^{\mu}\partial_{\mu} - \kappa)\Psi = g_s : \Psi\Phi : - g_v : \gamma^{\lambda}\varphi_{\lambda}\Psi : \equiv j_{\varphi}. \tag{4}$$

With the help of the (mathematical) Green's function G_o^s, G_0^v for the free meson equation (2, 3) the mesonic degree of freedom in Eq. (4) can be eliminated [7, 17, 10]

$$\begin{aligned} (i\gamma^{\mu}\partial_{\mu} - \kappa)\Psi_{\alpha} = & [g_s^2\delta_{\alpha\beta}\delta_{\gamma\delta}G_o^s - g_v^2(\gamma_{\mu})_{\alpha\beta}(G_0^v)^{\mu\rho}(\gamma_{\rho})_{\gamma\delta}] \\ & \times \bar{\Psi}_{\gamma}\Psi_{\delta}\Psi_{\beta} \end{aligned} \tag{5}$$

Here $\alpha, \beta, \gamma, \delta$ indicates the spinor indices. Recalling that $G_o^s = (-k^2 + m_s^2)^{-1}$ one may construct from (5) a 4-vector potential, which has the structure $U_s - U_v$. Therefore in any perturbation expression, where the potential enters squared, one obtains mixed coupling terms between the vector and scalar mesons. This usually used decoupling overlooks the problem of *meson initial correlations,* which here is assumed to be zero when the mesonic degrees are eliminated by inverting their differential equation into an integral one using the free Green's function. Otherwise some initial terms have to occur leading to a different result as will be shown.

We will develop a systematic many particle theory, which yields expressions without any mixed coupling terms with respect to the different meson contributions.

The physical properties of the system can be described by means of Green's functions or correlation functions. For the nucleon system we define (1 denotes the space, time, spin... etc variables)

$$\begin{aligned} iG(11') &= \langle T\Psi(1)\bar{\Psi}(1')\rangle = iG_{++} \\ iG^{<}(11') &= -\langle\bar{\Psi}(1')\Psi(1)\rangle = -iG_{+-} \\ i\bar{G}(11') &= -\langle T_{-}\Psi(1)\bar{\Psi}(1')\rangle = iG_{--} \\ iG^{>}(11') &= \langle\Psi(1)\bar{\Psi}(1')\rangle = iG_{-+}. \end{aligned} \tag{6}$$

and for the meson fields φ we introduce the correlation functions

$$\begin{aligned} id(11') &= \langle T\varphi(1)\varphi'(1')\rangle - \langle\varphi(1)\rangle\langle\varphi'(1')\rangle = id_{++} \\ i\bar{d}(11') &= -\langle\bar{T}\varphi(1)\varphi'(1')\rangle + \langle\varphi(1)\rangle\langle\varphi'(1')\rangle = id_{--} \\ id^{<}(11') &= -\langle\varphi'(1')\varphi(1)\rangle + \langle\varphi'(1')\rangle\langle\varphi(1)\rangle = -id_{+-} \\ id^{>}(11') &= \langle\varphi(1)\varphi'(1')\rangle - \langle\varphi(1)\rangle\langle\varphi'(1')\rangle = id_{-+}. \end{aligned} \tag{7}$$

Here we used the convenient matrix-notation where the $\pm$ terms of the 2×2 correlation matrix will be signed by *latin* letters. For the reason of legibility we write down only the scalar meson equations in the following. The vector meson Green's function carry spin indices. All final results will be presented for both, vector and scalar mesons. In order to include the possibility to consider pions and other pseudo- scalar (vector) mesons we indicate $\phi' = \phi^*$.

Using the definition (6) and (7) the equations of motion for the different Green's functions can be derived. The equation of motion for the causal one reads for instance

$$(i\gamma^{\mu}\partial_{\mu}-\kappa)G(11')=\delta(1-1')+\frac{1}{i}\langle Tj_{\varphi}(1)\bar{\psi}(1')\rangle \tag{8}$$

$$(\Box+m_s^2)d(11')=-\delta(1-1')+\frac{1}{i}\langle Tj_{\psi}(1)\varphi'(1')\rangle, \tag{9}$$

with j_{φ} and j_{Ψ} from Eq. (2) and (4). Up to now we have not specified the meaning of the average. Solutions, which coincide to some special averages, have to be chosen by appropriate boundary conditions. For equilibrium, which corresponds to the average with the grand canonical equilibrium density operator, the famous KMS condition [33, 26] holds

