Collective excitations and optical bistability in partially ionized hydrogen plasmas due to high electric fields

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The influence of an external electric field on the quantum distribution function and, consequently, on the nonequilibrium susceptibility is investigated in random phase approximation. The dispersion relation becomes bistable for certain wave vectors in a limited field-density-temperature plane. An instable acoustic mode is observed between two optical stable branches. These modes arise due to the applied field and can be controlled by the field strength. They are an expression of the nonequilibrium situation, which is established by an applied electric field.

I. INTRODUCTION

In this paper we investigate a partially ionized H-plasma in high electric fields. The properties of gaseous and solid state plasmas are essentially determined by the long range Coulomb interaction. This leads to collective behavior such as screening and plasma oscillation. The quantity which describes this collective behavior is the dielectric function (DF). The aim of the present paper is to investigate the influence of strong electric fields on the dielectric function in nonequilibrium systems.

This is motivated by some experimental results; e.g., a field-induced critical point in liquid–crystal phase transition was found,[1] and pattern formations in screened electrostatic fields were observed.[2]

Various attempts have been made to study screening in nonequilibrium, e.g., by Lowe and Barker.[3] Recently a treatment was presented in Refs. 4 and 5, where the different forms of the DF where examined using the Boltzmann equation in comparison with the one of Kadanoff–Baym. It was found that in the long wavelength limit, which corresponds to the situation where the wave vector is small compared with the inverse De Broglie wavelength, both results are identical. Interestingly, it was found that the Kadanoff–Baym equation leads to a Lindhard-like form of the DF with nonequilibrium distribution functions. This result can be verified in high electric fields under special restrictions, as we will show below. Further considerations are dealt with the influence of collisions on the DF itself.[6–8]

From one's knowledge of the DF it is possible to derive the spectrum of plasma oscillations and, if they exist, the region where the plasma becomes unstable. Some investigations were made concerning the stream induced plasma instability[9] for certain model systems. As the applied electric field causes a particle stream in the direction of the field, we can expect similarly unstable behavior in the plasma system found for stream instabilities. In this paper this problem is considered too, and we will show that it is possible to control the plasma instability through the applied field strength.

The outline of the paper is as follows. In the first section we will consider the one-particle distribution function in a high electric field, where we choose a field of a certain strength such that as kinetic effects become important, field emission still does not occur.[10] Then in Sec. II we solve the kinetic equation on the Fokker–Planck level. The dielectric properties of a partially ionized hydrogen plasma are discussed in Sec. III. The explicit influence of external fields are calculated and discussed together with selected properties. The main result of this work is the explicit evidence of field induced plasma instabilities, which are derived in Sec. IV and which turn out to be described by finite dispersion relations due to the quantum treatment of the dielectric function. A special region in the density-temperature plane is found where plasma excitations and instabilities can be controlled by applied electric fields. Optical bistability occurs due to the acting field and the dispersion spectra become manifold.

We want to start with the well known general nonequilibrium expression of retarded dielectric function in random phase approximation:[11]

$$\epsilon(p\tau RT)=\delta(\tau)+i\Theta(\tau)V(p)\int\frac{d\bar{\mathbf{p}}}{(2\pi\hbar)^3}\times(g^{>}(\bar{p},\tau)g^{<}(\bar{p}-p,-\tau)-g^{<}(\bar{p},\tau)g^{>}(\bar{p}-p,-\tau)). \quad (1)$$

The macroscopic variables R and T of Wigner coordinates

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were dropped for sake of simplicity. Here the correlation functions $g^>$ and $g^<$ can be obtained, e.g., by a nonequilibrium diagram technique. The problem of constructing a connection between the correlation function $g^>$ and $g^<$ to the Wigner distribution function f is solved in equilibrium by the spectral representation[12]

$$g^< = \mp ia(p\omega) f(p), \tag{2}$$

$$g^> = -ia(p\omega)(1 \pm f(p)) \tag{3}$$

with Bose–Fermi distribution functions f. This leads to the known Lindhard dielectric function for (1). In nonequilibrium this problem is much more complicated (as was investigated, e.g., by Lipavsky *et al.* in Ref. 13) and leads to a different ansatz, the generalized Kadanoff–Baym ansatz. The ordinary Kadanoff–Baym ansatz is obtained by neglecting the additional time retardations, and reads in the quasiparticle approximation

$$g^< = ia(p\omega RT) f(pRT), \tag{4}$$

$$g^> = -ia(p\omega RT)(1-f(pRT)). \tag{5}$$

Now the Wigner distribution function have to be determined by a kinetic equation derived on the same level of approximation as in obtaining (1), i.e. the Lenard–Balescu equation. Using the different Ansätze discussed in Ref. 14 one obtains different forms for the dielectric function in nonequilibrium.[15] While we get a Lindhard-like form for the dielectric function using the conventional ansatz, an additional time integration remains using the Lipavsky ansatz. As a first step we will neglect this correction by the Lipavsky ansatz, which is the aim of a following paper,[15] and consider the conventional Lindhard-like form of DF

$$\epsilon^R(p,\omega,R,T) = 1 + V(p) \int \frac{d\bar{\mathbf{p}}}{(2\pi\hbar)^3} \frac{1}{\epsilon\bar{p} - \epsilon_{\bar{p}-p} - \omega - i\eta} \times (f(\bar{p}-p,R,T) - f(\bar{p},R,T)). \tag{6}$$

II. KINETIC EQUATION FOR NONIDEAL PLASMAS

The most important value used in calculating the dielectric function (6) is the one-particle distribution function. In nonequilibrium one can find kinetic equations for partially ionized plasmas which contain bound states and medium effects, like self-energy. Such equations were given in three-particle collision approximation using the method of nonequilibrium real time Green's functions.[16,17] For the Wigner distribution function f_a of the free particles of species a in an external field $U(R,t)$, this equation can be written in the form

$$\left(\frac{\partial}{\partial t} + \frac{\partial E_a(p,Rt)}{\partial p}\frac{\partial}{\partial R} - \frac{\partial E_a(p,Rt)}{\partial R}\frac{\partial}{\partial p}\right) f_a(p,Rt) = \sum_b I^B_{ab}(p,Rt) + \sum_{bc} I_{abc}(p,Rt) + \sum_b \frac{\partial}{\partial t}\epsilon \frac{\partial}{\partial \epsilon}\frac{1}{\epsilon} I^B_{ab}(p,Rt). \tag{7}$$

On the right-hand side (RHS) the two- and three-particle scattering processes are taken into account by the corresponding collision integrals discussed below. On the left-hand side the drift of quasiparticles is described by energies following from the dispersion relation

$$E_a(pRt) = \frac{p^2}{2m_a} + \mathrm{Re}\,\Sigma^R_a(p\omega Rt)\,|_{\hbar\omega = E_a(pRt)} + U(Rt). \tag{8}$$

The real part of the retarded self-energy function Σ^R_a gives the energy shift of an isolated particle due to the influence of the surrounding plasma particles.

