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Section A

Transport theory with non-local corrections

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Abstract

A kinetic equation which combines the quasiparticle drift of Landau's equation with a dissipation governed by a nonlocal and non-instant scattering integral in the spirit of Snider's equation for gases is derived. Consequent balance equations for the density, momentum and energy include quasiparticle contributions and the second order quantum virial corrections and are proven to be consistent with the conservation laws.

The very basic idea of the Boltzmann equation (BE), to balance the drift of particles with dissipation, is used in gases, plasmas and condensed systems like metals or nuclei. In both fields, the BE allows for a number of improvements which make it possible to describe phenomena far beyond the range of validity of the original BE. In these improvements the theory of gases differs from the theory of condensed systems. In the theory of gases, the focus was on the so-called virial corrections that take into account a finite volume of molecules, e.g. Enskog included space non-locality of binary collisions [1]. In the theory of condensed systems, modifications of the BE are determined by the quantum mechanical statistics. A headway in this field is governed by the Landau concept of quasiparticles [2]. There are three major modifications: the Pauli blocking of scattering channels; underlying quantum mechanical dynamics of collisions; and quasiparticle renormalization of a single-particle-like dispersion relation. However, the scattering integral of the BE remains local in space and time. In other words, the Landau theory does not include a quantum mechanical analogy of the virial corrections. The missing link of these two major streams in transport theory is clearly formulated by Laloë and Mullin [3] in their comments on the Sinder's equation. Our aim is to fill this gap. We derive here, a transport equation that includes quasiparticle renormalizations in the standard form of Landau's theory and the virial corrections in the form similar to the theory of gases briefly. "Particle diameters" and other non-localities of the scattering integral are given in form of derivatives of phase shift in binary collisions [4,5].

A convenient starting point to derive various corrections to the BE is the quasiparticle transport

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equation first obtained by Kadanoff and Baym

$$\frac{\partial f}{\partial t}+\frac{\partial \varepsilon}{\partial k}\frac{\partial f}{\partial r}-\frac{\partial \varepsilon}{\partial r}\frac{\partial f}{\partial k}=z(1-f)\sum\nolimits^{<}-zf\sum\nolimits^{>}. \quad (1)$$

Here, f is the quasiparticle distribution, ε the quasiparticle energy and z the wave-function renormalization are functions of time t, co-ordinate r, momentum k and isospin a. The self-energy $\sum^{>,<}$ is moreover a function of energy ω, however it enters the transport equation only by its value at the pole $\omega=\varepsilon$. The drift terms in the l.h.s of Eq. (1) have the standard form of the BE expect that the single-particle-like energy ε is renormalized. This is exactly the form of the drift visualized by Landau. The scattering integral in the r.h.s. of Eq. (1) is, however, more general than that expected by Landau, in particular, it includes the virial corrections which emerge for complex self-energies [6]. The self-energy we discuss is constructed from a two-particle T-matrix in the Bethe–Goldstone approximation (for simplicity we have neglected the exchange term) $\sum^{<}(1,2)=T^{R}(1,\bar{3};\bar{5}\,\bar{6})T^{A}(\bar{7},\bar{8};2\,\bar{4})G^{>}(\bar{4},\bar{3})G^{<}(\bar{5},\bar{7})G^{<}(\bar{6},\bar{8})$, which is known to include the nontrivial virial corrections [7]. Here, G's are the single particle Green's functions, numbers are cumulative variables, $1\equiv(t,r,a)$, time, coordinate and isospin. Bars denote internal variables that are integrated over. The self-energy as a functional of Green's functions $\sum[G]$ is converted into the scattering integral $\sum_{\varepsilon}[f]$ via the quasiparticle approximation $G^{>}(\omega,k,r,t,a)=(1-f(k,r,t,a))2\pi\delta(\omega-\varepsilon(k,r,t,a))$ and $G^{<}(\omega,k,r,t,a)=f(k,r,t,a)2\pi\delta(\omega-\varepsilon(k,r,t,a))$. Omitting gradient contributions to collisions one simplifies the scattering integral, but on cost of the virial corrections. Indeed, the space and time nonlocality of the scattering integral is washed out in absence of gradients. To obtain the scattering integral with virial corrections we linearize all functions in a vicinity of (r,t) using $r^{i}-r$ and $t^{1}-t$ as small parameters to second order. Then the scattering integral of Eq. (1) results.

$$\sum_b\int\frac{\mathrm{d}p}{(2\pi)^3}\frac{\mathrm{d}q}{(2\pi)^3}\,2\pi\delta(\varepsilon_a^0+\varepsilon_b^3-\varepsilon_a^1-\varepsilon_b^2+2\Delta_E)$$
$$\times|T|^2\left(\varepsilon_a^0+\varepsilon_b^3-\Delta_E,k-\frac{\Delta_K}{2},p-\frac{\Delta_K}{2},q,t-\frac{1}{2}\Delta_t,r-\Delta_r\right)[f_a^1f_b^2(1-f_a^0)(1-f_b^3)$$
$$-(1-f_a^1)(1-f_b^2)f_a^0f_b^3]. \quad (2)$$

