Medium modification of two-particle scattering in nonideal Bose systems

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Medium modification of scattering properties in interacting Bose systems are considered by solving the Bethe-Salpeter equation. An equation of state for the normal phase (generalized Beth-Uhlenbeck formula) is given using the in-medium phase shifts to include two-particle correlations. Conclusions are drawn for systems of bosonic atoms with repulsive interaction such as sodium (^{23}Na) and rubidium (^{87}Rb). It is shown that the negative slope of the in-medium phase shift at zero momentum and the absolute value of the in-medium scattering phase shift for low scattering energies increase with density. [S1050-2947(97)06602-X]

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I. INTRODUCTION

Since the recent observation of Bose-Einstein condensation (BEC) in rubidium (^{87}Rb) [1], lithium (^{7}Li) [2], and sodium (^{23}Na) [3] in atom traps at extremly low temperatures, new interest has arisen in the investigations of interacting Bose gases. For a comprehensive overview about the current experimental and theoretical activities, see [4]. The effect of finite size on the critical temperature of an ideal Bose gas, which is confined in a small cube, was investigated in [5]. However, when performing experiments at higher densities, the question of the treatment of interaction in a nonideal Bose gas becomes essential.

In [6] the temperature dependence of the moment of inertia of a dilute Bose gas in a harmonic trap and the role of interaction is discussed. A common attempt to include interaction is the approximation of a hard sphere Bose gas which can be treated analytically [7]. There, the interaction can be replaced by a pseudopotential containing the free scattering length a_0 as the only parameter. For the atoms relevant in the recent BEC experiments, values for the scattering length are obtained in the low-density limit from measurements of the cross section [8] or from evaluation of spectroscopic data [9,10]. However, the isolated two-particle scattering properties should be modified in dense systems due to the medium as shown below. This means that the scattering processes in a dense medium cannot be interpreted in terms of the free scattering length only.

The medium modifications of two-particle properties leads to different effects. First, kinetic processes are influenced. Usually, kinetic equations are derived in random-phase approximation (RPA) or Born approximation [11–13]. The collision terms include final state Bose enhancement. It was demonstrated [13] that approaching the critical density of Bose condensation the scattering rates are increased due to Bose enhancement. In addition, the interaction can be included within the a mean-field approximation which leads to a quasiparticle concept. A mean-field quasiparticle approach using Bogoliubov transformation was proposed in [11,12], which is valid below the critical temperature.

However, additional effects arise if we go beyond a Born approximation on the basis of a mean-field quasiparticle picture. The effects of the medium on the scattering have been included in the ladder approximation of the T matrix in [14]. These effects are obtained by the solution of the Bethe-Salpeter equation containing intermediate Bose enhancement as shown below. This intrinsic medium effect causes a modification of the T matrix or the corresponding transition matrix elements which cannot be obtained using Born approximation only. It leads to medium-dependent effective scattering phase shifts and cross section which is an important input for collision integrals. In this paper we do not discuss kinetic theory in more detail (see [15]).

Another modification of two-particle properties is the influence on the thermodynamic properties. As is well-known, nonideality effects of thermodynamic quantities can be expressed in terms of two-particle scattering phase shifts. A relation to the second virial coefficient can be established using the Beth-Uhlenbeck formula [16]. It is possible to improve the isolated two-particle problem by using a quasiparticle approximation. In addition, it is possible to give a generalization for dense quantum gases. This can be achieved using the T matrix with intermediate Bose enhancement as discussed above.

Systematic attempts for the derivation of second virial corrections for dense quantum gases are given using the thermodynamic Green function technique for Fermi systems [17,18] or Ursell-operator technique [19]. In this paper we apply thermodynamic Green functions for Bose systems and study the microscopic two-particle scattering problem in medium and the resulting equation of state. We discuss the conclusions for two examples with repulsive interaction, sodium (^{23}Na) and rubidium (^{87}Rb), which play a major role in the BEC experiments.

