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Formation of correlations in strongly coupled plasmas

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Abstract

The formation of binary correlations in plasma is studied from the quantum kinetic equation. It is shown that this formation is much faster than dissipation due to collisions. In a hot (dense) plasma the correlations are formed on the scale of inverse plasma frequency (Fermi energy). We derive analytical formulae for the time dependency of the potential energy which measures the extent of correlations. We discuss the dynamical formation of screening and compare with the statically screened result. Comparisons are made with molecular dynamic simulations.

Recent lasers allow the creation of a high-density plasma within few femtoseconds and observe its time evolution on a comparable scale [1,2]. In this paper we discuss the very first time regime, the transient regime, in terms of the energy balance. Let us assume a typically set up of molecular dynamics. One takes N particles, distributes them randomly into a box and let them classically move under Coulomb forces due to their own charges. Their first movement thus forms correlations which lower the Coulomb energy $V_C = e^2/r$. This build up of screening stops when the effective Debye potential $V_D = e^2 e^{-\kappa r}/r$ is reached. We will discuss the formation of correlations in terms of correlation energy. To this end we can use a kinetic equation, which leads to the total energy conservation. It is immediately obvious that the ordinary Boltzmann equation cannot be used because the kinetic energy is an invariant of its collision integral. We have to consider non-Markovian kinetic equations of Levinson type [1]:

$$\frac{\partial}{\partial t} f_a(t) = \frac{2}{\hbar^2} \sum_b \int \frac{\mathrm{d}p\, \mathrm{d}q}{(2\pi\hbar)^6} V_D^2(q) \int_0^t \mathrm{d}\bar{t} \exp\left\{ -\frac{t-\bar{t}}{\tau} \right\} \times \cos\left\{ \frac{1}{\hbar}(t-\bar{t}) \Delta_E \right\} \{ f_a' f_b' \bar{f}_a \bar{f}_b - f_a f_b \bar{f}_a' \bar{f}_b' \} \tag{1}$$

where

$$\Delta_E = \frac{k}{2m} + \frac{p}{2m} - \frac{(k-q)}{2m} - \frac{(p+q)}{2m}$$

denotes the energy difference between initial and final states. The retardation of distributions, $f_a(k,\bar{t})$, $\bar{f}_a' = 1 - f_a'(k-q,\bar{t})$, etc., is balanced by the lifetime τ. The total energy conservation for Levinson's equation has been proved in Ref. [3].

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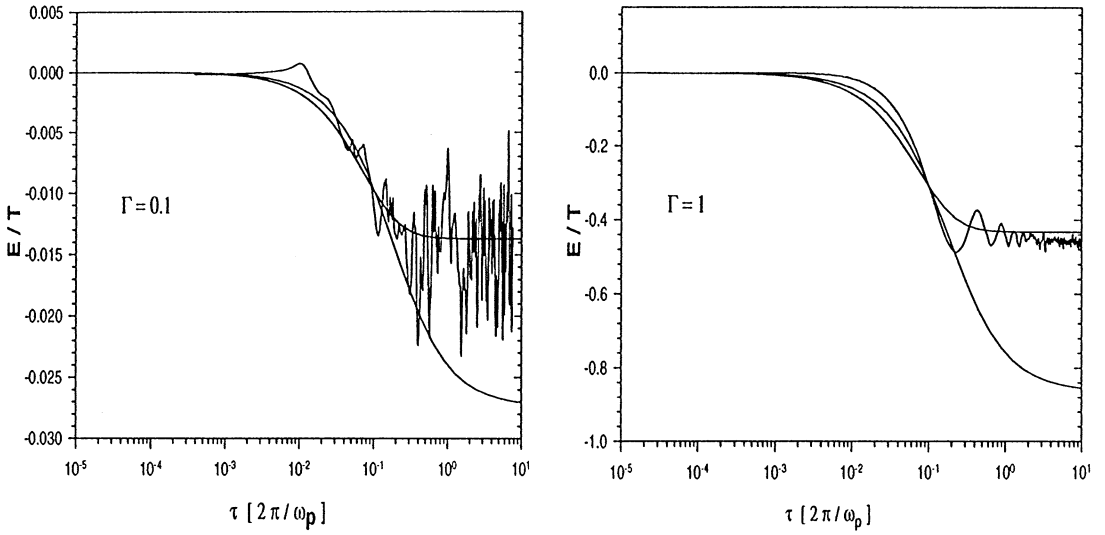

