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Suppression of the effect of collisions due to memory effects

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Abstract

Starting from the generalized kinetic equation including complete time evolution, the memory effect of the collision integral is calculated. It is found, that the influence of the collisions on the time development of the distribution function is diminished by the non-Markovian behavior of the collision integrals, which is an expression of collision broadening. Within the Lorentz model, the calculation shows a longer survival of the initial distribution due to this collision broadening compared with normal relaxation to the stationary distribution. The density dependence of this effect shows a minimum behavior.

Transport in strongly correlated systems, as may be found in dense plasmas [1,2] or nuclear physics [3], cannot be adequately described by the ordinary Markovian Boltzmann equation. The main free time between two collisions starts to interfere with the typical collision duration time [4]. This means that one collision event is no longer independent from the history. Memory effects which are described by a non-Markovian kinetic equation become important. At the same time the single quasi-particle energy is no longer a conserved quantity. This effect is also known as collision broadening in the literature [5–8]. Especially, this changes the transport properties of semiconductors interacting with femtosecond lasers. Investigations are made to derive quantum kinetic equations for laser-pulsed excited semiconductors [9–11]. It was found that the influence of memory effects changes the relaxation remarkably [11].

In this paper we present an estimate of this effect in a Lorentz plasma consisting of heavy ions interacting with light oppositely charged particles. This well-known model yields a physically transparent insight into the memory effect. Starting from the time-exact kinetic equation we derive expressions for the relaxation time with memory effects in the Laplace domain. This method was used, e.g., in Refs. [12,13], where the linear corrections of the memory effect were calculated. Here we go beyond the expansion used there.

We want to consider a system of charged fermions with density n_i and charge $Z_i e$ interacting by a static screened Coulomb potential via

$$V_s = \frac{e^2 Z_i Z_j}{\epsilon_0 r} \mathrm{e}^{-\kappa r} . \tag{1}$$

In the non-degenerated static limit κ is the Debye screening parameter

$$\kappa^2 = \frac{4\pi e^2}{\epsilon_0 k_B T} (Z_i n_i + Z_j n_j) .$$

In the general case, it should be a parameter which includes dynamical effects, see Ref. [14]. Here we use the static approximation in a first attempt and will investigate the interplay between dynamics and short time behavior in a forthcoming paper.

The nonequilibrium properties of such a system are

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determined in the well-known manner by correlation functions. We are interested in the single-particle correlations

$$g^{>}(1,2)=\frac{1}{\mathrm{i}}\langle\Psi(1)\Psi^{*}(2)\rangle,$$

$$g^{<}(1,2)=-\frac{1}{\mathrm{i}}\langle\Psi^{*}(2)\Psi(1)\rangle\,, \qquad (2)$$

where Ψ represents the destruction operator and the average $\langle\ \rangle$ is to be performed with respect to the unknown *nonequilibrium* statistical operator ρ.

The equations of motion for the nonequilibrium correlation functions may be derived in many different ways using, e.g., the technique of Kadanoff and Baym [15], Keldysh's nonequilibrium diagram technique [16] or the Martin–Schwinger hierarchy for the real-time Green function applying the condition of the weakening of initial correlations [17–20]. By subtracting the Kadanoff–Baym equation from their conjugate one and choosing the time diagonal part, one gets an equation for the time diagonal part of $g^{<}$, which is just the Wigner function $f_{\mathrm{W}}(pRT)$. For the considered homogeneous case, the R-dependence as center of mass coordinate is dropped and the kinetic equation reads [21]

$$\mathrm{i}\frac{\partial}{\partial T}f_{\mathrm{W}}(p,T)$$

$$=\int_{-\infty}^{0}\mathrm{d}\tau\,[\{g^{>}(p,T-\tfrac{1}{2}\tau,\tau),\Sigma^{<}(p,T-\tfrac{1}{2}\tau,-\tau)\}$$

$$-\{g^{<}(p,T-\tfrac{1}{2}\tau,\tau),\Sigma^{>}(p,T-\tfrac{1}{2}\tau,-\tau)\}]\,. \qquad (3)$$

Here $\{\ ,\ \}$ is the anti-commutator of integrals over Wigner coordinates. This equation is exact in time evolution and, therefore, conserves the energy [15].