Here, $v_a^0=(k,r,t,a)$, $v_a^1=(k-q-\Delta_K,r-\Delta_3,t-\Delta_t,a)$, $v_b^2=(p+q-\Delta_K,r-\Delta_4,t-\Delta_t,b)$, $v_b^3=(p,r-\Delta_2,t,b)$, and $\varepsilon_a^i=\varepsilon(v_a^i)$ and $f_a^i=f(v_a^i)$. One has to keep in mind that form (2) holds only up to its linear expansion in Δ's. All Δ's are given by derivatives of the phase shift $\phi=\mathrm{Im}\ln T_{SC}^{\mathrm{R}}(\Omega,k,p,q,t,r)$,

$$\Delta_t=\left.\frac{\partial\phi}{\partial\Omega}\right|_{\varepsilon_1+\varepsilon_2},\qquad \Delta_2=\left(\frac{\partial\phi}{\partial p}-\frac{\partial\phi}{\partial q}-\frac{\partial\phi}{\partial k}\right)_{\varepsilon_1+\varepsilon_2},$$
$$\Delta_E=-\frac{1}{2}\left.\frac{\partial\phi}{\partial t}\right|_{\varepsilon_1+\varepsilon_2},\qquad \Delta_3=-\left.\frac{\partial\phi}{\partial k}\right|_{\varepsilon_1+\varepsilon_2},$$
$$\Delta_K=-\frac{1}{2}\left.\frac{\partial\phi}{\partial r}\right|_{\varepsilon_1+\varepsilon_2},\qquad \Delta_4=-\left(\frac{\partial\phi}{\partial k}+\frac{\partial\phi}{\partial q}\right)_{\varepsilon_1+\varepsilon_2}, \quad (3)$$

and $\Delta_r=\frac{1}{4}(\Delta_2+\Delta_3+\Delta_4)$. After differentiation, Δ's are evaluated at the energy shell $\Omega\to\varepsilon_1+\varepsilon_2$. The Δ's are effective shifts and they represent mean values of various non-localities of the scattering integral. These shifts enter the scattering integral in form known from the theory of gases [1], however, the set of shifts is larger than the one intuitively expected. The full set (3) is necessary to guarante gauge invariance. One can see that setting all Δ's to zero, the scattering integral (2) simplifies to the one used in the BE for quasiparticles. The scattering integral is interpreted as a collision at time t and coordinate r in which two particles (holes) a and b of momenta k and p scatter into final states of momenta $k-q$ and $p+q$. This interpretation is correct for the weak-coupling limit $T^{R}\approx\mathrm{V}$, where the phase shift in the dissipative channels vanishes, $\phi=0$, and no virial corrections appear. With nontrivial Δ's, the interpretation has to be slightly modified due to finite collision duration and finite "particle diameters". For instant potential, the particles a and b enter the collision at the same time instant (there is no time shift between the arguments v_a^0 and v_b^3) and leave the collision process together (there is no time shift between v_a^1 and v_b^2).

The only time shift Δ_t is between the beginning and the end of collision. This time shift is just the collision delay discussed by Danielewicz and Pratt [8]. Due to the finite duration of the collision, the pair of particles a and b can gain an energy $2\Delta_\omega$ from external fields. The momentum shift $2\Delta_k$ describes an acceleration the pair of particles picks up during their correlated motion.

With respect to a general form of the transport equation we have already fulfilled our task: the quasiparticle transport equation (1) with the nonlocal scattering integral (2) is our final result. This transport equation has a complicated self-consistent structure: (i) quasiparticle energy depends on the distributions via the real part of the self-energy, (ii) scattering rate depends on the distributions via Pauli blocking of two-particle propagation in the T-matrix, (iii) Δ's depend on the distributions also due to Pauli blocking. One meets the same complexity for the quasiparticle BE, except for neglected Δ's. In fact, Δ's do not represent much additional work as the T-matrix has to be evaluated within the BE anyway. To summarize, we have derived a Boltzmann-like transport equation for quasiparticles that includes virial corrections to the scattering integral via set of shifts in time, space, momentum and energy. We have been able to prove the conservation laws for density, momentum and energy [9,10]. The presented theory extends the theory of quantum gases [11,12] and non-ideal plasma [13] to generated systems.

With respect to numerical implementations the presented theory is as simple as possible; the scattering integral (2) includes only a six-dimensional integration as the standard BE, the virial corrections in form of Δ's are friendly to simulation using Monte Carlo methods. Numerical tractability of the presented transport equation documents Ref. [14], where space shifts estimated from ground state have been used.

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