$$G^{>}|_{t=0}=\pm e^{\beta\mu}G^{<}|_{t'=-i\beta}. \tag{10}$$

In the nonequilibrium situation and for real time Green's functions this condition is not valid. Especially $G^{<}$ and $G^{>}$ are independent functions. One powerful possibility to derive real time functions is the condition of weakening of initial correlation [14], which is to be expressed in systems with finite densities. If the condition for the correlation time τ_{corr} and the mean free collision time τ_{coll}

$$\tau_{\text{corr}}\ll\tau_{\text{coll}}$$

is valid, we have a condition for the right sides of (8)

$$\lim_{t\to-\infty}\langle j_{\varphi}(1)\bar{\psi}(1')\rangle\langle\psi(1)\bar{\psi}(1')\rangle\{g_s\langle\Phi(1)\rangle-g_v\gamma^{\lambda}\langle\varphi_{\lambda}(1')\rangle\}. \tag{11}$$

This is an asymptotic condition, which breaks the time-symmetry and provides irreversible evolution in nonequilibrium systems.

In the next step the correlated self energy Σ_c is introduced formally by subtracting the mean field parts (11) from the right side of Eq. (8)

$$\frac{1}{i}\langle Tj_{\varphi}(1)\bar{\psi}(1')\rangle-\lim_{t\to-\infty}\langle Tj_{\varphi}(1)\bar{\psi}(1')\rangle$$
$$\equiv\int_c d\bar{1}\Sigma_c(1\bar{1})G(\bar{1}1'). \tag{12}$$

The way of integration c has to be determined in such a way that (11) is fulfilled. This can be found by

$$\int_c d\bar{1}\Sigma(1,\bar{1})G(\bar{1},1')=\int_{-\infty}^{+\infty}d\bar{1}\{\Sigma(1,\bar{1})G(\bar{1},1')$$
$$-\Sigma^{<}(1,\bar{1})G^{>}(\bar{1},1')\}$$
$$=\sum_{d=\pm}\int d\bar{1}\Sigma_{+d}(1\bar{1})G_{d+}(\bar{1}1'). \tag{13}$$

Here and in subsequent texts we indicate only the time-integration range explicitly. The space integration is complete. It is easy to see that the boundary condition is fulfilled, since the contribution (13) vanishes in the limit $t_1'=t_1^{\pm}\to-\infty$. For the case $t_1<t_1'$ (and vice versa) we can write e.g.

$$\int_{-\infty}^{+\infty}d\bar{1}\{\Sigma(1,\bar{1})G(\bar{1},1')-\Sigma^{<}(1,\bar{1})G^{>}(\bar{1},1')\}$$
$$=\int_{-\infty}^{t_1}\Sigma^{>}(1,\bar{1})G^{<}(\bar{1},1')+\int_{t_1}^{t_1'}\Sigma^{<}(1,\bar{1})G^{<}(\bar{1},1')$$
$$+\int_{t_1'}^{\infty}\Sigma^{<}(1,\bar{1})G^{>}(\bar{1},1')-\int_{-\infty}^{\infty}\Sigma^{<}(1,\bar{1})G^{>}(\bar{1},1'), \tag{14}$$

which tends to Zero for $t_1\to t_1\to t_0=-\infty$. If we split the last integral on the right of (14) into two parts according to

$$\int_{-\infty}^{+\infty}d\bar{t}_1=\int_{-\infty}^{t_1'}d\bar{t}_1+\int_{t_1'}^{-\infty}d\bar{t}_1$$

a contour of time integration follows which is equal to the Keldysh-contour [12, 8]. To summarize, the weakening of initial correlations and consequently the breaking of time symmetry is equivalent to the Keldysh contour.

Now we are able to enclose the equation of motion (8) resulting in the nonequilibrium Dyson equation as matrix equation (6) in the following form

$$(i\gamma^{\mu}\partial_{\mu}-\kappa)G_{bc}(11')=\delta_{bc}(11')+\int_{-\infty}^{\infty}d\bar{1}\Sigma_{b\bar{a}}(1\bar{1})G_{\bar{a}c}(\bar{1}1') \tag{15}$$

and for the meson correlation functions (9) one has

$$(\Box+m^2)d_{bc}(11')=\delta_{bc}(1-1')+\sum_d\int d2\Pi_{bd}(12)d_{bc}(21'). \tag{16}$$

Equations (15) and (16) are matrix equations, where latin letters remark $(+,-)$ in the following.