To obtain an explicit form for the kinetic equation we have to specify the self-energy $\Sigma^{\gtrless}$. For this, we use the screened Born approximation,

$$\Sigma^{\gtrless}(pR\tau T)$$

$$=\int\frac{\mathrm{d}\bar{p}\,\mathrm{d}\bar{p}'\,\mathrm{d}p'}{(2\pi)^{9}}(2\pi)^{3}\delta(p+p'-\bar{p}-\bar{p}')\,[V_{\mathrm{s}}(p-\bar{p})]^{2}$$

$$\times g^{\lessgtr}(p',-\tau,R,T)g^{\gtrless}(\bar{p},\tau,R,T)g^{\gtrless}(\bar{p}',\tau,R,T)\,. \qquad (4)$$

In order to close Eq. (3) one has to find a connection between the correlation functions $g^{\gtrless}$ and the Wigner distribution, known as ansatz. A systematic way of constructing such an ansatz can be found in Ref. [22]. There, a generalized Kadanoff–Baym ansatz (GKB) was proposed which is an exact relation, if the self-energy is diagonal in time (e.g. for Hartree–Fock approximations). The ansatz reads in convenient time domain

$$g^{<}(p\tau RT)$$

$$=-\mathrm{i}\exp[-(\mathrm{i}/\hbar)(\epsilon_{p}\tau)]f_{\mathrm{W}}(p,R,T-\tfrac{1}{2}\tau) \qquad (5)$$

and can be deduced from the operator technique as well. In general, the quasi-particle energy reads $\epsilon_{\mathrm{p}}=p^{2}/2\mathrm{m}+\mathrm{Re}\,\Sigma(p,\epsilon)$. Here we consider free particles interacting via the collision integral. Inserting (5) in Eq. (4) for Eq. (3), the kinetic equation for the two particle scattering reads [23,21,24]

$$\frac{\partial}{\partial T}f_{a}=\sum_{b}I_{ab}\,,$$

with

$$I_{ab}=\frac{2}{\hbar^{2}}\int\frac{\mathrm{d}p'_{a}\,\mathrm{d}p_{b}\,\mathrm{d}p'_{b}}{(2\pi\hbar)^{9}}\delta(p_{a}+p_{b}-p'_{a}-p'_{b})$$

$$\times V_{\mathrm{s}}^{2}(p_{a}-p'_{a})\int_{0}^{\infty}2\,\mathrm{d}\tau\cos\left(\frac{1}{\hbar}(\epsilon_{a}+\epsilon_{b}-\epsilon'_{a}-\epsilon'_{b})\tau\right)$$

$$\times[f'_{a}f'_{b}(1-f_{a})(1-f_{b})-f_{a}f_{b}(1-f'_{a})(1-f'_{b})]\,. \qquad (6)$$

For simplicity we have written, e.g., f_{b} for $f_{b}(p_{b},T-\tau)$. The usual Boltzmann collision integral is modified in (6) by a broadening of the δ-distribution function of the energy conservation and an additional retardation in the center-of-mass times of the distribution functions. This is known as collision broadening and is a result of the finite collision duration [5]. If the retardation in the distribution functions is neglected, the time integral over the cosine reduces to the familiar δ-function of energy conservation and one gets the usual Boltzmann equation. The kinetic equation generalized to external fields can be found, e.g., in Refs. [21,25,26].

To estimate the influence of the memory effects we repeat the steps leading to the ordinary relaxation

time approximation in a Lorentz plasma. In account to the great mass difference between electrons and ions, the heavy particles can be assumed Maxwellian distributed. Then the integration over the ion momentum can be carried out in (6) yielding the ion density. The structure of the time integral in the remaining equation is

$$\int_0^\infty 2\,\mathrm{d}\tau \cos\{(1/\hbar)[(\epsilon_a-\epsilon'_a)\tau]\}$$

$$\times[f_a(p'_a, T-\tau)-f_a(p_a, T-\tau)]\,. \qquad (7)$$

With respect to the evolution of our system starting at a given time $t_0=0$ with an initial distribution f_0, it is justified to truncate the time integral to the (macroscopic) time T. This means that we do not take into account memory before initial conditions, which should be physically reasonable.