To work with the energy shifts given by (8) is a difficult problem. But it is sufficient in many cases to approximate $E(pRt)$ according to a momentum-independent quasiparticle shift.[18] Then we have instead of (8)

$$E_a(pRt) = \frac{p^2}{2m_a} + \Delta_a(Rt) + U(Rt).$$

In this approximation, the shift is related to the chemical potential

$$\mu_a = \mu^{id}_a + \Delta_a, \quad \text{i.e. } \Delta_a = \mu^{\mathrm{corr}}_a. \tag{9}$$

On the RHS of the kinetic equation (7) the term I^B_{ab} is the quantum mechanical Boltzmann-like collision term

$$I_{ab}(p_a,r,t) = \frac{1}{\hbar V}\int \frac{d^3p_b}{(2\pi\hbar)^3}\frac{d^3\bar{p}_a}{(2\pi\hbar)^3}\frac{d^3\bar{p}_b}{(2\pi\hbar)^3} |<p_a p_b|T_{ab}|\bar{p}_a\bar{p}_b>|^2 2\pi\delta(E_{ab}-\bar{E}_{ab}) \times \{\bar{f}_a\bar{f}_b(1-f_a)(1-f_b) - (1-\bar{f}_a)(1-\bar{f}_b)f_a f_b\}. \tag{10}$$

The explicit form of the three-particle collision integral I_{abc} can be found in Refs. 16, 17, and 19. This collision term contains all the possible three-body scattering processes with free and bound particles. The last contribution on the rhs of Eq. (7) follows from the first order gradient expansion of the kinetic equation. This ensures energy conservation in binary collision approximation and the compensation of these secular divergencies in I_{abc}.[19] Here the ϵ signs the infinitesimal expression of the energy δ-function in (10). The kinetic equations for the bound particles can be derived in a way very similar to that of free particles.[20,21]

At this point we have to remark that the collision integrals for plasmas in strong electric fields are modified by the field, resulting in an explicit field dependence. What we do know is the field dependence of the two-particle collision integral in first Born approximation.[22–26] This field

dependent collision integral contains two important effects:

(i) Collisional broadening, which is a memory effect also existing in the zero field case.

(ii) Intra-collisional field effect, which is determined by field dependent two-particle dynamics.

In the following we will consider the collision integrals in the form given by (10) and account for the field dependence of the collision integral in a simplified manner, introducing a relaxation field.[22,26–28]

A. Diffusion approximation

As an illustrative example we discuss now a spatially homogeneous hydrogen plasma of electron density n_e, proton density n_p, and atom density n_a in a constant electric field with arbitrary field strength E. Several methods are presented in the literature[29–37] to solve the field dependent kinetic equation (7). We will restrict ourselves to the Fokker–Planck approximation of the collision integrals. We follow the pioneer papers,[38–40] where Davydov has assumed the explicit influence of the field to the isotropic part. This gives rise to nonlinear effects in the transport properties. For the considered hydrogen plasma we account for anisotropy in first approximation:

$$f_e(\mathbf{p},E)=f_e^0(p,E)+f_e^1(p,E)\cos\vartheta \tag{11}$$

where f_e^0 is the isotropic part of the electron distribution function which explicitly includes the field dependence. This is an extension of the more commonly-used Chapman–Enskog method given in Refs. 41 and 42. If we insert (11) in the electron kinetic equation, we arrive at the stationary case[43–45]

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\left(\frac{eE}{3}f_e^1-\frac{m_e}{m_H}\frac{1}{\tau_e}\left[pf_e^0+mkT\frac{\partial}{\partial p}f_e^0\right]\right)\right\}=0, \tag{12}$$

$$eE\frac{\partial}{\partial p}f_e^0+\frac{1}{\tau_p}f_e^1=0. \tag{13}$$

Here the contributions of collisions are included in the energy- and momentum-relaxation time

$$\tau_e^{-1}=\left(\nu_a+\nu_p+\nu_{ao}+\frac{m_H}{m_e}\nu_e\right);\quad \tau_p^{-1}=(\nu_a+\nu_p+\nu_{a1}) \tag{14}$$

where the collision frequencies ν_j are connected with the quantum mechanical transport cross-sections $\sigma_j^T=\int(1-\cos\theta)d\sigma$ by

$$\nu_j=\frac{p}{m_j}n_j\sigma_j^T. \tag{15}$$

Thus the determination of the field dependent distribution function requires the knowledge of the quantum transport cross sections and the composition of the plasma.[43,44] In the case of the electron–electron ($j=e$) and the electron–proton ($j=p$) collisions the cross sections were calculated from the scattering phase shifts by numerical solution of the radial Schrödinger equation. The elastic scattering of electrons on hydrogen atoms ($j=a$) in the ground state was dealt with by applying the adiabatic exchange model. In addition to elastic electron–atom scattering we have included excitation ($j=a_o$, $j=a_1$) and ionization processes ($j=i$) from the atomic ground state by electron impact. Details of the derivation can be found in Refs. 10, 43–45.