To study the structure of the self energy Σ and the polarization function Π explicitly we want to use a technique developed by Schwinger [31]. Therefore we introduce an infinitesimal meson generating flux j_{Φ} for scalar mesons and j_{λ} for vector mesons, as a new type of interaction

$$L_{\text{int}}=-j_{\Phi}\Phi-j_{\lambda}\varphi^{\lambda}. \tag{17}$$

In our presented formalism it is not necessary to introduce a generating functional for nucleons. It turns out that the infinitesimal interaction (17) is sufficient to obtain the Kadanoff-Baym [33] equations. Therefore they are valid for any density or correlations in the system.

At this point it has to be remarked, that we suppress the nondiagonal terms of the vector mesons by choosing the generating functional in diagonal form. This is reasonable, because they must not contribute to physical observables such as S-matrices as a result of coupling to the baryon conserving flux. The influence to non-observable quantities such as self energy should be considered in principle [34]. But they are neglected here for the reason of simplicity.

Now we introduce an interaction picture with respect to the infinitesimal interaction (17). Further, we distinguish between the upper and lower branch [25] of the contour because in such a way we can find relations for the time evolution operator of this (infinitesimal) interaction on the Keldysh contour, which reads

$$S_c = T_c \exp\{-ic \int (-j_\Phi \Phi - j_\mu \varphi^\mu)_c\}. \tag{18}$$

Here T_c is the time-ordering operator on the branch $c = \pm$ of the contour. Now special relations can be established by variational technique

$$\frac{\delta S_c}{\delta j_b} = -ibT_b \varphi' S_b \delta_{bc}, \tag{19}$$

where j stand for j_Φ or j_λ. Latin letters b, c remark $(+, -)$ respectively. In a straight forward manner one can express all correlation functions (7) through variations with respect to j [25]. The causal Green's function e.g. can be expressed as

$$i\frac{\delta\langle\varphi(1)\rangle_+}{\delta j_+(1')} = \langle T_+ \varphi(1)\varphi(1')\rangle - \langle\varphi(1)\rangle_+ \langle\varphi(1')\rangle_+ \equiv d_{++}. \tag{20}$$

With some manipulations the introduced polarization function (16) of mesons can be derived from (9) in the form

$$\Pi_{bd}(12) = -g_s^2 b \sum_{a\bar{a}} \int Tr\{G_{ba}(1\,\bar{1})\Gamma_{a\bar{a}d}(\bar{1}\,\bar{\bar{1}}\,2) G_{\bar{a}b}(\bar{\bar{1}}\,1^+)\} \tag{21}$$

where we have introduced the vertex function

$$\begin{aligned}\Gamma_{a\bar{a}d}(\bar{1}\,\bar{\bar{1}}\,2) &= -\frac{1}{g_s}\frac{\delta G_{a\bar{a}}^{-1}(\bar{1}\,\bar{\bar{1}})}{\delta\langle\varphi(2)\rangle_d} \\ &= -i\delta_{a\bar{a}}(\bar{1} - \bar{\bar{1}})\delta_{ad}(\bar{1} - 2) + \frac{1}{g_s}\frac{\delta\Sigma_{a\bar{a}}(\bar{1}\,\bar{\bar{1}})}{\delta\langle\varphi(2)\rangle_d}.\end{aligned} \tag{22}$$

The second line of (22) is quite easy to verify by means of (15). Concerning the nucleons we find that the right hand side of equation (8) can be expressed by the help of variations in such a way that the equation (8) takes the form

$$\begin{aligned}&[i\gamma^\mu \delta_\mu - \kappa_0 + g_s\langle\phi\rangle_b - g_v\langle\varphi_\lambda\rangle\gamma^\lambda] G_{bc}(1\,1') \\ &= \delta_{bc}(1\,1') + \left\{\frac{bg_s}{i}\frac{\delta}{\delta j_{\phi b}(1)} - \frac{bg_v}{i}\gamma_\lambda \frac{\delta}{\delta j_{\lambda b}(1)}\right\} G_{bc}(1\,1').\end{aligned} \tag{23}$$

After the introduction of the same vertex function (22), the structure of the self energy introduced in (15) can be written finally as

$$\Sigma_{b\bar{a}}(1\bar{\bar{1}}) = -\sum_{ad} \int d\bar{1}\, d2 : bg_s^2 G_{ba}(1\,\bar{1})\Gamma_{a\bar{a}d}(\bar{1}\,\bar{\bar{1}}\,2) d_b(2\,1) :. \tag{24}$$

Equations (15), (16), (21), (22) and (24) form a complete quantum statistical description of the many particle system in nonequilibrium and serves as a starting point for any further approximated treatment.