First, let us recall some definitions and results of the scattering of two interacting particles in vacuum. The free scattering length a_0 is defined in the limit of vanishing relative momentum k

$$a_0 = -\lim_{k\to 0} \frac{\delta_0(k)}{k}. \tag{1}$$

This follows from the phase shift $\delta_0(k)$ between the asymptotic incoming and outgoing plane wave of the scattered particle, which can be expanded for small relative momentum

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$$\cot\delta_0(k) = -\frac{1}{a_0 k} + \frac{1}{2} k r_0 + O(k^3). \tag{2}$$

The coefficient r_0 in the second term characterizes the range of the potential.

Let us consider an imperfect Bose gas with $a_0 \ll \lambda$ where $\lambda = (2\pi\hbar^2/mk_BT)^{1/2}$ is the thermal wavelength. In this case, a zero range pseudopotential can be employed which is parametrized by the free scattering length a_0 only. For densities below the critical density of the ideal Bose-Einstein condensation $n_{\mathrm{BEC}} = 2.612\lambda^{-3}$, a first-order correction of the ideal Bose gas pressure p_{id} can be derived [7]

$$p(n,T) = p_{\mathrm{id}}(n,T) + \frac{4\pi\hbar^2 a_0}{m} n^2. \tag{3}$$

In the nondegenerate limit ($n \ll n_{\mathrm{BEC}}$), the equation of state (3) can be rewritten in terms of a virial expansion either of pressure or of density

$$p(n,T) = nk_BT + B_p(T) n^2 k_B T,$$

or (4)

$$n(\mu,T) = n_{\mathrm{id}}(\mu,T) + B_n(T) n_{\mathrm{id}}^2(\mu,T),$$

with the second virial coefficient $B_p(T) = 2a_0\lambda^2$ or $B_n(T) = -4a_0\lambda^2$. In the following, we present a method of deriving the in-medium generalizations of the scattering phase shifts and the scattering length as well as an equation of state for a dense Bose gas with respective virial corrections in the normal phase.

II. GREEN FUNCTION APPROACH TO IN-MEDIUM SCATTERING AND THERMODYNAMICS

In a dense medium, the single-particle, two-particle, etc., energies are modified compared to vacuum values. For instance, quasiparticle energies and exchange processes with the medium have to be included. A systematic quantum statistical approach can be given employing thermodynamic Green functions. In the case of fermions, the in-medium scattering problem was investigated in [17,18]. We follow the method as outlined in [18] and apply it for the case of bosons.

We start from the Dyson equation

$$G_1 = G_1^0 + G_1^0 \Sigma G_1, \tag{5}$$

which relates the free one-particle Green function $G_1^0(1;z) = (z-\varepsilon_1)^{-1}$ and the the full one-particle Green function G_1 in a self-consistent way. The latter one can be written as

$$G_1(1;z) = [z - \varepsilon_1 - \Sigma(1;z)]^{-1}, \tag{6}$$

where $1 = \{p_1, \sigma_1\}$ denotes quantum numbers momentum and spin and $\varepsilon_1 = \hbar^2 k_1^2/2m$ is the free single-particle energy.

The self energy Σ can be expressed in terms of the T matrix and the spectral function A as

$$\Sigma(1;z) = \sum_2 \int_{-\infty}^{+\infty} \frac{dE_2}{2\pi} A(2,E_2) \Bigg[g_1(E_2) V_{ex}(12,12) + \int_{-\infty}^{+\infty} \frac{dE}{\pi} \mathrm{Im} T_{ex}(12,12;E) \frac{g_1(E_2) - g_2(E)}{E - z - E_2} \Bigg], \tag{7}$$

where the subscript ''ex'' denotes the sum of direct and exchange matrix elements in the respective quantity and $g_n(E) = \{\exp[(E-n\mu)/T] - 1\}^{-1}$ is the n-particle Bose distribution function.

