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Exercises Theoretical Optics 2012 (Exam.: 6+1 Exercises, 30+5 points)

1/18 The Stefan-Boltzmann law should be derived from a reversible process with the help of the first law of thermodynamics and the relation between light pressure and energy density $p = u(T)/3$.

2/18 What is the temperature of the sun surface if the intensity of sun radiation has a maximum at 500nm ? (A black body at room temperature 290K has a maximum at 10μm.)

3/18 Up to which temperature can the sun heat up a black body on earth maximally if the intensity of sun radiation is 0.1W/cm^2 and the Stefan-Boltzmann-constant is 7.55×10^{-16}Ws/m^{3}K^4 ?

4/18 Derive an expression similar to the Stefan Boltzmann law but for the number of photons per unit volume excited in a cavity at temperature T. Show that the cosmic background radiation contains about 5×10^5 photons per litre. ($\int\limits_0^\infty \frac{x}{e^x-1}dx = \pi^2/6$, $\int\limits_0^\infty \frac{x^2}{e^x-1}dx = 2\zeta_3 \approx 2.40411$, $\int\limits_0^\infty \frac{x^3}{e^x-1}dx = \pi^4/15$, $k_B = 1.38 \times 10^{-23}$J/K, $c = 3 \times 10^8$m/s, $\hbar = 1.055 \times 10^{-34}$Js)

5/18 How do the Maxwell equations look like in differential and integral form ?

6/18 Show with the help of the Gauß law, that the electric field is perpendicular directed on the surface of a conductor and that the value is equal to the charge surface density σ/ϵ.

7/18 Show the equivalence of the following expressions if the phase velocity is $v(k) = c/n(\omega(k))$:

$$v_g = \frac{d\omega}{dk} = \frac{c}{n + \omega\frac{dn}{d\omega}} = v - \lambda\frac{dv}{d\lambda}.$$

8/18 From the vector potential of the dipole radiation

$$\vec{A}(\vec{r}) = -i\frac{\mu_0\omega}{4\pi}\,\vec{p}\,\frac{e^{ikr}}{r}$$

with the dipole moment $\vec{p}$ calculate the magnetic and electric field in the far-distance zone. (Help: $\nabla \times \vec{p}f(r) = -\vec{p} \times \nabla f(r)$, direction of travel $\vec{e} = \vec{r}/r$)

9/18 Calculate the intensity distribution of diffraction of monochromatic light at a rectangular obstacle (length a, width b) in Frauenhofer approximation.

10/18 Show from the Hamiltonian of the harmonic oscillator in second quantization $H = \hbar\omega(a^+a + \frac{1}{2})$, that it holds

$$[H, a^+] = \hbar\omega a^+$$

11/18 Calculate for the dimensionless (quadrature) spatial operator $x = (a + a^{+})/2$ the mean value and the fluctuation in number-state representation. (Calculate first the matrix representation of the operator.)

12/18 From the definition of the coherent state as eigenvector of the annihilation operator $a|\alpha >= \alpha|\alpha >$ show the representation of the coherent state in number-state representation and show that the probability to find n photons in a coherent (Glauber) state. (Hint: Use $|n >= \frac{(a^+)^n}{\sqrt{n!}}|0 >$ and the normalization to 1)

13/18 Calculate the photon fluctuation in coherent state.

14/18 Calculate for the dimensionless (quadrature) operators $\hat{x} = (\hat{a}+\hat{a}^+)/2$ and $\hat{y} = i(\hat{a}^+ - \hat{a})/2$ the mean value and the fluctuation in coherent representation. How does look like the standard deviation?

15/18 For the dimensionless photon mode $\hat{E} = \frac{1}{2}\left(\hat{a}e^{-i\chi} + \hat{a}^+ e^{+i\chi}\right)$ calculate the mean value in the coherent state.

16/18 How does look like the second-order correlation function for single modes as function of the mean value of the photon number and the fluctuation? Calculate the correlation function for coherent states.

17/18 With the help of a Hanbury Brown-Twiss interferometer the photon number $1 + n$ is measured for a light beam, where n is Poisson-distributed. Show that the second-order correlation function is then given by

$$g^{(2)} = 1 - \frac{1}{(1+ < n >)^2}$$

18/18 Consider a light beam by superposition of two stable waves with the electric fields

$$E(z,t) = E_1 e^{ik_1 z - i\omega_1 t} + E_2 e^{ik_2 z - i\omega_2 t}.$$

Show that the light is coherent in first order in all pairs of points.

19/18 How large must be the volume that the electric vacuum-field amplitude has the value of 1V/m? ($c = 3 \times 10^8$m/s, $\epsilon_0 = 8.85 \times 10^{-12}$F/m, $\hbar = 1.055 \times 10^{-34}$Js, $\lambda = 0.5\mu$m)