We have to point out, that the different contributions of mesons are linearly additive in the self energies and therefore quadratic additive in the coupling constants as it is to be seen from (23). This differs from other treatments, where on the stage of Eq. (2) the different parts are decoupled approximately resulting in a linear additive coupling [10, 11]. Since we have not used any restriction or approximation in deriving the general structure of (23), it has to be considered as a more general treatment. The physical reason is, that initial correlations of different meson are neglected during the derivation of the normal simple additive coupling, if one like to describe long range correlations. By the scheme proposed here this is done resulting in a quadratic additive behavior. Of course, if one goes beyond the RRPA-approximation, i.e. to include higher vertex corrections in (22), one would obtain crossed terms. This indicates that the crossed coupling terms arise by higher order interactions besides self-consistent RPA. Especially treating bound states, the common used decoupling works well by this reason.

In principle one has to admit that in the straight forward derivation of the above expressions one has to care about divergent contributions ("vacuum polarization graphs"). As we have already pointed out after Eq. (1) and as discussed in several articles (see [18] and the citations therein on page 475), one may neglect such contributions for the calculation of many-body properties to get first insight in the structure of equations. In principle the renormalization scheme is given in [31].

III. Spectral information

Because a complete description of a nonequilibrium situation demands *two* independent correlation functions instead of one, as it is the case in equilibrium, we have some relations between the *four* Green functions

$$\begin{aligned}&G^r - G^> = G^a - G^< = \bar{G} \\ &G^r + G^< = G^a + G^> = G,\end{aligned} \tag{25}$$

where we introduce the retarded and advanced functions [12]

$$G^{r/a} = G_\delta \pm \Theta(\pm(x_0 - x_0'))\{G^> - G^<\}. \tag{26}$$

This function determines the spectral information [26] and we get the equation for them from (15)

$$(i\gamma^\mu \partial_\mu - \kappa)G^r = \delta(11') + \int d\bar{1}\Sigma^r(1\bar{1})G^r(\bar{1}1'). \tag{27}$$

After the Wigner transformation, the variables are splitted in macroscopic and microscopic ones and it is assumed

$$\Sigma^r(X, p) \gg \left|\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial p_\mu}\Sigma^r(X, p)\right| \gg \cdots \tag{28}$$

This means that $\Delta x^\mu \Delta p_\mu \gg 1$ where the characteristic length Δp, at which the self energy varies in four-momentum space, corresponds to the inverse space-time interaction range. Therefore the approximation (28) demands the shortness of the space-time interaction range when compared with the systems space-time inhomogeneity scale [20].

All this is the physical background, when we applied the so called gradient expansion at this place, which yields now for (27)

$$(\gamma^\mu p_\mu - \kappa - \Sigma^r(pX))G^r(pX) = 1. \tag{29}$$

To make further progress, we write the self energy in characteristic parts in terms of the γ-matrices, which can be verified by general considerations [35, 36]

$$\Sigma^r = \gamma^\mu \Sigma_\mu^r + I\Sigma_I^r. \tag{30}$$

Further, it is useful to split up the self energy in real $\mathrm{Re}\,\Sigma$ and imaginary Γ parts, between which the dispersion relation exists

$$\mathrm{Re}\,\Sigma^r(pR\omega T) = P\int \frac{d\bar{\omega}}{2\pi}\frac{\Gamma(p\bar{\omega}RT)}{\omega-\bar{\omega}}. \tag{31}$$

Introducing the following medium-dressed variables

$$\tilde{\mathbf{P}}_\mu = p_\mu - \mathrm{Re}\,\Sigma_\mu$$

$$\tilde{\mathbf{M}} = \kappa + \mathrm{Re}\,\Sigma_I$$

$$\tilde{\mathbf{G}} = \left(\gamma_0 p_0 \varepsilon + \frac{\Gamma}{2}\right)(\gamma\tilde{\mathbf{P}} + \tilde{\mathbf{M}}) \tag{32}$$

one obtains the complete spectral function from (29)

$$A = i\{G^r(\omega + i\varepsilon) - G^a(\omega - i\varepsilon)\}$$

$$= (\gamma^\mu \tilde{\mathbf{P}}_\mu + \mathbf{I}\tilde{\mathbf{M}})\frac{2\tilde{\mathbf{G}}}{(\tilde{\mathbf{P}}^2 - \tilde{\mathbf{M}}^2)^2 + \tilde{\mathbf{G}}^2}. \tag{33}$$

This derivation underlines the correct expression using $\varepsilon\gamma^0$ instead of ε in the denominator as it was pointed out in [37]. In the case of vanishing damping Γ we get the spectral function, which determines the quasiparticle energies in the quasiparticle picture

$$A = 2\pi\{\gamma^\mu \tilde{\mathbf{P}}_\mu + I\tilde{\mathbf{M}}\}\delta(\tilde{\mathbf{P}}^2 - \tilde{\mathbf{M}}^2)\,\mathrm{sgn}(\tilde{\mathbf{P}}_0). \tag{34}$$