The solution of the homogeneous equations (12) and (13) is the well known Davydov expression

$$f_e^0=C\exp\left(-\int_0^\varepsilon\frac{d\varepsilon}{kT+m_He^2E^2\tau_p\tau_e/3m_e^2}\right) \tag{16}$$

with the normalization to the density

$$\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}f_e(\mathbf{p})=n_e.$$

From the observation of non-normalizable distribution functions, e.g. by including only Coulomb interactions, we conclude that there exist electrons whose acceleration is, theoretically, unlimited. Following the idea of Gurevitch[46] we can account for this fact by the observation of a non-vanishing flux in momentum space, whose divergence is equal to zero according to Eq. (12). In order to determine this flux we return to the differential equation (12) and use the complete solution f_e^G of Eq. (12) instead of the solution f_e^o (16) of the homogeneous equation (12). This complete solution reads:

$$f_e^G=C_1f_e^o\left(1+C_2\int_0^\epsilon\frac{\tau_p\tau_e(m_H/m_e)\,d\varepsilon}{(2m_e\varepsilon)^{3/2}(e^2E^2\tau_e\tau_pm_H/3m_e^2+kT)f_e^o(\varepsilon)}\right). \tag{17}$$

Now we have two constants due to the second order differential equation (12). Besides the normalization we choose for the secondary boundary condition that the momentum of electrons are limited somewhere by the boundaries

$$\lim_{p\to p_{\max}}\frac{f_e^G(p)}{f_e^0(p)}=0. \tag{18}$$

Using the quantum mechanical scattering cross sections calculated in Ref. 45, we can discuss the main features of the field dependence of the electron distribution function (17). We assume a fixed plasma composition determined by a field-independent mass-action law valid in thermodynamic equilibrium. In Fig. 1 the distribution function is shown for different field-strengths. As can be seen the electric field causes a broadening in the direction of higher energies. At energies above the first excitation threshold of electron–atom scattering the electrons cool down. This leads to a decrease in the distribution function providing stationarity. Further, it causes a second maximum at energies near 13.6 eV, which is the low density ionization threshold.

Above the critical field strength, which was here 10^7 V/m, the inelastic scattering is no longer effective enough to produce a stationary solution. The result is a nonrenormalizable distribution function as was mentioned above.

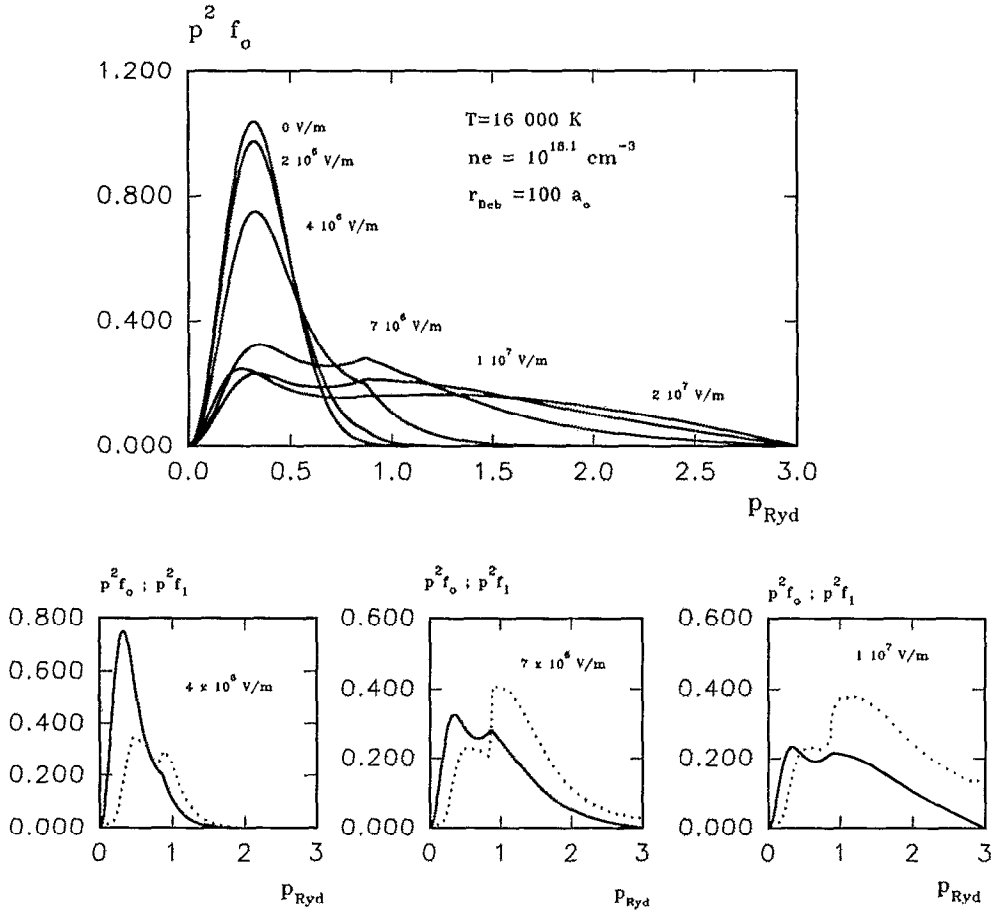

FIG. 1. Isotropic distribution function (above) in artificial units vs momentum for a free electron density of $10^{18.1}$ cm^{-3}, an ionization degree of 0.1 and a temperature of 16 000 K with six different field strengths. The corresponding Debye-radius is $r_{Deb}=100a_o$ with a_o the atomic radius. In the below 3 fields are picked out together with the corresponding anisotropic part (dotted line).

With the help of the Gurevitch boundary conditions (18) this can be expressed by a certain runaway-electron current, which can be interpreted as the stream of runaway electrons reaching every upper limit in momentum space.[29,44] In Fig. 2 this stream is shown for different plasma compositions, which are derived in the next section. Reaching the critical field strength the runaway current increases rapidly due to the long tail of the distribution function. It also indicates the breakdown of the system.

B. Plasma composition

The equations of ionization kinetics used in determining the plasma composition may be obtained from the generalized Boltzmann equation for reactive system in external fields in which many particle effects are taken into account.[17,21,45,47,48] Following these papers we obtain for the electron number density n_e

$$\frac{\partial n_e}{\partial t}=\sum_{a=e,p}\sum_{j}(\alpha_a^j n_a n_j-\beta_a^j n_a n_e n_p) \tag{19}$$

where n_p and n_j are the number densities of the protons and hydrogen atoms in the state $|j>$, respectively. If de-

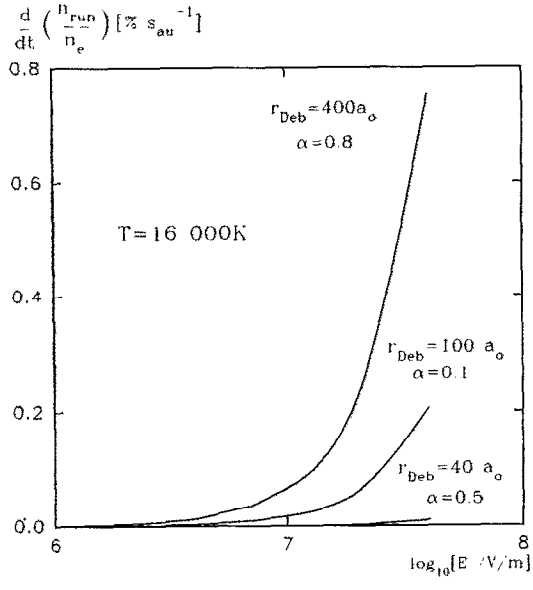

FIG. 2. The runaway electron stream in % of the electron density per atomic time units vs electric fields for different screening parameters in the partially ionized hydrogen plasma and a temperature of 16 000 K.

generacy effects are neglected one gets the statistical expressions for the rate coefficients of ionization and recombination as given in Refs. 17, 21, 47, and 48.