The time integral is factorized by Laplace transform. Therefore we transform (6) and proceed in the Laplace domain $L[f(t), t](s)$ is the same way as one does without collision broadening in the time domain. Linearizing the distribution

$$f(\boldsymbol{p}, t)=f_g(p)+f_1(p, t)\cos\theta$$

the kinetic equation reads

$$sf(\boldsymbol{p}, s)-f_0(\boldsymbol{p})=-\nu(p, s)\left(f(\boldsymbol{p}, s)-\frac{f_g(p)}{s}\right). \qquad (8)$$

Here f_0 is the initial distribution and f_g the equilibrium distribution function. The now complex collision frequencies $\nu(p, s)$ are derived as follows,

$$\nu(s, p)=n_i\frac{2e^4}{p\hbar}\int_0^\infty \mathrm{d}y\left(\ln\left|\frac{(y+1)^2+b^2}{(y-1)^2+b^2}\right|\right.$$

$$\left.-\frac{4y}{(y+1)^2+b^2}\right)\Phi(a, y, s)\,,$$

with

$$\Phi(a, y, s)=\frac{s}{s^2+a^2(y^2-1)^2}$$

and the constants $a=p^2/2m\hbar$ and the screening parameter $b=\hbar\kappa/p$. Neglecting the collision broadening we obtain $\Phi(a, y)=\pi\delta(a(y^2-1))$, which yields just the known Brooks–Herrings result [27,28]. The memory effect is therefore a broadening of the energy conserving δ-distribution function to a Lorentzian shape, which illustrates the name "collision broadening" literally.

Now, the kinetic equation is solved in relaxation time approximation in the Laplace domain as follows,

$$f(\boldsymbol{p}, s)=f_0(\boldsymbol{p})\frac{1}{s+\nu(p, s)}$$

$$+f_g(p)\frac{\nu(p, s)}{s[s+\nu(p, s)]}\,. \qquad (9)$$

Without collision broadening $\nu(p, s)\to\nu_0(p)$, one can transform (9) immediately back to the time domain which results in the known relaxation time approximation. Including the collision broadening, we can evaluate (9) numerically and cast it therefore in a similar form, but now with time dependent collision frequencies $\nu(p, t)$,

$$f(\boldsymbol{p}, t)=f_0(\boldsymbol{p})\mathrm{e}^{-\nu(p,t)t}+f_g(p)(1-\mathrm{e}^{-\nu(p,t)t})\,. \qquad (10)$$

In Fig. 1, the ratio of the collision frequencies with collision broadening and without $\nu(p, t)/\nu_0$ is plotted for six different densities versus time. The time is normalized to the relaxation time $\nu_0(p)^{-1}$ without collision broadening. It is to be seen that the collision frequencies are suppressed at time shorter than the relaxation time without collision broadening, whereas for times larger than the static relaxation time the effect is opposite. This means the initial distribution f_0 is stabilized at times before ν_0^{-1} and is destroyed faster at times larger than ν_0^{-1} compared with normal relaxation processes. In other words the original assumptions about molecular chaos are weakened. Due to the memory effect the collisions cannot be considered independently. The collisions are not complete. This is sometimes investigated with the collision duration time [29].

Comparing this result with the result of Malfliet [4], who takes into account the second order gradient expansion we can show that the behavior of the quantum time dependent relaxation agrees for times larger than ν_0^{-1}. For shorter time scales, the behavior is just opposite. This becomes more obvious if higher densities are considered. At higher densities ($>10^{23}$ cm^{-3}) the effect of lowering the collision frequency and therefore stopping returns. This means that for higher densities the initial distribution is destroyed