The δ-function in (34) represents the defining equation for the quasiparticle energies

$$E_{1/2} = \mathrm{Re}\,\Sigma_0^r \pm \sqrt{(\vec{p} - \mathrm{Re}\,\vec{\Sigma}^r)^2 + (\kappa_o + \mathrm{Re}\,\Sigma_I)^2}\,\Big|_{\omega = E_{1/2}} \tag{35}$$

This is a nonlinear equation by the many particle influence and shows immediately the particle/antiparticle symmetry. Therefore the single particle and antiparticle states become dressed by the medium and yield a natural generalization of the Dirac–Bruckner Theory. It can be seen that (35) is determined only by the retarded self energy, which yields a mass offshell behavior of the quasiparticles. This is important to note for the discussion in Chap. 4.

Let us now consider the meson equations, which read for the retarded correlated function in operator notation

$$(\Box + m_0^2)(-d^r) = 1 - \Pi^r d^r, \tag{36}$$

where integration about inner variables is assumed. Moreover, we have for the correlation function

$$(\Box + m_0^2)(-d)^{\gtrless} = -\Pi^r d^{\gtrless} - \Pi^{\gtrless} d^a. \tag{37}$$

Combining (36) and (37) we obtain the following important relation

$$-d^{\gtrless} = d^r \Pi^{\gtrless} d^a, \tag{38}$$

which presents the generalized optical theorem [8, 33]. Following the same arguments as in deriving (29), one gets for the product $d^r d^a$ in gradient expansion

$$d^r d^a = \frac{1}{(\omega^2 - E_d^2)^2 + (\frac{\Gamma_\pi}{2} + 2\varepsilon\omega)^2}. \tag{39}$$

Here the quasi-particle energy of mesons is introduced by

$$E_d^2 = p^{-2} + m^2 - \mathrm{Re}\,\Pi. \tag{40}$$

The spectral function of mesons in the case of nearly vanishing damping reads

$$B = i(d^r - d^a) \to 2\pi\delta(\omega^2 - E_d^2)\,\mathrm{sgn}(\omega) \quad \text{for} \quad \Gamma \to 0. \tag{41}$$

In static case with vanishing damping one obtains from (39) the Fourier-transformed Yukawa-potential for the meson exchange

$$d^r d^a \approx \frac{1}{g^4(4\pi)^4} V^2(p)$$

$$V(r) = g^2\frac{e^{-r/r_0}}{r} \qquad r_0^{-2} = m^2 - \mathrm{Re}\,\Pi. \tag{42}$$

When Π is determined by (21) we have the possibility to construct a self consistent system including fluctuation phenomena.

IV. Kinetic equations

The starting point are the generalized KB equations [38] which are exact and read in the space-time notation

$$[\mathrm{Re}\,G^{r^{-1}}, G^{\gtrless}] - [\Sigma^{\gtrless}, \mathrm{Re}\,G] = \tfrac{1}{2}\{G^{<}\Sigma^{>}\} - \tfrac{1}{2}\{G^{>}\Sigma^{<}\}. \tag{43}$$

Here we used operator notations, where the integration is assumed over inner variables and the brackets sign the commutator [ ] and anticommutator { } respectively.

Using the gradient expansion here we have a different physical content than the one yielding (29) and (36). If we suppose here

$$G(X,p) \gg \left|\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial p_\mu}G(X,p)\right| \gg \cdots \tag{44}$$

we have to demand that $\Delta X \Delta p \gg 1$. This would be justified in the case of one-particle systems only by classical description. But, we have a many particle system and G contains information averaged over space-time cells, which can be in principle smaller than the one determined by the single particle de Broglie wave length [15]. Because in nuclear physics time gradients occur, which are in principle nonvanishing, this assumption is the most restrictive one and is now under investigation and will soon be published [39]. We have from (43) in $p\omega RT$ variables

$$[\mathrm{Re}\,G^{r^{-1}}, G^{<}] = G^{<}\Sigma^{>} - G^{>}\Sigma^{<} \tag{45}$$

where the bracket means the Poisson-bracket

$$[A,B] = \frac{\partial}{\partial\omega}A\frac{\partial}{\partial T}B - \frac{\partial}{\partial\omega}B\frac{\partial}{\partial T}A$$

$$+ \nabla_R A \nabla_p B - \nabla_p A \nabla_R B. \tag{46}$$

As a first step of approximation we neglect higher vertex corrections in agreement with the gradient expansion and derive from (22) for the vertex function

$$\Gamma_{a\bar{a}d}(\bar{1}\,\bar{1}\,2) \approx \delta_{a\bar{a}}(1-\bar{1})\delta_{ad}(\bar{1}-2). \tag{47}$$