The evaluation of the plasma composition is critically dependent on the distribution function for the quasiparticle shifts as well as for the reaction coefficients, see Refs. 10 and 45. If we assume Maxwellian distributions, which is justified in a situation where energy relaxation is much faster than momentum relaxation, we get the conventional mass action law for nonideal plasmas, also known as the Saha equation:

$$\frac{n_a}{n_e n_p}=\left(\frac{\lambda_e\lambda_p}{\lambda_a}\right)^3 e^{[-(E_n+\Delta_j-\Delta_p-\Delta_e)/kT]}. \tag{20}$$

The influence of field-dependent distribution functions on the impact of ionization can be found in Refs. 10 and 45.

III. RESULTS FOR THE DIELECTRIC FUNCTION

Let us now discuss briefly the dielectric function (6). Starting from Eq. (6) and using the field-dependent distribution function (11) we obtain for the real part and the imaginary part:

$$\epsilon(p\omega)=1+\frac{V(p)m}{(2\pi)^2\hbar^3 p}\sum_{p_1-p_o}\int_0^\infty dk \times\left\{(kf_e^o-p_jF_1)\ln\left|\frac{k+p_j}{k-p_j}\right|\right\} +i\frac{V(p)}{(2\pi)^2\hbar^3 p}\pi\sum_{p_1-p_o}\int_{|p_j|}^\infty dk(kf_e^o-p_jF_1) \tag{21}$$

where we have already introduced the two term expansion (11) of the distribution function and carried out the azimuthal integration. So the remaining terms are $F_1=f_e^1\cos\theta(p,E)$. The abbreviation $p_{o/1}=\mp p/2(1\pm 2m\xi/p$ is introduced, where the parameter $\xi=\hbar\omega/p$ will play an important role during the analysis of the plasma oscillations.

The limiting behavior of (21) is now

$$\lim_{\omega->0}\epsilon(0,\omega)\to 1-\frac{\omega_p^2}{\omega^2}, \qquad \lim_{p->0}\epsilon(p,0)\to 1+\frac{\hbar^2\kappa^2}{p^2}A(E) \tag{22}$$

with κ the inverse Debye screening length and ω_p the plasma frequency

$$\kappa^2=\frac{4\pi e^2 n_e}{\epsilon_o kT}, \quad \omega_p^2=\kappa^2\frac{kT}{m}.$$

The field-dependent constant $A(E)$ turns out to be[11]

$$A(E)=\frac{mkT}{2\pi^2\hbar^3 n_e}\int_0^\infty dk\, f_e^o(k,E) \tag{23}$$

where the distribution function is given by (16). The result of our distribution function (16) is shown in Fig. 3. One sees that the Debye screening parameter is diminished by an applied electric field until it vanishes completely. This

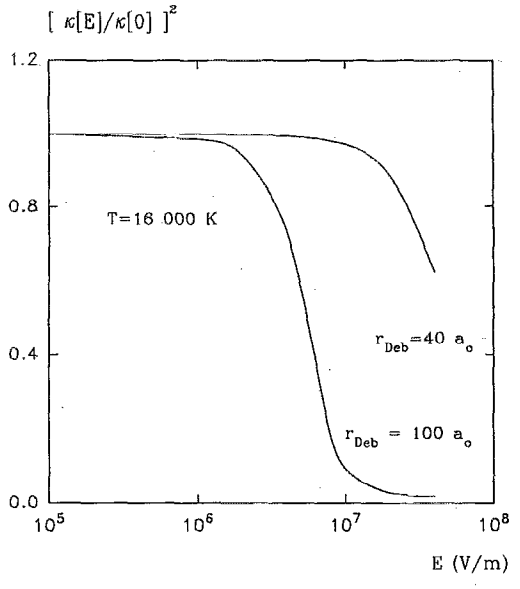

FIG. 3. The ratio of field-dependent Debye-screening parameters to field-free one vs field strength, a temperature of 16 000 K and two different Debye lengths.

coincides with the breakdown of the system as stated earlier. This becomes more obvious if we had used a heated Maxwellian with a field dependent electron temperature T_E. Then the constant A is equal to the ratio T/T_E which vanishes with increasing fields. This decreasing effect is in agreement with the results found in Ref. 49.

A. The excitation spectra

As pointed out above the excitation spectrum of the plasma is crucially determined by the imaginary part of the inverse DF:

$$-\operatorname{Im}\epsilon^{-1}(p,\omega)=\frac{\operatorname{Im}\epsilon(p,\omega)}{((\operatorname{Re}\epsilon(p,\omega))^2+(\operatorname{Im}\epsilon(p,\omega))^2)}. \tag{24}$$

Assuming the imaginary part of ϵ to be small we can expand the spectral function as follows:

$$-\operatorname{Im}\epsilon^{-1}(p,\omega)=\sim\pi\delta(\operatorname{Re}\epsilon(p,\omega)) +\operatorname{Im}\epsilon(p,\omega)P'\left(\frac{1}{\operatorname{Re}\epsilon(p,\omega)}\right) \tag{25}$$

where P' denotes the derivation of the principal value and Re ϵ, Im ϵ the real and imaginary part of the DF from Eq. (21). It is easy to conclude from this equation that the excitation spectra will peak sharply at the zero of the real part, whereas the broadening is determined by the imaginary part, which is therefore to be considered as the damping of the plasmons.