Using thermodynamic Green functions, the T matrix of the quantum scattering theory is generalized to finite temperature and density. It contains a partial summation of the interaction. The resulting Bethe-Salpeter equation for the T matrix within the ladder approximation reads [17,18,15]

$$T(12,1'2';z) = V(12,1'2') + \sum_{3456} V(12,34) G_2^0(34,56;z) \times T(56,1'2';z). \tag{8}$$

The T matrix describes the two-particle properties and corresponds to the Schrödinger equation but contains the influence of the medium in the uncorrelated two-particle Green function G_2^0. The uncorrelated two-boson Green function reads in spectral representation

$$G_2^0(121'2',z) = \int_{-\infty}^{+\infty} \frac{dE_1}{2\pi} \int_{-\infty}^{+\infty} \frac{dE_2}{2\pi} \frac{1 + g_1(E_1) + g_1(E_2)}{z - E_1 - E_2} \times A(1,E_1) A(2,E_2) \delta_{11'} \delta_{22'}. \tag{9}$$

The one-particle spectral function is defined as $A(1,E) = i[G_1(1,E+i0) - G_1(1,E-i0)]$ and reads by using the Dyson equation (5)

$$A(1,E) = \frac{2\ \mathrm{Im}\Sigma(1,E)}{[E - \varepsilon_1 - \mathrm{Re}\Sigma(1,E)]^2 + [\mathrm{Im}\Sigma(1,E)]^2}. \tag{10}$$

The set of equations (7)–(10) has to be solved self-consistently.

The T matrix is directly related to the two-particle scattering properties. For example in the continuum of scattering states the phase shifts δ can be expressed as

$$\cot\delta(12,12;E) = \frac{\mathrm{Re}T(12,12;E)}{\mathrm{Im}T(12,12;E)}, \tag{11}$$

which contain the influence of the medium. These in-medium scattering phase shifts are a direct generalization of the standard definition of free phase shifts which are included in the low-density limit [18].

From the spectral function thermodynamic properties of the system can be derived. The one-particle density of a Bose system reads

$$n(\mu,T) = \frac{1}{\Omega} \sum_1 \int \frac{dE}{2\pi} g_1(E) A(1,E), \tag{12}$$

where Ω is a normalization volume.

III. EXTENDED QUASIPARTICLE APPROXIMATION AND GENERALIZED BETH-UHLENBECK FORMULA

The full self-consistent solution of Eqs. (7)–(10) is a complicated task. Therefore, we consider the spectral function (10) in quasiparticle approximation $A_{\rm qp}(1,E)=2\pi\delta(E-\varepsilon_1-\Delta(1))$ which shows δ-peaked spectral weight at sharp quasiparticle energies $\epsilon_1=\varepsilon_1+\Delta(1)$ of the single bosons. The single-particle energy ε_1 is shifted by $\Delta(1)=\mathrm{Re}\Sigma(1,z)|_{z=\epsilon_1}$, which is the real part of the self-energy (7) at the quasiparticle energy. The quasiparticle spectral function is applied to the self-energy (7) and to the uncorrelated two-boson Green function G_2^0 (9) which enters the ladder T matrix (8). From this a spectral function can be obtained which goes beyond the quasiparticle approximation. The so-called extended quasiparticle approximation is obtained by assuming that the imaginary part of the self-energy is small. The resulting spectral function can be represented as a δ function and an additional contribution due to correlated particles in form of a derivation of the principal value [17,18]

$$A_{\rm eqp}(1,E)=\frac{2\pi\delta(E-\epsilon_1)}{1-\frac{d}{dz}\mathrm{Re}\Sigma(1,z)|_{z=\epsilon_1}} -2\ \mathrm{Im}\Sigma(1,E)\frac{d}{dE}\frac{\mathrm{P}}{E-\epsilon_1}. \quad (13)$$

Applying Eq. (13) in Eq. (12) results in an equation of state for the total density which contains quantum statistical spectral weight from single bosons in quasifree or in possible two-particle bound and scattering states. Similar to the classical equation of Beth and Uhlenbeck [16], the total density splits into density contributions due to free quasiparticles and correlated particles in bound and scattering states. In an analogous way as done in [18] for a fermionic system we give a generalized Beth-Uhlenbeck formula for the total density in a bosonic system which is only valid in the normal phase for temperatures above the onset of Bose-Einstein condensation,