In the framework of this relativistic Random Phase Approximation (RRPA) we get from (15), (16), (21), (24), (38) and (47) the complete set of equations

$$[G^{r^{-1}}, G^<](p,X) = G^<(p,X)\Sigma^>(p,X) - G^>(p,X)\Sigma^<(p,X)$$

$$\Sigma^{\gtrless}(xX) = -ig_0^2 G^{\gtrless}\left(X+\frac{x}{2}, X-\frac{x}{2}\right) \times d^{\gtrless}\left(X-\frac{x}{2}, X+\frac{x}{2}\right)$$

$$d^{\gtrless}(p,X) = \frac{1}{|4\pi g_0^2|^2} V^2(p,X)\Pi^{\gtrless}(p,X)$$

$$\Pi^{\gtrless}(xX) = ig^2 \mathrm{Tr}\left\{G^{\gtrless}\left(X+\frac{x}{2}, X-\frac{x}{2}\right) \times G^{\gtrless}\left(X-\frac{x}{2}, X+\frac{x}{2}\right)\right\}, \quad (48)$$

where all variables mean 4-vectors and p is the Fourier-transform of x. It has to be stressed that the *dynamical* potential introduced by the optical theorem (38) now contains an infinitesimal sum of nucleon fluctuations by the polarization function resulting in an effective meson exchange mass. Therefore this approximation can be considered as a modified first Born approximation including collective effects, especially density fluctuations.

The set of equations (48a–d) forms a closed set and determines the correlation function $G^{\gtrless}$ and therefore the kinetic description proposed we find a connection between $G^{\gtrless}$.

Therefore we proceed on the kinetic level of description by introducing the generalized distribution

$$G^>(p\omega RT) = -iA(p\omega RT)(1-f(p\omega RT))$$

$$G^<(p\omega RT) = iA(p\omega RT)f(p\omega RT), \quad (49)$$

which is until now an exact variable change without approximations. Especially it fulfills the conditions (33).

As far as the generalized distribution should describe physical particles and antiparticles we use the Dirac interpretation in the quasiparticle picture. There an empty state of particle with negative energy equals to an antiparticle state with positive energy and we can write

$$f(-\omega) = 1 - F(\omega), \quad (50)$$

where F signs the antiparticle distribution. In equilibrium this is identical with the fact, that the chemical potential has to chosen with opposite signs for particles and antiparticles, which follows immediately from the conserved baryon density [2].

With the help of (34) and (35) we obtain for $G^<$

$$G^< = \frac{i\pi}{\sqrt{\mathbf{P}_1^2 + \mathbf{M}^2}}\{(\gamma^0 E_1 - \gamma^1 \mathbf{P}_1 + \mathbf{M})\mathbf{f}(E_1)\delta(\omega - E_1) - (-\gamma^0 E_2 - \gamma^1 \mathbf{P}_1 + \mathbf{M})\mathbf{f}(-E_2)\delta(\omega + E_2)\}. \quad (51)$$

The choice of (50) coincides with Bezzerides and DuBois [25] setting $F^<(\omega) = f(\omega)$ and $F^>(\bar\omega) = 1 - F(\omega)$. The expression for spin polarized nucleons are given in [24].

If we now introduce (50) and (51) into the kinetic equation (48a) we have to perform the Poisson brackets on the left side carefully. After partial integration one can show, that the renormalization denominator, which arise from the spectral function (34) cancels exactly with the factors following from the gradient expansion on the left side so that the drift term takes the form

$$\frac{\partial}{\partial T} f + \nabla_R E \nabla_p f - \nabla_p E \nabla_R f.$$

we want to restrict to virtual meson exchange and neglect all density terms in the meson Green's function. After somewhat extensive but straight forward calculation using the set of equations (48a–d) we finally arrive at the kinetic equation for the particle distribution, if we integrate both sides over positive frequencies. The equation for antiparticles are obvious by replacing $\mathbf{f}_p \leftrightarrow F_p$.