The numerical results for the real and imaginary part as well as the spectral function of the DF are plotted in Figs. 4–6. In the field free case we observe two zeros of the real part of the DF in Fig. 4. While the lower one is strongly damped by the imaginary part in Fig. 5, the higher one can be interpreted as a plasma excitation mode,

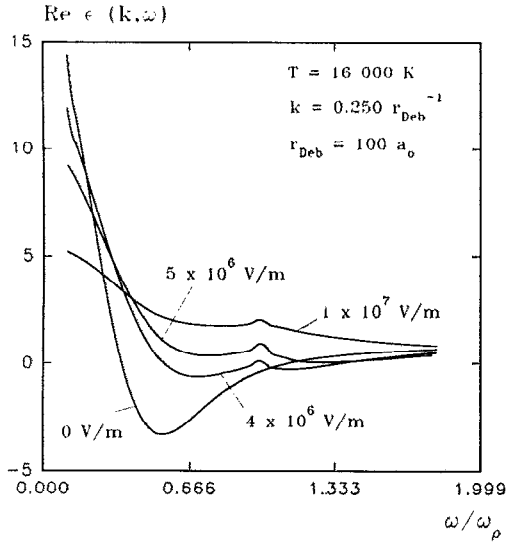

FIG. 4. The real part of the DF for one momentum of $k=0.25\ \hbar r_{\mathrm{Deb}}^{-1}$ (r_{Deb} Debye radius) vs frequency for the case of four different field strengths, a temperature of 16 000 K and an electron density of $10^{18.1}$, which corresponds to the situation of the distribution functions in Fig. 1.

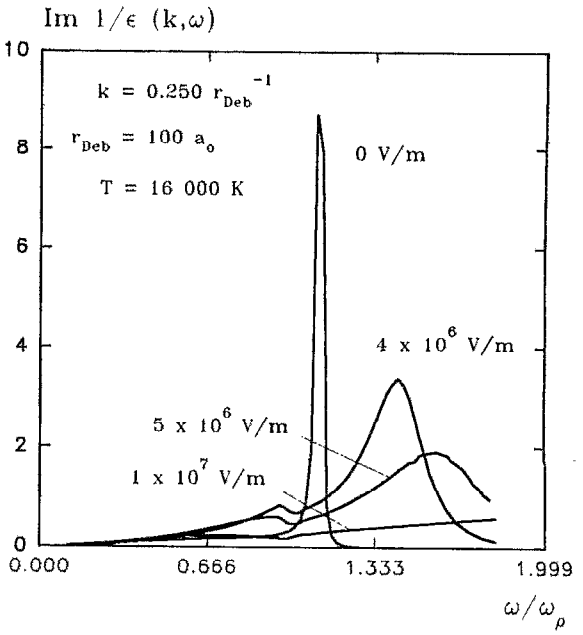

FIG. 6. The excitation spectra of the DF of one momentum of $k=0.25\ \hbar r_{\mathrm{Deb}}^{-1}$ vs frequency for the case of four different field strength and a temperature of 16 000 K and an electron density of $10^{18.1}$, which corresponds to the situation of the distribution functions in Fig. 1.

because the imaginary part becomes sufficiently small in this case and the excitation spectrum in Fig. 6 is well recognizable. As is well known there is a maximum momentum above which no zeros of the real part are possible. For a momentum higher than this maximum one the imaginary part of DF approaches the spectral function, see Eq. (24).

Now we consider the influence of strong applied electric fields on the excitation spectrum itself. In Fig. 6 the spectral function is plotted for four field strengths. One recognizes that for higher values of the applied electric field a new maximum arises. This may be interpreted as a new excitation due to applied electric fields.

This behavior can be understood if we consider the real and imaginary part of the DF for this situation represented in Figs. 4 and 5. We observe the following behavior:

- The acoustic zero of the real part tends to higher values of ω and at the same time the imaginary part is lowered.
- At special critical values of the electric field additional zeros occur.

Both effects result in the additional excitation described above, which is unstable as may be seen in the following. Following (24) the condition of the occurrence of a collective mode $\omega(q)$ with wave vector q is

$$\epsilon^R(q,\omega(q))=0. \tag{26}$$

The sign of the imaginary part of these collective modes decides whether it is a damped mode or an exponentially increasing mode. If we look for the complex zero $\omega=\omega_o - i\gamma$, with the corresponding Landau damping γ, the linearized solution of (26) reads

$$\gamma=\frac{\mathrm{Im}\,\epsilon(\omega_o)}{\mathrm{Re}\,\epsilon'(\omega_o)} \tag{27}$$

where ω_o is the zero of the real part of DF. This is valid as long as the increment is small in comparison with the real part of ω, which is true throughout the parameter used in this paper.

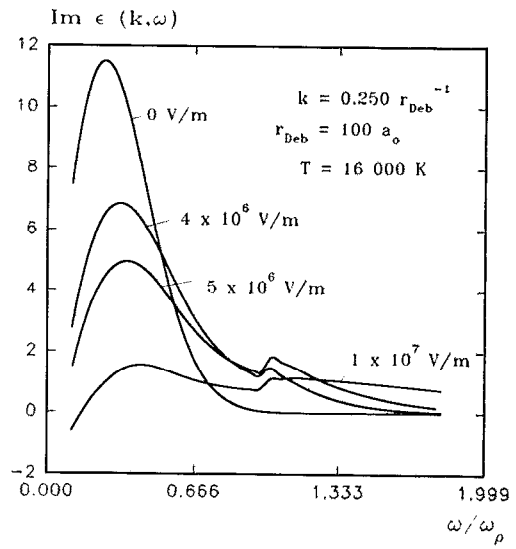

FIG. 5. The imaginary part of the DF of one momentum of $k=0.25\ \hbar r_{\mathrm{Deb}}^{-1}$ vs frequency for the case of four different field strengths and a temperature of 16 000 K and an electron density of $10^{18.1}$, which corresponds to the situation of the distribution functions in Fig. 1.

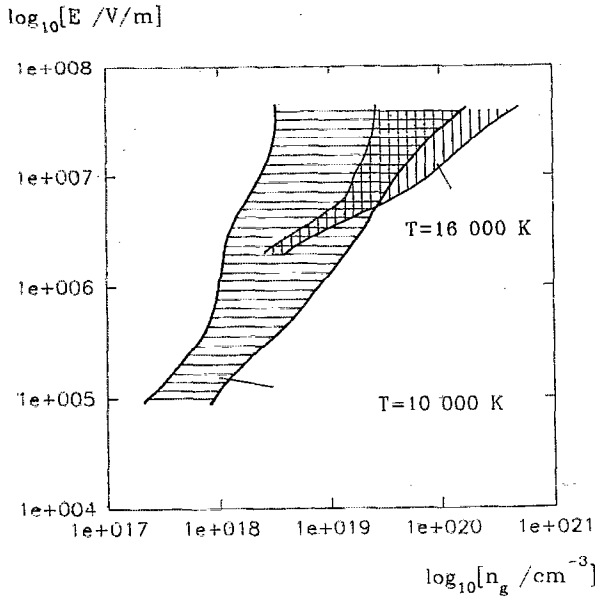

FIG. 7. The electric field-total electron density plane, where plasma instabilities can occur following Penrose for two different temperatures. The temperature of 16 000 K corresponds to the situation calculated in Fig. 1.