Fig. 1. The formation of correlation energy due to molecular dynamic simulations [8] together with the result of Eq. (3) for a plasma parameter $\Gamma = 0.1$ (left) and $\Gamma = 1$ (right). The upper curve is the static and the lower the dynamical calculation of (3). The latter one approaches the Debye–Hückel result.

The solution in the short-time region $t \ll \tau$ can be written down analytically. In this time domain we can neglect the time evolution of distributions, $f_a(\bar{t}) = f_a(0)$, and the lifetime factor, $\exp\{-(t-\bar{t})/\tau\} = 1$. The resulting expression for Eq. (1) then describes how two particles correlate their motion to avoid the strong interaction regions. This very fast formation of the off-shell contribution to Wigner's distribution has been found in numerical treatments of Green's functions [4,5]. Of course, starting with a sudden switching approximation we have the Coulomb interaction and during the first transient time period the screening is formed. This can be described by the non-Markovian Lenard – Balescu equation [6] instead of the statically screened Eq. (1) leading to the dynamical expression of the correlation energy. To demonstrate its results and limitations, we use Maxwell initial distributions at the high temperature limit, where the distributions are non-degenerate. From Eq. (1) we find with

$$\frac{\partial}{\partial t}E_{\rm corr} = -\sum_a \int \frac{{\rm d}k}{(2\pi\hbar)}\frac{k}{2m}\frac{\partial}{\partial t}f_a,$$

$$\frac{\partial}{\partial t}\frac{E^{\rm static}_{\rm corr}(t)}{n} = -\frac{e^2\kappa T}{2\hbar}{\rm Im}\left[(1+2z^2)\,{\rm e}^z(1-{\rm erf}(z)) - \frac{2z}{\sqrt{\pi}}\right],$$

$$\frac{\partial}{\partial t}\frac{E^{\rm dynam}_{\rm corr}(t)}{n} = -\frac{e^2\kappa T}{\hbar}{\rm Im}[{\rm e}^{z_1}(1-{\rm erf}(z_1))], \qquad (2)$$

where we used $z = \omega_p\sqrt{t^2 - {\rm i}t\frac{\hbar}{T}}$ and $z_1 = \omega_p\sqrt{2t^2 - {\rm i}t\frac{\hbar}{T}}$. This is the analytical quantum result of the time derivative of the formation of a correlation for statically as well as dynamically screened potentials. For the classical limit we are able to integrate expression (2) with respect to time and arrive at

$$E^{\rm static}_{\rm corr}(t) = -\tfrac{1}{4}e^2 n\kappa\left\{1 + \frac{2\omega_p t}{\sqrt{\pi}} - (1+2\omega_p^2 t^2)\exp(\omega_p^2 t^2)\,[1-{\rm erf}(\omega_p t)]\right\},$$

$$E^{\rm dynam}_{\rm corr}(t) = -\tfrac{1}{2}e^2 n\kappa\left\{1 - \exp\left(\frac{\omega_p^2}{2}t^2\right)\left[1 - {\rm erf}\left(\frac{\omega_p}{\sqrt{2}}t\right)\right]\right\}. \qquad (3)$$