$$n_{\rm tot}(\mu,T)=n_{\rm free}(\mu,T)+2n_{\rm corr}(\mu,T), \quad (14)$$

with

$$n_{\rm free}(\mu,T)=\frac{1}{\Omega}\sum_1 g_1(\epsilon_1),$$

$$n_{\rm corr}(\mu,T)=n_{\rm bound}(\mu,T)+n_{\rm scatt}(\mu,T)$$
$$=\frac{1}{\Omega}\sum_K\sum_i g_2(E_{\rm cont}+E_i^b(K))$$
$$+\frac{1}{\Omega}\sum_K\int_0^\infty\frac{dE}{\pi}g_2(E_{\rm cont}+E)$$
$$\times\sum_\alpha c_\alpha\left\{\frac{d}{dE}\left[\delta_\alpha(K;E)-\frac{1}{2}\sin\{2\delta_\alpha(K;E)\}\right]\right\},$$

where, if necessary, summations over bound states i and scattering channels α with a corresponding degeneracy factor c_α have to be performed. The in-medium bound-state energy $E_i^b(K)$ depends on total momentum K, chemical potential, and temperature. Like the scattering energy E, it is taken relative to the energy at the continuum edge $E_{\rm cont}(K)=\hbar^2K^2/4m+2\Delta(K)$ which separates bound state $[E_i^b(K)<0]$ and scattering state region $(E>0)$.

Rewriting Eq. (14) as

$$n_{\rm tot}(\mu,T)=n_{\rm free}(\mu,T)[1+B^*(\mu,T)n_{\rm free}(\mu,T)] \quad (15)$$

for $(n_{\rm free}<n_{\rm BEC})$, we can define a quantity $B^*(\mu,T)=2n_{\rm corr}(\mu,T)/[n_{\rm free}(\mu,T)]^2$ which represents the nonideality contribution in analogy to the virial expansion of the total density (4). We remark that B^* contains nonideality contributions due to two-particle interaction and degeneracy in any order of density, but no three-particle interaction contributions. While the second virial coefficient $B_n(T)$ per definitionem contributes to second order in density [see Eq. (4)] and depends on temperature only, B^* also depends on chemical potential and hence on density. In the low-density and low-temperature limit, or more exact for $a_0\ll\lambda$ and $n_{\rm free}\lambda^3\ll1$, the nonideality coefficient $B^*(\mu,T)$ coincides with the second virial coefficient $B_n(T)$ from Eq. (4).

IV. MODEL CALCULATION

In the case of a separable representation of the interaction, the Bethe-Salpeter equation (8) can be solved and the T matrix can be written in an analytic form. The choice of a separable potential is motivated by the fact that there is a rigorous method to replace an arbitrary potential by a rank-N separable potential [20]. Separable potentials are applied in the description of different physical systems, e.g., by Nozières and Schmitt-Rink [21] for electron-hole interaction or in nuclear physics for the nucleon-nucleon interaction [22]. As shown below the contact interaction which is widely used in atomic physics is a special case of a separable potential.

For our model calculation we choose a rank-one separable potential of the form

$$V(12,1'2')=V(kK,k'K')=V_0v(k)v(k')\delta_{KK'}, \quad (16)$$

with a form factor

$$v(k)=\frac{1}{k^2+\beta^2}. \quad (17)$$

The potential depends on the square of the incoming and outgoing relative wave vector $k=|\vec{k}_1-\vec{k}_2|/2$ and $k'=|\vec{k}_1'-\vec{k}_2'|/2$ and the two parameters β (approximately the inverse range) and V_0 (strength), but it is independent of the center-of-mass momentum $\hbar K$. For $\beta\to\infty$ the special case of a contact potential, i.e., a zero range interaction, is included.

Now, using relative coordinates, we can solve the in-medium Bethe-Salpeter equation (8) explicitly

$$T(kK,k'K';z)=\frac{V(kK,k'K')}{1-J(K;z)}, \quad (18)$$

with

$$J(K;z)=\sum_{k',K'} V(k'K,k'K')G_2^0(k'K;z). \tag{19}$$

The two-boson Green function in quasiparticle approximation reads

$$G_2^0(kK;z)=\frac{Q(K,k)}{z-\epsilon_1-\epsilon_2}. \tag{20}$$

The medium effects enter via the Bose enhancement factor $Q(K,k)$ and via the self-energy shift of the quasiparticle energies. The self-energy is considered in a ladder approximation for the real part of the T matrix $\mathrm{Re}\Sigma(1,z)=\Sigma_2 g_1(\epsilon_2)\,\mathrm{Re}T(1212,z+\epsilon_2)$. For our calculations at low temperatures we replace the self-energy by a rigid shift $\bar{\Delta}=\mathrm{Re}\Sigma(0,0)$. This rigid shift depends on μ and T only and can be incorporated in an effective chemical potential $\mu^*=\mu-\bar{\Delta}$.