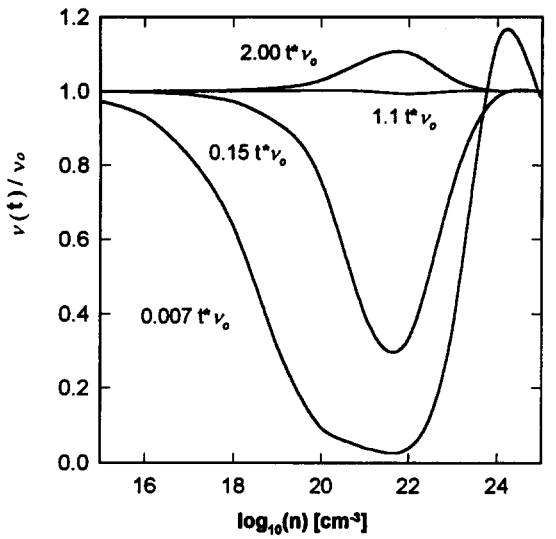

Fig. 1. The ratio of collision frequencies with and without memory effects versus time for different densities.

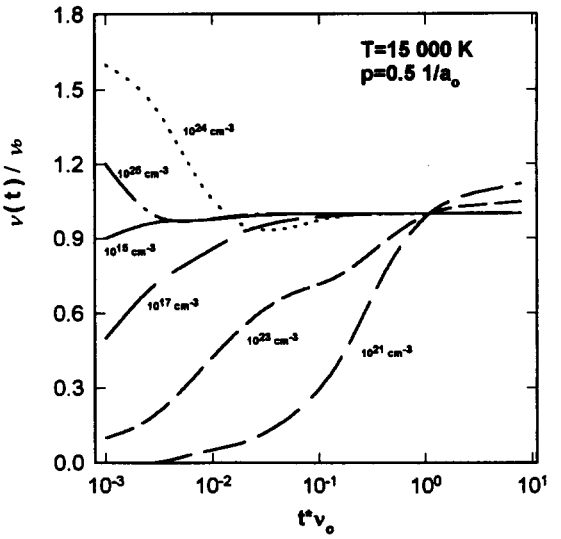

Fig. 2. The suppression of collision frequencies versus density for different scaled time points in respect to the static relaxation time.

faster than one would expect from normal relaxation. This is understandable by the high accounts of collisional events.

The different behavior of the effect in dependence on the density and momentum is in agreement with the calculation of Ref. [30]. There it was found that for the high-temperature region which corresponds to our low-density region by the Debye screening parameter, stopping power is reduced.

In order to compare the different density regions we scale the time by the static relaxation time $\nu_0(p)^{-1}$ without collision broadening and obtain in Fig. 2 the behavior that with increasing times the suppression is lowered. It is interesting to remark that the density dependence of this suppression is nonlinear. We get a density range where the suppression has a maximum. This changes at higher densities, because there the mean distance between the particles becomes smaller than the Debye screening length. The resulting collective behavior is in contradiction to the single particle picture we used so far. For short times this leads to an enhancement in agreement with the behavior explained in Fig. 1. For times larger than ν_0^{-1} the density dependence changes and leads to an increase which indicates the enhancement of stopping and in such a way to a subsequent faster decay of the initial distribution.

It can be seen that there exists a density region where memory is important. This area is limited in the low-density region by too less collisions and in the high-density region by too much collisions. Here we should admit that at this high densities the single particle picture used so far becomes questionable and the ansatz (5) certainly fails. But this should be regarded as a fingerprint to improve this ansatz.

A related phenomenon is the Debye–Onsager relaxation effect as known from electric field transport problems, where the time retardation decreases the electrical conductivity. For details and citations see e.g. Ref. [8].

To summarize, we present the known relaxation time approximation including memory effects, which occur due to high correlated dense systems. They are described by non-Markovian collision integrals. This collision integrals are presented in the complete time behavior and are calculated in relaxation time approximation. The memory results in a remarkable suppression of collisions, which leads to a stabilization of initial distribution functions and a later equilibrium state at times shorter than the usual relaxation time.

Especially in dense nuclear systems these effects should be considered, because there very short time scales interfere with the here presented effects. In a forthcoming paper we will present the results for nuclear matter calculations.

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