In Fig. 1, these formulae are compared with molecular dynamic simulations [8] for two values of the plasma parameter $\Gamma = 0.1$ and 1. This parameter $\Gamma = e/aT$, where $a_e = (3/4\pi n)^{1/3}$ is the interparticle distance or Wigner–Seitz radius which measures the strength of the Coulomb coupling. Ideal plasma are found for $\Gamma \ll 1$. In this region the static formula (3) closely follows the major trend of the numerical result, see Fig. 1. The agreement is in fact surprising, because the static result underestimates the dynamical long time result of Debye–Hückel $\frac{\sqrt{3}}{2\Gamma^{3/2}}$ by a factor of two, which can be seen from the long time and classical limit

$$b^2 = (\hbar\kappa)^2 \frac{m_a + m_b}{8 m_a\ m_b}\frac{}{T} \to 0,$$

$$E_{\rm corr}^{\rm dynam}(\infty) = -\frac{e^2\kappa}{2}\frac{\sqrt{\pi}}{b}(1 - e^b \operatorname{erfc}(b)) = -\tfrac{1}{2}e^2 n\kappa + o(b),$$

$$E_{\rm corr}^{\rm static}(\infty) = -\frac{e^2\kappa}{4}(1 - \sqrt{\pi}\operatorname{erfc}(b)) = -\tfrac{1}{4}e^2 n\kappa + o(b). \quad (4)$$

The first result represents the Montroll correlation energy [9,10]. The explanation for this fact is that we can prepare the initial configuration within our kinetic theory such that sudden switching of interaction is fulfilled. However, in the simulation experiment we have initial correlations which are due to the set up within the quasiperiodic boundary condition and Ewald summations. This obviously results in to an effective statically screened Debye potential, or at least the simulation results allow for this interpretation.

For $\Gamma = 1$, see Fig. 1, non-ideal effects become important and the formation time is underestimated by Eq. (3). This is due to non-ideality which was found to be an expression of memory effects [11] and leads to a later relaxation.

The characteristic time of formation of correlations at high temperature limit is given by the inverse plasma frequency $\tau_c \approx \frac{1}{\omega} = \sqrt{2}/v\kappa$. The inverse plasma frequency indicates that the long-range fluctuations play the dominant role. This is equivalent to the time a particle needs to travel through the range of the potential with a thermal velocity $v_{\rm th}$. This confirms the numerical finding of [12] that the correlation or memory time is proportional to the range of interaction. In the low-temperature region, i.e., in a highly degenerated system $\mu \gg T$, one finds a different picture [13]. Unlike in the classical case, the equilibrium limit of the degenerated case is rapidly built up and then oscillates around the equilibrium value. We can define the build up time τ_c as the time where the correlation energy reaches its first maximum, $\tau_c = 1.0\,\hbar/\mu$ with the Fermi energy μ. Note that τ_c is in agreement with the quasiparticle formation time known as Landau's criterion. Indeed, the quasiparticle formation and the build up of correlations are two alternative views of the same phenomenon. The formation of binary correlations is very fast on the time scale of dissipative processes. Under extremely fast external perturbations, like the massive femto second laser pulses, the dynamics of binary correlations will hopefully become experimentally accessible.

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References

[1] H. Haug, A.P. Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, Springer, Berlin, 1996.
[2] W. Theobald, R. Häßner, C. Wülker, R. Sauerbrey, Phys. Rev. Lett. 77 (1996) 298.
[3] K. Morawetz, Phys. Lett. A 199 (1995) 241.
[4] P. Danielewicz, Ann. Phys. 152 (1984) 305.
[5] H.S. Köhler, Phys. Rev. C 51 (1995) 3232.
[6] K. Morawetz, Phys. Rev. E 50 (1994) 4625.
[7] G. Zwicknagel, C. Toepffer, P.G. Reinhard, in: W.D. Kraeft, M. Schlanges (Eds.), Physics of strongly coupled plasmas, World Scientific, Singapore, 1995, p. 45.
[8] W.D. Kraeft, D. Kremp, W. Ebeling, G. Röpke, Quantum Statistics of Charged Particle Systems, Akademie Verlag, Berlin, 1986.
[9] J. Riemann et al., Physica A 219 (1995) 423.
[10] K. Morawetz, R. Walke, G. Röpke, Phys. Lett. A 190 (1994) 96.
[11] M. Bonitz et al., J. Phys.: Condens. Matter 8 (1996) 6057.
[12] K. Morawetz, H.S. Koehler, Phys. Rev. C, 1997, submitted.