The integration of angles in the Bose distributions $Q(K,k)=\int(d\Omega/4\pi)[1+g_1(\epsilon_1)+g_1(\epsilon_2)]$ gives an analytic expression for the Bose enhancement factor

$$Q(K,k)=1+\frac{2mk_BT}{\hbar^2Kk}\times\ln\left|\frac{1-\exp\left(-\frac{\hbar^2\left(\frac{K}{2}+k\right)^2}{2mk_BT}+\frac{\mu^*}{k_BT}\right)}{1-\exp\left(-\frac{\hbar^2\left(\frac{K}{2}-k\right)^2}{2mk_BT}+\frac{\mu^*}{k_BT}\right)}\right|. \tag{21}$$

Now, the quantity J reads explicitly

$$J(K,E(k)+i0)=\frac{V_0}{2\pi^2}\int_0^\infty dk'k'^2\frac{v(k')^2Q(K,k')}{E(k)-\hbar^2k'^2/m+i0}. \tag{22}$$

Here, the scattering energy $E(k)$ is taken relative to the energy at the continuum edge $E_{\mathrm{cont}}(K)$. The in-medium scattering phase shift follows according to Eqs. (11) and (18),

$$\cot\delta(K;E(k))=\frac{1-\mathrm{Re}J(K;E_{\mathrm{cont}}(K)+E(k))}{\mathrm{Im}J(K;E_{\mathrm{cont}}(K)+E(k))}. \tag{23}$$

The small k expansion (2) can be applied to the in-medium phase shifts δ in the same way as for the free phase shift. Correspondingly, the negative slope of the in-medium scattering phase shift for zero momentum is given as

$$a(\mu,T)=-\lim_{k\to0}\frac{\mathrm{Im}J(0,E_{\mathrm{cont}}(0)+E(k))}{k[1-\mathrm{Re}J(0,E_{\mathrm{cont}}(0)+E(k))]}. \tag{24}$$

Since $a(\mu,T)$ coincides in the low-density limit with the free scattering length a_0 it can be considered as a medium modification of the scattering length.

We stress that in contrast to the free scattering length a_0 the quantity $a(\mu,T)$ is not a measure for the strength of interaction. A common measure of the strength of interaction in the medium would be the zero energy and zero momentum limit of the T matrix [14].

If we consider a repulsive potential (no bound states) and cooled spin-polarized atoms (triplet scattering only) as used in magnetic traps, the generalized Beth-Uhlenbeck formula (14) simplifies after partial integration to

$$n_{\mathrm{tot}}(\mu,T)=\frac{1}{\Omega}\sum_1 g_1(\epsilon_1)+2\int_0^\infty\frac{dKK^2}{2\pi^2}\int_0^\infty\frac{dE}{\pi}\frac{\delta(K;E)}{2k_BT[\cosh\{(E+E_{\mathrm{cont}}(K)-2\mu)/k_BT\}-1]}. \tag{25}$$

Now we focus on the case of a contact potential. This is done to compare with classical results as Eq. (4) which are also obtained with a zero range pseudopotential. A physical reason is the assumption $r_0\ll a_0\ll\lambda$ which makes the first term in expansion (2) the dominant one. It is valid for the atomic systems considered below if we have low temperatures and a potential range r_0 in the order of the atomic hard core.

For a zero range potential [$\beta\to\infty$ in Eq. (16)] the T matrix takes the form

$$T\left(K;E=\frac{\hbar^2k^2}{m}\right)=\frac{4\pi\hbar^2a_0}{m}\left(1+ia_0k+\frac{a_0}{\pi}\times\int_0^\infty dy\sqrt{\frac{y}{y-k^2}}[Q(K,\sqrt{y})-1]\right)^{-1}. \tag{26}$$

Here, the free T matrix is first integrated out and then the limit $\beta\to\infty$ is taken [23]. This procedure avoids the intro-

duction of a cutoff parameter to regularize the integration. In the limit of zero relative momentum the expression (26) coincides with the result of the T matrix found in Ref. [14].