IV. FIELD INDUCED PLASMA INSTABILITIES

The behavior of collective modes depends on the sign of the imaginary part of the frequency zeros of the dielectric function. As a first overview we may follow the idea of Penrose,[50] who derived the following criterion for instability:

> Exponentially growing modes exist if, and only if, there is a minimum of the distribution function $f(u)$ at a value $u=\xi$ such that

$$\int_{-\infty}^{\infty} \frac{f(u)-f(\xi)}{(u-\xi)^2} > 0.$$

For the quantum dielectric function (21) one cannot conclude from the case where Im $\epsilon(p, \omega)$ change the sign to the fact that the distribution function has a minimum. Therefore the only remaining condition is Re $\epsilon(p, \omega) < 1$ and Im $\epsilon(p, \omega)$ change the sign from $+$ to $-$.

It is instructive to look at the density-electric field plane, where we show the Penrose criteria to be fulfilled for the case of our partially ionized plasma. As can be recognized from Fig. 7, we find for fixed density, both a low-field limit, above which plasma instability should occur as well as an upper limit. This means that exponentially growing modes can only occur in a very small region in the density-field strength plane. This is a surprising result which was nevertheless suggested by the change in behavior of the excitation spectra at a certain field strength with increasing fields. We will explain the existence of the upper limit in Sec. IV A.

The area is of course larger for small temperatures, because electrons are stronger accelerated by the field before they scatter with another particle. For temperatures above 30 000 K no area was found. This can be explained by the thermal maximum of the distribution function,

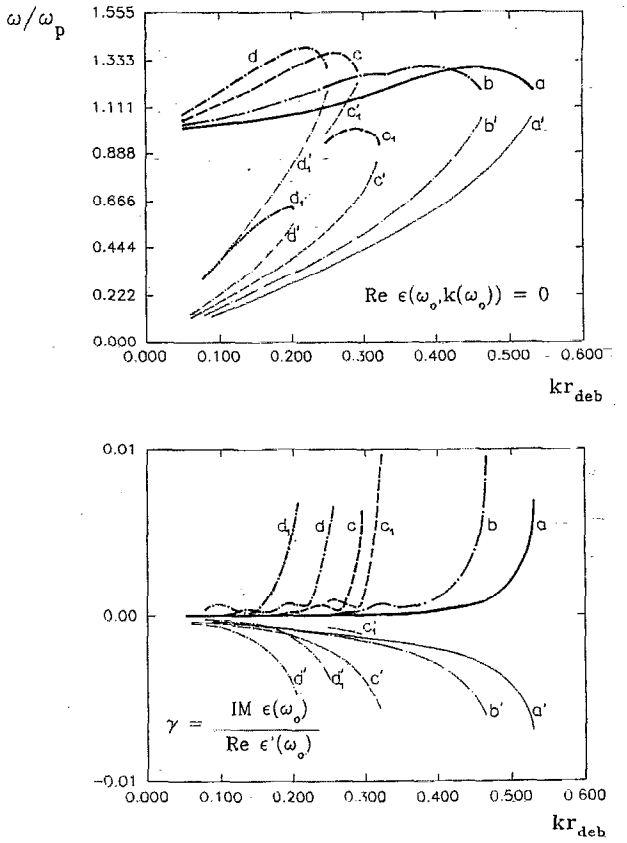

FIG. 8. The plasma oscillation dispersion (above) for four different field strengths vs wave vector together with the corresponding increment (below). Different branches are signed with different lines belonging to the upper ones. The letters a–d stand for the field strengths: 0 V/m, $2*10^6$ V/m, $4*10^6$ V/m, $5*10^6$ V/m. The prime indicates the acoustic modes and the subscript 0.1 labels the doubled mode by the field.

which is then very close to the one which occurs due to the acceleration of electrons by fields. This maximum is around the ionization energy of hydrogen, where excitation processes take place. Because of this relatively close situation, the resulting minimum cannot satisfy the second Penrose criterion, and therefore it is not successful enough for an exponentially growing mode.

The numerical solution of the quantum-mechanical dispersion relation (24) can be found in Fig. 8. In the case of zero field we obtain the well known dispersion curve, which is characterized by a critical momentum and two branches, the optical and the acoustic one. By quantum effects these two branches are closed as was already found in Ref. 51. The acoustic mode is plotted for completeness. There the imaginary part of the dielectric function is larger than the one of the optical branch. Further, the acoustic mode belongs to the lower zero of the real part of DF and to quasiparticle excitation spectra. Therefore this mode is not a collective one.

Here we want to focus on the field influence on electron transport and neglect the dynamics of protons in agreement with the approximation made deriving at the distribution function (16). The ion dynamics can be found within a hydrodynamical model in Refs. 52 and 53.

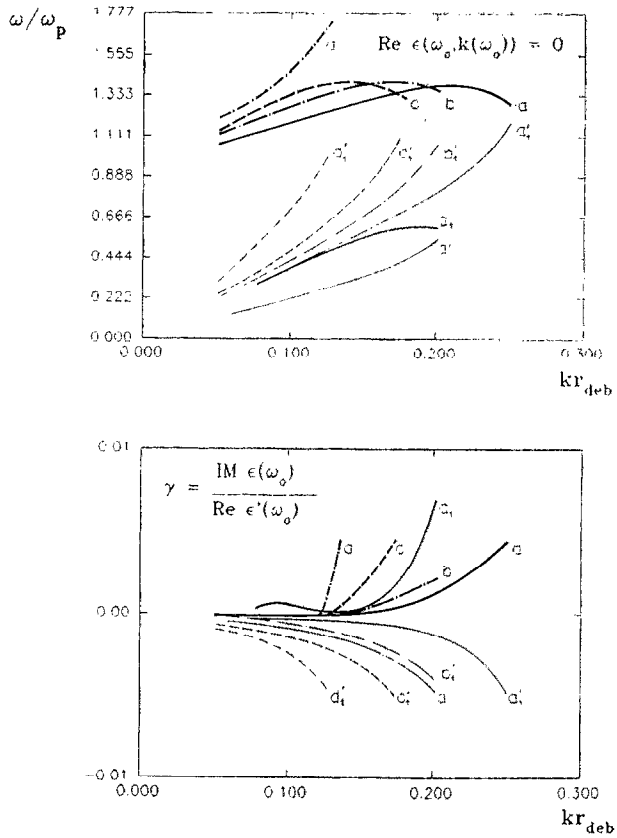

FIG. 9. The plasma oscillation dispersion (above) for four different field strengths vs wave vector together with the corresponding increment. The letters a–d stand for the field strengths: $5*10^6$ V/m, $6*10^6$ V/m, $7*10^6$ V/m, $1*10^7$ V/m. The prime indicates the acoustic modes and the subscript 0.1 labels the doubled mode by the field.