We use for shortness the following abbreviation of the *dynamical* potential (39) without static approximation

$$V^2(p,\bar p, p_1, \bar p_1) = (2(p\bar p_1)(\bar p p_1) + 2(pp_1)(\bar p \bar p_1) - 2\kappa^2[(p_1\bar p_1) + (p\bar p)] + 4\kappa^4)V_v^2(\bar p - p) - (p\bar p + \kappa^2)(p_1\bar p_1 + \kappa^2)V_s^2(\bar p - p), \quad (52)$$

with $V_v^2(p) = g_v^4/(p^2 - m_v^2 + \mathrm{Re}\,\Pi_v^R)$ and correspondingly for V_s^2. As already mentioned following (24) no mixed terms between g_v^2, g_s^2 occur in the cross sections due to the additive behaviour of the different self energies. The kinetic equation reads now

$$\frac{\partial}{\partial T} f + \nabla_R E \nabla_p f - \nabla_p E \nabla_R f = \mathrm{Tr}\int [g^<\Sigma^> - g^>\Sigma^<]\frac{d\omega}{2\pi} + \frac{\partial}{\partial T}\mathrm{Tr}\int P\frac{1}{(\omega-\omega')^2}[g^<\Sigma^> - g^>\Sigma^<]\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} \quad (53)$$

with the collision integral up to first *two* order gradient expansions. The second part of the right side of (53) is obtained by the second order gradient expansion or equivalently by the expansion of the time retardation up to first order. It ensures the complete energy conservation whereas the first part of (53) ensures only the quasiparticle energy [39]. Because we restrict here to the case, where the quasiparticle picture is valid, only the first part will be discussed. In a later paper we will present the results obtained for the second part [39]. The first part of the right side from (53) contains 8 processes. The first one, which describes elastic particle-particle scattering reads

$$\int \frac{d\bar p^3}{(2\pi)^3}\frac{dp_1^3}{(2\pi)^3}\frac{d\bar p_1^3}{(2\pi)^3}(2\pi)^4\delta^{(4)}(\bar p_1 + \bar p - p_1 - p)\frac{1}{|E_1\bar E_1 E\bar E|} \times V^2(p,\bar p,p_1,\bar p_1)\{f_p f_{p_1} N(f_{\bar p} f_{\bar p_1}) - f_{\bar p} f_{\bar p_1} N(f_p f_{p_1})\}. \quad (54)$$

Here and in the following we used the Pauli blocking factors

$$N(F) = 1 - F$$

$$N(Ff) = (1-F)(1-f)$$

$$N(FfG) = (1-F)(1-f)(1-G).$$

The next two processes are the crossing symmetric processes of 54 in the corresponding t and u channel. They are obvious by the crossing symmetry and describe the elastic particle-antiparticle scattering. The next 5 processes are only possible due to the off-shell character of the quasiparticle energies (35)

$$\int \frac{d\bar{p}^3}{(2\pi)^3}\frac{dp_1^3}{(2\pi)^3}\frac{d\bar{p}_1^3}{(2\pi)^3}(2\pi)^4\frac{1}{|E_1\bar{E}_1E\bar{E}|}$$
$$\times[-V^2(p,\bar{p},-p1,\bar{p}1)\delta^{(4)}(\bar{p}_1+p_1+\bar{p}-p)$$
$$\times\{f_pN(f_{\bar{p}}F_{p_1}f_{\bar{p}_1})-f_{\bar{p}}f_{\bar{p}_1}F_{p_1}N(f_p)\}$$
$$-V^2(p,\bar{p},p_1,-\bar{p}_1)\delta^{(4)}(-\bar{p}_1-p_1+\bar{p}-p)$$
$$\times\{f_pf_{p_1}F_{\bar{p}_1}N(f_{\bar{p}})-f_{\bar{p}}N(f_pf_{p_1}F_{\bar{p}_1})\}$$
$$-V^2(p,\bar{p}_1,p_1,-\bar{p})\delta^{(4)}(-\bar{p}-p_1+\bar{p}_1-p)$$
$$\times\{f_pf_{p_1}F_{\bar{p}}N(f_{\bar{p}})-f_{\bar{p}}N(f_pf_{\bar{p}_1}F_{\bar{p}})\}$$
$$-V^2(p,\bar{p},-p_1,-\bar{p}_1)\delta^{(4)}(-\bar{p}_1+p_1-\bar{p}-p)$$
$$\times\{f_pF_{\bar{p}}F_{\bar{p}_1}N(F_{p_1})-F_{p_1}N(f_pF_{\bar{p}}F_{\bar{p}_1})\}$$
$$+V^2(p,-\bar{p},p_1,-\bar{p}_1)\delta^{(4)}(-\bar{p}_1-p_1-\bar{p}-p)$$
$$\times\{f_pF_{\bar{p}}f_{p_1}F_{\bar{p}_1}-(1-f_p)(1-F_{\bar{p}})(1-f_{p_1})(1-F_{\bar{p}_1})\}]. \tag{55}$$