The in-medium scattering phase shift for the zero range interaction reads

$$\delta\left(K;E=\frac{\hbar^2k^2}{m}\right) = -\arctan\left(\frac{a_0kQ(K,k)}{1+\frac{a_0}{\pi}\int_0^\infty dy\frac{\mathrm{P}}{y-k^2}\sqrt{y}[Q(K,\sqrt{y})-1]}\right). \tag{27}$$

It differs from the free scattering phase, which is simply $\delta_0(k)=-\arctan(a_0k)$, by the occurrence of the Bose enhancement factor Q according to Eq. (21) in the nominator and in the principal value integration in the denominator. Then, the negative slope of the in-medium phase shift at $k=0$ for the zero range interaction is

$$a(\mu,T)=a_0\left(\frac{1+2g_1(0)}{1+\frac{4a_0}{\pi}\int_0^\infty dk' g_1(\epsilon_{k'})}\right). \tag{28}$$

The free scattering length a_0 is modified by the Bose functions in the nominator and denominator. They disappear in the low-density limit and $a(\mu,T)$ coincides with a_0. For $\mu^*\to 0$ the Bose pole in the nominator dominates the expression (28) and the slope of the in-medium phase shift at $k=0$ diverges as one approaches the onset of Bose-Einstein condensation.

To apply the considerations above we focus on two systems which play an important role in the current discussion on atomic Bose gases. For the first one, sodium (^{23}Na), a free ground state scattering length of $\pm(92\pm25)a_B$ (in units of the Bohr radius a_B) is obtained from cross section measurement [8]. Spectroscopic data are in agreement with $64<a_0/a_B<152$ [9], i.e., repulsive interaction is favored and bound state formation is excluded. For the second example, rubidium (^{87}Rb), the ground state triplet scattering length $a_0\approx 5$ nm is of the same order as for sodium [1]. Spectra are analyzed to yield values $85<a_0/a_B<140$ [10]. For our exploratory calculation we have chosen a free scattering length $a_0=92a_B$ for sodium and $a_0=95a_B$ for rubidium (^{87}Rb) to parametrize the T matrix (26).

V. RESULTS AND DISCUSSION

Figure 1 shows the result for the in-medium scattering phase shift of sodium according to Eq. (27). For some densities the dependence on relative momentum is plotted and compared to the free scattering phase shift. The temperature $T=5\ \mu$K has been chosen to have similar values as reached in the recent experiments. For example, in the sodium experiment [3] densities exceeding $10^{14}\mathrm{cm}^{-3}$ have been produced and a condensate has been observed at temperatures below $2\,\mu$K. The densities in our calculation are below the critical density of $n_{\mathrm{BEC}}=6\times10^{14}\mathrm{cm}^{-3}$ at $T=5\ \mu$K. In general, medium modifications of the phase shift compared to

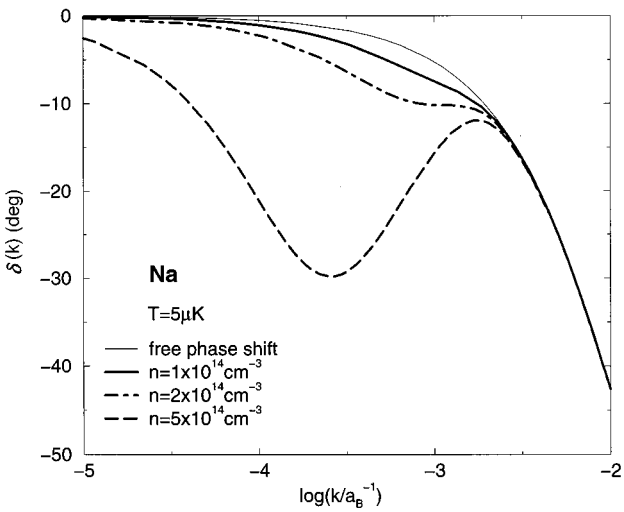

FIG. 1. Free and in-medium scattering phase shift versus decadic logarithm of relative momentum for sodium. Temperature is $T=5\,\mu$K.

the free scattering phase shift become small ($<1\%$) for scattering energies $\hbar^2k^2/m>(10\text{–}20)k_BT$. The influence of the medium enlarges the absolute value of the phase shift and leads to an extra minimum if one increases the density towards the critical value. Furthermore, we see that the medium increases the slope of the phase shift at small momenta, as demonstrated in Fig. 2. Since the free scattering length appears to be of the same size for sodium and rubidium, the major difference in the results originates from the different atomic mass. Consequently, compared to sodium at the same temperature a similar behavior of scattering shift and length occurs for rubidium at densities about seven to eight times higher.