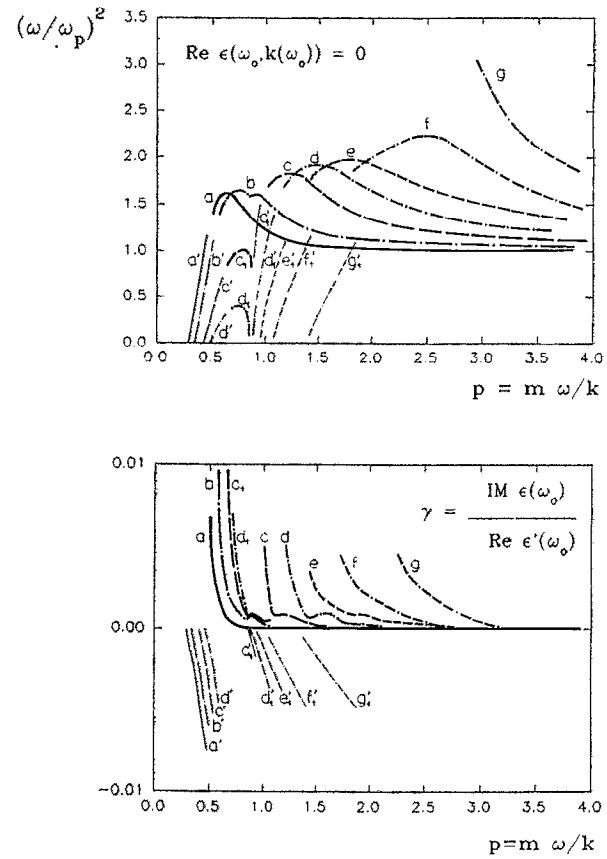

FIG. 10. The plasma oscillation dispersion (above) for seven different field strengths vs transfer momentum together with the corresponding increment. The letters a–g stand for the field strengths: 0 V/m, $2*10^6$ V/m, $4*10^6$ V/m, $5*10^6$ V/m, $6*10^6$ V/m, $7*10^6$ V/m, $1*10^7$ V/m. The prime indicates the acoustic modes and the subscript 0.1 labels the doubled mode by the field.

In the next step we discuss the influence of the electric field. Corresponding to the behavior of the real part of the DF in Fig. 4, which shows the appearance of four zeros instead of two, we observe now four branches with two critical momenta. It is quite surprising that the optical mode bends back with increasing fields to produce a butterfly-like structure. The corresponding increments show that the upper branches belong to positive and therefore exponentially damped increments, whereas the lower ones are exponentially increased. This behavior vanishes, if we go to higher fields as is to be seen in the next Fig. 9.

That means we obtain a maximum and a minimum field strength, between which exponentially growing modes exist, confirming the Penrose criterion. The appearance of a maximum limit becomes understandable in Sec. IV A.

It is justified to identify the turning back of the optical branch with optical modes itself, because the hidden 'acoustic' modes are clearly separated from the behavior found for the optical modes, as will be evident from the plot of frequency versus the ratio of $p=m\omega/k$ in Fig. 10, whose p corresponds to the momentum transfer of the isotropic distribution function calculated in Fig. 1. This frequency to wave vector ratio coincides with the additional maximum of the isotropic distribution function Fig. 1 at the point where the curves incline. This has been found already from the criteria of Penrose. It is very satisfactory that the limiting field strengths, where the notch starts and ends, correspond very well to the predicted two limiting field strengths for plasma instability as calculated from the Penrose criteria in Fig. 7. This minimum occurs in the distribution function due to the anisotropic part, caused by the applied electric field, as one sees from Fig. 1.

From the behavior described above it follows that for a test wave with wave vector $k/\hbar$ the plasma oscillations show a bifurcation for these special field strengths. Thus it can be interpreted as an optical bistability. Between these two stable modes one acoustic instable mode is found.

The bending back of the plasma dispersion relation or, equivalently the turning of the curve in Fig. 8 are effects which arise by quantum mechanical effects. If we consider the classical expression instead we would get singularities and a gap in the dispersion relation. This fact can be considered as analogous with cases which occur very often, e.g. in collision theory where classical divergencies become finite by quantum mechanical corrections.

A. Model explanation

Whereas the complicated behavior of plasma dispersion can only be derived numerically, it is instructive to

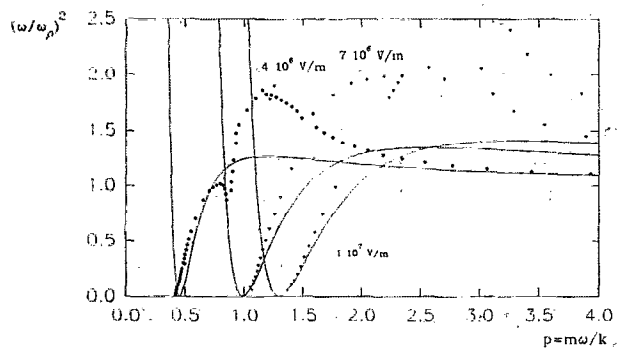

FIG. 11. The plasma oscillation dispersion for three different field strengths vs transfer momentum together with the linearized model.