Analyzing the energy and momentum conserving δ-function on finds in connection with (35) that these processes occur if the c.m. energy $\sqrt{s}$ fulfills the condition

$$\sqrt{s}=-2\,\mathrm{Re}\,\Sigma_o^r. \tag{56}$$

This means, that in a nucleus system, where the zero part of the selfenergy becomes negative, new reaction channels will be opened by many particle influence. One may think on pseudo-scalar and pseudo-vector meson coupling, where the absence of mean field terms of the vector mesons ensures the condition (56). In a subsequent paper it will be shown that this leads to the description of the Δ-resonance as an in-medium effect [41].

Further, it can be seen explicitly that the relativistic treatment yields some forefactors (52) to the dynamical potentials (39). The dynamical potentials have to be determined self-consistently with the kinetic equation by means of (53) and the polarization function (48d).

Consequently, we arrived at a generalized relativistic Lennard-Balescu equation. The dynamical potentials reflect the influence of virtual mesons and are written for each process with the influence of the dynamical density fluctuations by (39).

The first 3 terms are presenting the elastic scattering between particles and antiparticles respectively. The next 5 processes are only possible, if the particles are off shell, which is ensured by the quasiparticle energies shown in (35). This can be understood as particle creation and destruction by means of virtual meson exchange. It is important to recognize, that these processes are special effects of many particle treatment and are forbidden by momentum and energy conserving δ-functions in the case of vanishing many particle influence.

The equation (53) possesses all properties of a self-consistent quantum-mechanical kinetic equation which describes the relativistic behaviour of a meson-nucleon system from the many- particle point of view. Particularly, it contains Pauli blocking and inelastic processes by the medium. This is the main result of the present paper and should be the starting point to kinetic description of hot nuclear matter.

To complete, we give the in-medium cross sections derived from the elastic process (54). For shortness we denote $\mathrm{Re}\,\Sigma_o^r=\Delta$ and $\tilde{\kappa}=k+\mathrm{Re}\,\Sigma_i^r$. The proton-proton cross section reads

$$\begin{aligned}\frac{d\sigma}{d\Omega}=\frac{1}{32\pi^2}\Bigg\{&\frac{2g_v^4}{s(m_v^2+t)^2}\Bigg[\left(\Delta(\sqrt{s}+\Delta)+\kappa^2-\frac{u}{2}\right)^2\\&+\left(\Delta(\sqrt{s}+\Delta)-\kappa^2+\frac{s}{2}\right)^2\\&-4\tilde{\kappa}^2\left(\Delta(\sqrt{s}+\Delta)-\frac{t}{2}\right)+4\tilde{\kappa}^2(4\tilde{\kappa}^2-\kappa^2)\Bigg]\\&-\frac{2g_s^4}{s(m_s^2+t)^2}\left(\Delta(\sqrt{s}+\Delta)+\tilde{\kappa}^2+\kappa^2-\frac{t}{2}\right)^2\Bigg\}.\end{aligned} \tag{57}$$

Here s,t,u are the Mandelstam variables. This means that the in medium cross sections are effected by the dressed Baryon mass and an additional term arises which proportional to the vector part of the self energy.

The in-medium proton-antiproton elastic cross sections as well as the discussed medium caused inelastic processes can be derived from the corresponding collision integrals (54) and (55) in the same way. It turns out that they are identical to the cross sections one obtains from (57) by using crossing symmetric relations in the t or u channel or to inelastic crossing. This means that we end up with an expression for the medium dependent cross section for nucleons, which possesses crossing symmetries and includes an infinitesimal series of meson interactions by the RPA polarization function resulting in an effective meson mass (40).

V. Conclusion

We have analysed the structure of equations for a nonequilibrium situation in a relativistic nucleon-meson system. New features arise due to the consequent nonequilibrium treatment as pair creation and destruction and dynamically interacting meson masses. With one formalism of real time Green function one can determine the self-energy, the quasiparticle behaviour and kinetic equations of the system and therefore the scattering measures. The kinetic equations are derived of Lennard-Balescu type by V_s-approximation which coincide with the RPA screened potentials in nonrelativistic treatment. These obtained potentials may serve as a starting point for constructing optical potentials [20]. On the other hand in-medium cross sections can be derived from the kinetic equation, which show no mixed terms of different meson contributions. This follows immediately from the more general decoupling of equations by the self energies. In a forthcoming work we will present the numerical results for the in-medium cross sections derived from (53) for the new inelastic scattering channels.

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