Figure 2 shows the negative slope of the in-medium phase shift at zero momentum as calculated according to Eq. (28). We have plotted the density dependence of the negative slope given in units of a_0 in the case of sodium for two

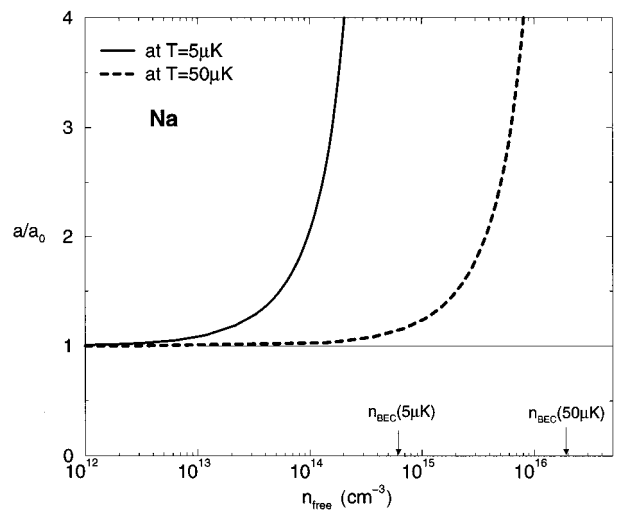

FIG. 2. The negative slope $a(\mu,T)$ of the in-medium phase shift at $k=0$ in units of the free scattering length a_0 in a parametric plot versus free density $n_{\mathrm{free}}(\mu,T)$. Considered is sodium at temperatures $T=5\ \mu$K and $T=50\ \mu$K. The critical densities are marked.

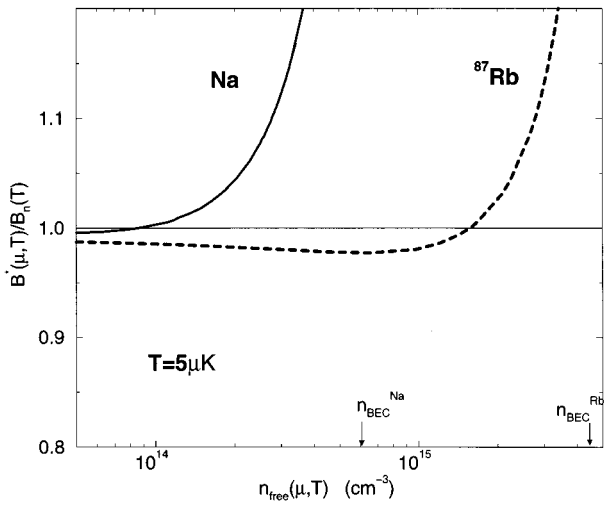

FIG. 3. The nonideality coefficient $B^*(\mu,T)$ in units of the classical second virial coefficient $B_n(T)$ for sodium and rubidium in a parametric plot versus free density $n_{\text{free}}(\mu,T)$ at temperature $T=5\ \mu$K.

different temperatures. Due to the Bose enhancement it shows a strong increase if the density rises towards the critical density n_{BEC}. As discussed above $a(\mu,T)$ can be understood as a scattering length modified due to medium effects. According to our model calculation for sodium the value of $a(\mu,T)$ is doubled for $T=5\ \mu$K at a density of 1×10^{14} cm^{-3} ($n_{\text{BEC}}=6\times10^{14}$ cm^{-3}) and for $T=50\mu$K at a density of 3.7×10^{15} cm^{-3} ($n_{\text{BEC}}=1.9\times10^{16}$ cm^{-3}). For ^{87}Rb at $T=5\ \mu$K $a(\mu,T)$ is doubled at 8×10^{14} cm^{-3} ($n_{\text{BEC}}=4.5\times10^{15}$ cm^{-3}). It should be mentioned that as a consequence of the medium modification of the T matrix, the in-medium scattering cross section deviates from the free cross section with increasing density, cf. [18]. According to our estimation, this modification should be observable already at densities about five times smaller than the critical density for Bose-Einstein condensation.