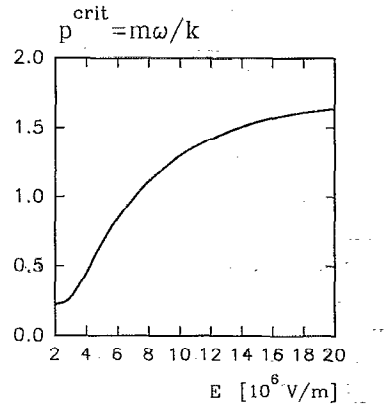

FIG. 12. The minimal critical value of transfer momentum $p=m\omega/k$, below which no plasma oscillation occurs vs electric field.

estimate the behavior according to microscopically interpretable values. Thus the expression for the DF (21) is expanded in the momentum p in order to derive at an expansion of the distribution function in the first moments. If we take the first three moments, we obtain the expansion:

$$\epsilon(p,\omega)=1-\frac{\omega_p^2}{\omega^2}\left[1+\frac{2}{m\omega^2\hbar^2}\left\langle\frac{p^2}{2m}\right\rangle p^2+\frac{2}{m\omega\hbar}\langle p\cos\theta\rangle p-\frac{4}{5m^3\omega^3\hbar^3}\langle p^3\cos\theta\rangle p^3\right]. \quad (28)$$

The even terms coincides with the expansion of Refs. 12 and 54. The odd terms arise from the applied field and describe the mean stream of particles due to the field. These terms are dependent on the cosine between p and E. Introducing the constants

$$a=\frac{4\langle p^2/2m\rangle}{kT},\quad b=\frac{4\langle p\cos\theta\rangle}{\sqrt{mkT}},\quad f=\frac{12\langle p^3\cos\theta\rangle}{(mkT)^{3/2}}, \quad (29)$$

we find the dispersion relation

$$y^2=1+\frac{a}{2}\left(\frac{x}{y}\right)^2+\frac{b}{2}\left(\frac{x}{y}\right)-\frac{f}{15}\left(\frac{x}{y}\right)^3, \quad (30)$$

where dimensionless variables $x=p/(\kappa\hbar)$, $y=\omega/\omega_p$ are used. This can be linearized according to the expansion $O(p^3)$

$$y=1+\frac{b}{2}\frac{x}{2y}+\left[a-\frac{b^2}{8}\right]\left(\frac{x}{2y}\right)^2-\left(\frac{b}{2}\left[a-\frac{b^2}{8}\right]+\frac{8}{15}f\right)\left(\frac{x}{2y}\right)^3. \quad (31)$$

The result of (31) is plotted versus momentum transfer in comparison with the numerical result for three different field strengths in Fig. 11.

While the expansion works well for small momentum or for high momentum transfer, it fails in the region where the dispersion function has a maximum and shows some interesting behavior due to the appearance of a bending back. The branch below the zero of our model must be neglected. It can be seen that the minimum ascent of $\omega(k)$, which corresponds to the minimal possible value in $\omega(p=m\omega/k)$ is well defined by the positive zero of our linearized model. The result of this critical momentum is shown in the next Fig. 12. While it starts at a certain value for small fields, it reaches a maximum value for fields higher than $1*10^7$ V/m. That is the field region where no plasma instability occurs.

To illustrate this maximal limit of the critical momentum and field more explicitly we consider the expansion (31) only to orders $O(p^2)$, which yields a condition necessary for the existence of zeros: $a > b^2/8$, which we may rewrite from (29) as follows:

$$\left\langle\frac{p^2}{2m}\right\rangle\geq\frac{\langle p\cos\theta\rangle^2}{2m}. \quad (32)$$

This has a direct interpretation. As long as the mean kinetic energy of the system is smaller than the energy built up from the mean stream of particles following the field, we can obtain plasma oscillations. The behavior of the constants described above is plotted in Fig. 13. Here the criteria for existing plasma oscillations are fulfilled as long as the fields are smaller than 10^7 V/m. Comparing this with the discussion following the solution of the kinetic equation (16), it is just the critical field strength at which breakdown occurs. Therefore we derived a very clear cri-

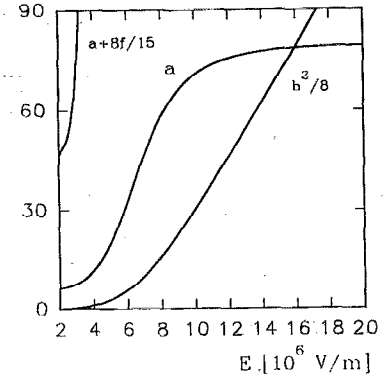

FIG. 13. The two constants of mean kinetic energy a and the mean stream energy b, which decide, whether plasma oscillations occur, vs electric fields.

terion for the point of breakdown of plasma oscillations from the expansion of plasma dispersion relation.

V. SUMMARY

We have investigated the explicit influence of electric fields on a partially ionized hydrogen plasma, which is on one hand simple enough to perform the complete quantum calculations and on other hand brings about a number of interesting effects, like ionization and dissociation. Therefore the kinetic equations are solved in Fokker–Planck approximation, including electron–atom, electron–proton, and electron–electron interaction as well as the inelastic excitation and ionization processes. The stationary nonequilibrium solution can be given as a general integral expression containing all known special cases of analytic hot electron distribution functions.[44,45]

With the help of these distribution functions the existence of a critical field strength is shown, above which the interaction mechanism is not fast enough to keep the system stable. Following an idea of Gurevitch we can evaluate the runaway electron current explicitly, describing the electron flux to be accelerated to an extent which is not limited by elastic scattering but rather only by inelastic processes and the boundaries of the system. In order to evaluate the distribution function the plasma composition is determined from a nonequilibrium Saha–equation.

The dielectric properties of the plasma are discussed with the help of the nonequilibrium field-dependent RPA approximation. The explicit influence of external fields is calculated. The main result is a small region in which plasma instabilities can occur. Thus it is possible to produce and control plasma instabilities by an applied electric field. The quantum treatment of the screening leads to a new picture of dispersion relations. The optical branch can be shown to turn back and become multi-valued for a certain range of wave vectors and frequencies and special field strengths. This is determined by the directed stream of electrons following the applied electric field. As a consequence it leads to a bifurcation of the dispersion $\omega(k)$ at special wave vectors k. Therefore we obtain an optical bistable region. Between two optical modes one instable acoustic one is found.

On account of the relatively simple requirements to produce a high electric field, this should suggest an experimental investigation of electric-field controlled plasma instability and bifurcation.

The statical screening is described by a field-dependent screening length, which breaks down for high fields and consequently would result in an increase of scattering rates.

In a forthcoming paper the RPA approximation will be improved, as it is known from normal transport theory that the local field corrections by Hubbard[55] and Singwi-Sjölander[56] provide a more realistic description of dielectric properties.

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