In Fig. 3 we have evaluated the generalized Beth-Uhlenbeck equation as given in Eq. (25). The self-energy is considered as a rigid shift $\bar{\Delta}(\mu,T)$ containing the real part of the ladder T matrix. We have compared the nonideality coefficient $B^*(\mu,T)$ of Eq. (15) to the classical second virial coefficient $B_n(T)$ of Eq. (4). The relation of both is plotted for ^{23}Na and ^{87}Rb versus the free quasiparticle density at a fixed temperature $T=5\ \mu$K. Figure 3 shows an increase of the nonideality coefficient $B^*(\mu,T)$ with increasing density. The mass difference causes the respective behavior for rubidium to occur at roughly seven times higher densities than for sodium at this temperature. In the low-density limit B^* approaches the second virial coefficient B_n. Figure 3 shows a minor deviation especially for ^{87}Rb from the expected low-density limit $B^*/B_n=1$. It can be traced back to the limited validity of the condition $a_0\ll\lambda$ necessary to derive Eq. (3). The classical second virial coefficient B_n as the low-density limit is slightly overestimated for higher temperatures and greater masses. We stress that the absolute value of the nonideality coefficient $B^*(\mu,T)$ is monotonically increasing with density.

The present approach should be considered as a low-density expansion since it is based on the extended quasiparticle approximation of the spectral function which assumes a small imaginary part of the self-energy. This is no longer true for higher densities where a self-consistent determination of the spectral function must be performed. In this way we limit our calculations of the nonideality coefficient B^* to densities well below the critical density as shown in Fig. 3.

We have also performed calculations with a separable potential of finite range. The model parameter V_0 and β of the potential (16) have been fitted to the free scattering length a_0 and the potential range r_0. The latter one characterizes the short range behavior of the interaction. Since no exact data is available, we have taken a value in the order of the hard core radius of the atoms. A theoretical value of the hard core radius is about $4a_B$ (from [24] at higher temperatures). The results with this finite range interaction have been found to be not sensitive if compared to the results presented in Fig. 1–3 for zero range interaction. This is not very surprising since r_0 is much smaller than the free scattering length a_0. But we are cautious to state that the results are in general independent of the potential range since we know neither r_0 nor a hard core radius at this very low temperatures.

We remind the reader that we restricted our calculations to the normal phase. To continue the description of the dilute, weakly interacting Bose gas to the Bose-condensed phase, a self-consistent Hartree-Fock-Bogoliubov approximation is necessary (see [25] for the equation of motion for an inhomogeneous weakly interacting Bose gas with condensate).

Furthermore, our calculations are restricted to the case of repulsive interacting bosons. The consideration of an attractive interaction leads to the possibility of an additional liquid-gas phase transition and possible formation of bound states. The question of a possible superposition of a gas-liquid or gas-solid phase transition and the Bose-Einstein condensation in an attractive atomic Bose gas like lithium (^{7}Li) is discussed in [26]. There, the equation of state is extended to the Bose-condensed region. In the normal phase it is considered in the approximation of Eq. (3), but below the critical temperature a BCS transition is obtained. A treatment in the framework of the generalized Beth-Uhlenbeck equation (14) would provide a better understanding of the possible phase transitions.

In a generalization of the derived results to nonequilibrium processes, transport coefficients should be modified not only due to final state Bose enhancement [12,13], but as well due to intrinsic medium effects. The ladder T matrix should serve as an input in more advanced solutions of kinetic equations including strong correlations [15].

In conclusion, we suggest that in the recent experiments with Bose gases, such as sodium and rubidium in atom traps, density and temperature regions are within reach where the scattering properties and the thermodynamic properties are influenced by the medium. We expect that the medium modifications have direct impact on the experimentally extracted values of the in-medium cross section.

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